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NORMAL DEPTH COMPUTATION IN A RECTANGULAR OPEN-CHANNEL WITH CIRCULAR SIDES USING THE ROUGH MODEL METHOD (RMM)

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ABSTRACT

The computation of normal depth in open channels is a topical field in the practice of hydraulic engineers. Many methods consider Chezy's resistance coefficient or Manning's roughness coefficient as given data when calculating the normal depth. This seems unjustified since these coefficients depend in particular on the normal depth sought. The objective of this study is to propose an explicit method allowing the calculation of the normal depth in a rectangular channel with a horizontal bottom and circular walls using the rough model method (RMM), which is based on parameters that are easily measurable in practice. These parameters are the discharge, longitudinal bed slope, absolute roughness and kinematic viscosity. After establishing the equations governing the geometric and hydraulic characteristics of the referential rough model, the study shows that the normal depth sought is equal to the normal depth in the referential rough model corrected for the effects of a nondimensional correction factor.

Keywords: Discharge, Normal depth, Rectangular Open Channel with Circular Sides, Slope, Turbulent flow, Uniform flow.

INTRODUCTION

The channel shape profile can be of a compound form or a simple prismatic one where the cross-section is uniform throughout its length and the bottom slope is constant. Prismatic channels can be rectangular, triangular, trapezoidal, circular or even parabolic. For smaller discharges, triangular-shaped channels with circular bottoms are the most commonly used shape for line canals (Chow, 1959). For carrying water over long distances, the trapezoidal section is favored (Das, 2007). Rectangular sections are commonly used in urban areas as aqueducts (Swamee and Chahar, 2015).

Many types of compound open channels have been proposed in the literature. Formal design of the parabolic channel has been introduced by Mironenko et al. (1984) considering that parabolic sides of the cross section improve slope stability compared to trapezoidal sections and that parabolic buckets can be used to excavate parabolic channels (Das, 2007)

Easa and Vatankhah (2014) present a new general elliptic section, similar to the general power-law section that can produce several special sections. The special forms are a section with both circular sides and a horizontal bottom, a circular section, and a rectangular section.

In this paper, one may propose a special case of an elliptical section with a horizontal bottom and circular sides. The proposed section could be advocated as a viable alternative to the trapezoidal section.

The following advantages of this section have been recognized: 1) circular sides improve slope stability since the slope gradually increases from a horizontal slope at the channel bottom to a depth less than the channel height; 2) circular channels do not have sharp edges at which cracks may occur due to stress concentration; and 3) channels with circular sides and a horizontal bottom have been found to be more economical, providing a lower construction cost per unit length than trapezoidal channels.

Other channel models have been developed to address special aspects, such as slope stability, hydraulic efficiency, and normal or critical flow conditions (Froehlich, 1994).

Normal depth plays an important role in the classification of varied flow and in the design of channels and conduits (Achour, 2015b). According to the literature, Manning and Chézy resistance equations have been widely used for the calculation of normal depth (Raikar et al., 2010; Easa, 2011; Vatankhah, 2015), but the computation is often iterative, which requires a trial-and-error method.

However, the problem is not in the iterative nature of the calculation but lies in the fact that Chezy and Manning coefficients are considered to be the data of the problem, whereas they depend on the normal depth sought (Achour, 2015a; Loukam et al., 2018; Beboukha et al., 2019; Achour and Amara, 2020). This implies that these coefficients can only be known if the normal depth is given.

If the normal depth is not given, it takes much experience to estimate these resistance coefficients.

In practice, the absolute roughness characterizing the state of the internal walls of the channel is a measurable parameter. This characteristic is used as data for the problem, replacing the coefficients of Chezy and Manning (Achour, 2014c; Lakehal and Achour, 2014). In this context, the present study is suggested based on the rough model method (RMM) (Achour, 2007). This method is based on a rough model having the same shape as the current channel. A strong relative roughness arbitrarily chosen is assigned to this rough model so that the flow regime is in the rough turbulent domain involving a constant friction factor.

On one hand, this feature makes it possible to deduce the characteristics of this flow explicitly because the coefficient of friction is no longer an unknown variable. In a second step, these characteristics are multiplied by a coefficient that makes it possible to calculate the hydraulic characteristics sought, such as the normal depth.

The resulting RMM relationships are applicable throughout the entire domain of turbulent flow, as depicted by Moody's diagram, corresponding to Reynolds number $R \geq 2300$ and are valid for the following wide range of relative roughness ε/D_h values [0; 0.05], where ε: absolute roughness and D_h: hydraulic diameter (Achour, 2014a).

Unlike traditional implicit methods, this study will highlight the explicit nature of the calculation of the normal depth in the considered channel given the minimum of known practical parameters.

An example of calculation will be proposed to show the reader how to apply the method and, above all, to highlight both its ease of execution and its remarkable precision.

BASIC EQUATIONS

The universal formulas of Darcy-Weisbach (1854) and Colebrook-White (1939) and the Reynolds number equation constitute the basis of the theoretical development for the main proposed approach equations. The following Colebrook-White formula is used to determine the friction factor f as a function of both the Reynolds number R and the relative roughness ε/D_h:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon / D_h}{3.7} + \frac{2.51}{R\sqrt{f}}\right) \tag{1}$$

The Reynolds number R is expressed as:

$$R = \frac{4Q}{P\nu} \tag{2}$$

Q: Discharge, P: Wetted perimeter and ν: Kinematic viscosity.

The Darcy-Weisbach relationship expresses the longitudinal channel bed slope S_0 as follows:

$$S_0 = \frac{f}{D_h} \frac{Q^2}{2gA^2} \tag{3}$$

REFERENTIAL ROUGH MODEL

For the rest of the study, the symbol " $\overline{}$ " is used to denote all the geometrical and hydraulic parameters of the flow in the rough model. Figure 1 shows the geometric and hydraulic characteristics of the studied channel as well as its rough model. The rough model shape is similar to that of the current channel and is characterized by an arbitrarily chosen relative roughness $\bar{\varepsilon}/\overline{D_h} = 0.037$, where $\bar{\varepsilon}$ and $\overline{D_h}$ represent the absolute roughness and hydraulic diameter, respectively, in the rough model. The chosen value of the relative roughness is so great that the flow in the rough model is in the fully rough turbulent domain characterized by a Reynolds number $\bar{R} \to \infty$. Therefore, applying Eq. (1) yields a friction factor $\bar{f}$ in the rough model such that $\bar{f} = 1/16$ (Achour, 2007).

Let us consider the following four equalities:

1. $\bar{b} = b$, where $\bar{b}$ and b are the widths of the bottom of the rough model and that of the current channel, respectively (Fig. 1).
2. $\overline{D} = D$, where $\overline{D}$ and D are the diameters of the lateral walls of the rough model and that of the current channel, respectively (Fig. 1).
3. $\overline{S_0} = S_0$, where $\overline{S_0}$ and S_0 are the channel longitudinal bed slopes of the rough model and that of the current channel, respectively.
4. $\overline{Q} = Q$ as the equality between the discharge in the rough model and that in the current channel.

Taking all these considerations into account, it makes sense to write that $\overline{y_n} \neq y_n$ and even $\overline{y_n} > y_n$, where $\overline{y_n}$ and y_n are the normal depths of the flow in the rough model and the corresponding one in the current channel. Therefore, the relative normal depth is $\bar{\eta} = \overline{y_n}/b \neq \eta = y_n/b$, and the filling rates are $\bar{\xi} = \overline{y_n}/\overline{D} \neq \xi = y_n/D$.

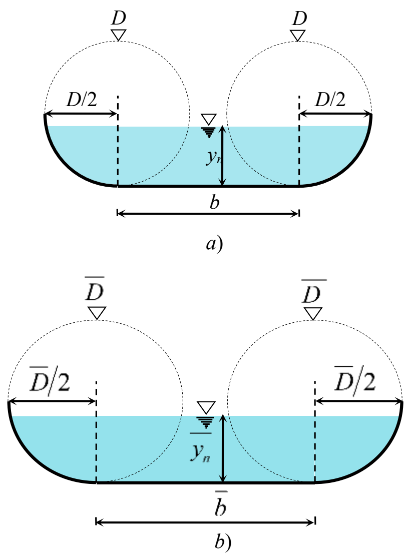

Figure 1: Schematic representation of the considered channels. *a*) Current channel; *b*) Referential rough model

Thus, applying Eq. (3) to the rough model, one may write the following:

$$S_0 = \frac{\overline{f}}{\overline{D_h}} \frac{Q^2}{2g\overline{A}^2} \tag{4}$$

where $\bar{A}$ is the water area in the rough model.

Taking into account that $\overline{D_h} = 4\bar{A}/\bar{P}$, where $\bar{P}$ is the wetted perimeter in the rough model, and knowing that $\bar{f} = 1/16$, Eq. (4) becomes:

$$S_0 = \frac{1}{128g} \frac{\overline{P}}{\overline{A}^3} Q^2 \tag{5}$$

Considering the previous first equality, i.e., $\bar{b} = b$, the water area of the rough model (Fig. 1b) is written as:

$$\overline{A} = b^2 \left[\frac{\sigma\left(\overline{\xi}\right)\varphi\left(\overline{\xi}\right)}{4\beta^2} + \frac{\overline{\xi}}{\beta} \right] \tag{6}$$

where:

$$\sigma\left(\overline{\xi}\right)=\cos^{-1}\left(1-2\,\overline{\xi}\,\right) \tag{7}$$

$$\varphi\left(\overline{\xi}\right)=1-\frac{2\left(1-2\,\overline{\xi}\,\right)\sqrt{\overline{\xi}\left(1-\overline{\xi}\,\right)}}{\cos^{-1}\left(1-2\,\overline{\xi}\,\right)} \tag{8}$$

$$\beta=b/D \tag{9}$$

The parameter β can be considered the relative width channel.

The wetted perimeter of the rough model is expressed as follows (Fig. 1*b*):

$$\overline{P}=b\left[\frac{\sigma\left(\overline{\xi}\right)}{\beta}+1\right] \tag{10}$$

By virtue of Eqs. (6) and (10), Eq. (5) can be written as:

$$\frac{\beta^5\left[\sigma\left(\overline{\xi}\right)+\beta\right]}{2\left[\sigma\left(\overline{\xi}\right)\varphi\left(\overline{\xi}\right)+4\beta\,\overline{\xi}\,\right]^3}\left(\frac{Q^2}{gS_0b^5}\right)=1 \tag{11}$$

Let us define the relative conductivity Q^* as:

$$Q^*=\frac{Q}{\sqrt{gS_0b^5}} \tag{12}$$

Therefore, Eq. (11) can be rewritten as:

$$\frac{\beta^5\left[\sigma\left(\overline{\xi}\right)+\beta\right]}{2\left[\sigma\left(\overline{\xi}\right)\varphi\left(\overline{\xi}\right)+4\beta\,\overline{\xi}\,\right]^3}Q^{*\,2} \tag{13}$$

Since Q, i and b are given in practice, the relative conductivity can then be determined according to Eq. (12). What is needed is to deduce the filling rate $\overline{\xi}$ using Eq. (13) provided the relative width β is given. However, Eq. (13) is implicit in $\overline{\xi}$ and thus requires a trial-and-error method or any iterative or graphical procedure.

To avoid this constraint, one may propose an approximate relationship to replace Eq. (13). After testing different mathematical models, it was found that the polynomial type equation is the most suitable. This is written explicitly as follows:

$$\overline{\xi}=a_6Z^6+a_5Z^5+a_4Z^4+a_3Z^3+a_2Z^2+a_1Z+a_0 \tag{14}$$

The Z parameter depends solely on the relative conductivity Q^* such that:

$$Z = \log Q^* \tag{15}$$

On the other hand, an in-depth study of Eq. (14) showed that the coefficients a_0 to a_6 depend exclusively on the relative width β.

The values of the coefficients are grouped together in Table 1 as a function of the relative width β. For greater precision, β values are given in terms of a fraction.

Table 1: Values of the adjustment parameters of Eq. (14)

		a_6	a_5	a_4	a_3	a_2	a_1	a_0
β	1	$\frac{109}{2389}$	$-\frac{89}{13051}$	$\frac{47}{1351}$	$\frac{202}{2311}$	$\frac{545}{3286}$	$\frac{328}{1423}$	$\frac{836}{5199}$
	1.2	$\frac{289}{6616}$	$\frac{137}{4389}$	$\frac{245}{5501}$	$\frac{673}{6113}$	$\frac{401}{1907}$	$\frac{1090}{3821}$	$\frac{304}{1545}$
	1.4	$\frac{155}{3713}$	$\frac{96}{1603}$	$\frac{193}{2692}$	$\frac{543}{3929}$	$\frac{985}{3844}$	$\frac{364}{1065}$	$\frac{361}{1547}$
	1.6	$\frac{124}{3109}$	$\frac{307}{3752}$	$\frac{437}{4110}$	$\frac{327}{1874}$	$\frac{904}{2961}$	$\frac{549}{1373}$	$\frac{815}{3013}$
	1.8	$\frac{84}{2203}$	$\frac{181}{1832}$	$\frac{659}{4595}$	$\frac{386}{1771}$	$\frac{533}{1487}$	$\frac{307}{668}$	$\frac{269}{873}$
	2	$\frac{293}{8029}$	$\frac{355}{3168}$	$\frac{935}{5183}$	$\frac{187}{701}$	$\frac{435}{1046}$	$\frac{358}{687}$	$\frac{974}{2813}$
	2.5	$\frac{120}{3647}$	$\frac{2553}{19024}$	$\frac{437}{1646}$	$\frac{387}{964}$	$\frac{1631}{2832}$	$\frac{2983}{4367}$	$\frac{173}{390}$
	3	$\frac{179}{5977}$	$\frac{505}{3444}$	$\frac{756}{2245}$	$\frac{1009}{1866}$	$\frac{818}{1087}$	$\frac{89}{104}$	$\frac{845}{1554}$
	3.5	$\frac{199}{7236}$	$\frac{472}{3073}$	$\frac{369}{934}$	$\frac{577}{854}$	$\frac{979}{1044}$	$\frac{2162}{2085}$	$\frac{1154}{1785}$
	4	$\frac{102}{4007}$	$\frac{281}{1786}$	$\frac{654}{1477}$	$\frac{7621}{9495}$	$\frac{778}{691}$	$\frac{2070}{1691}$	$\frac{2337}{3110}$
	4.5	$\frac{533}{22465}$	$\frac{807}{5072}$	$\frac{552}{1145}$	$\frac{1358}{1475}$	$\frac{1563}{1190}$	$\frac{1261}{891}$	$\frac{115}{134}$
	5	$\frac{155}{6967}$	$\frac{224}{1403}$	$\frac{2471}{4800}$	$\frac{861}{836}$	$\frac{418}{279}$	$\frac{7630}{4743}$	$\frac{633}{655}$

Eq. (14) is applicable in the range of relative conductivity indicated in Table 2 and for the values of the filling rate $\bar{\xi}$ varying in the following wide range $0.1 \le \bar{\xi} \le 0.5$. Note that the extreme values of $\bar{\xi}$ correspond to the practical minimum and maximum values.

Table 2: Limits of applicability of Eq. (14)

β	1	1.2	1.4
Q^*	$0.4666 \le Q^* \le 5.9515$	$0.3409 \le Q^* \le 4.2616$	$0.2625 \le Q^* \le 3.2330$
β	1.6	1.8	2
Q^*	$0.2100 \le Q^* \le 2.5557$	$0.1728 \le Q^* \le 2.08355$	$0.1454 \le Q^* \le 1.7395$
β	2.5	3	3.5
Q^*	$0.1013 \le Q^* \le 1.1947$	$0.07567 \le Q^* \le 0.8840$	$0.05927 \le Q^* \le 0.6876$
β	4	4.5	5
Q^*	$0.04803 \le Q^* \le 0.5544$	$0.03994 \le Q^* \le 0.4592$	$0.03389 \le Q^* \le 0.3884$

Fig. 2 graphically represents the variation in the relative deviation $\Delta\bar{\xi}/\bar{\xi}$ between the exact Eq. (13) and the approximate Eq. (14) for different values of $\bar{\xi}$ (0.1 to 0.5) and β (1 to 5).

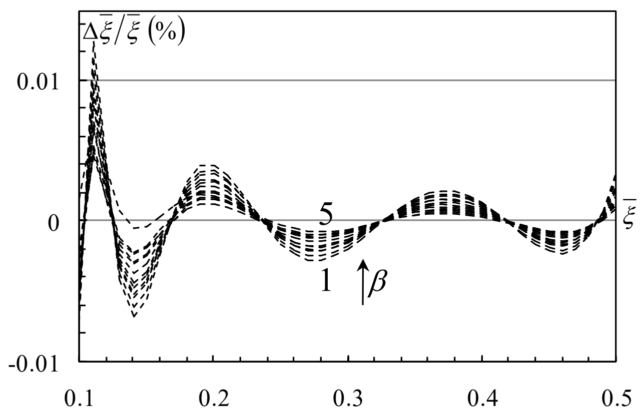

Figure 2: Relative deviation $\Delta\bar{\xi}/\bar{\xi}$ for different values of β (Q^* listed in Table 2)

Fig. 2 clearly shows that the maximum relative deviation $\Delta\bar{\xi}/\bar{\xi}$ is less than 0.013%. This confirms the reliability of the approximate relation (14). Once the filling rate is determined, the relative depth in $\bar{\eta}$ the rough model can be worked out by simply writing:

$$\bar{\eta} = \bar{\xi}/\beta \tag{16}$$

NON-DIMENSIONAL CORRECTION FACTOR OF LINEAR DIMENSION

The rough model method suggests that any linear dimension L of a channel is related to its homolog $\overline{L}$ in the rough model by the following relationship (Achour, 2014b):

$$L = \psi \, \overline{L} \tag{17}$$

The linear dimension L can correspond to the width of a rectangular channel or to the diameter of a circular pipe.

It is worth noting that Eq. (17) is applicable to the whole turbulent flow regime, as depicted in Moody's diagram.

With regard to Eq. (17), it makes sense to define ψ as the dimensionless correction factor for the linear dimension. As the linear dimension $\overline{L}$ of the rough model is always greater than the linear dimension L of the current channel, one may then write $0 < \psi < 1$.

An in-depth study of both the Darcy-Weisbach and Colebrook-White relationships made it possible to deduce that the dimensionless coefficient ψ could be expressed by the following improved explicit equation (Achour and Bedjaoui, 2006):

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D_h}}{4.75} + \frac{8.5}{\overline{R}}\right)\right]^{-2/5} \tag{18}$$

where $\overline{R}$ is the Reynolds number characterizing the flow in the rough model expressed as:

$$\overline{R} = \frac{4Q}{\overline{P}\,\nu} \tag{19}$$

where ν is the kinematic viscosity.

COMPUTATION STEPS OF NORMAL DEPTH

To calculate normal depth y_n in a rectangular open-channel with circular sides, an organization chart (Fig. 3) has been constructed where the input data are the following known parameters: the discharge Q, the bottom width b, the diameter D of the circular lateral side, the longitudinal bed slope S_0, the absolute roughness ε and the kinematic viscosity ν. Note that all these parameters are measurable in practice.

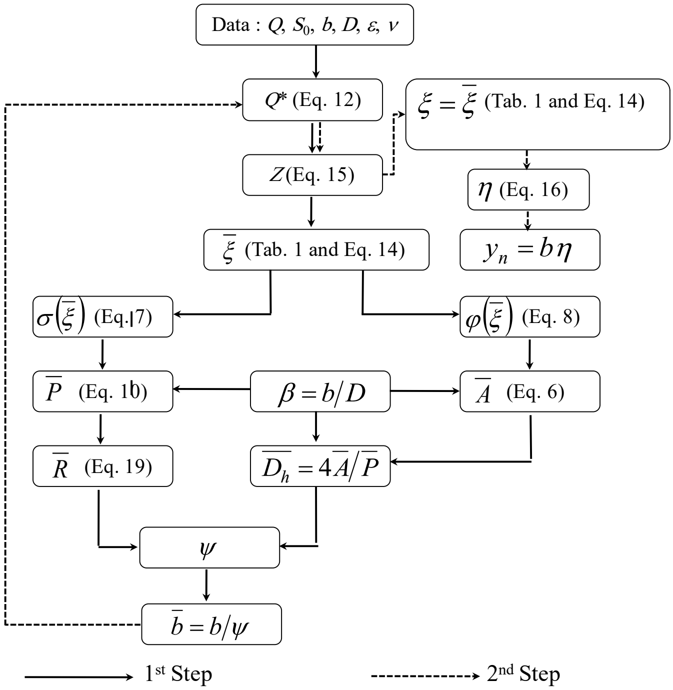

Figure 3: Flow chart for normal depth calculation

PRACTICAL EXAMPLE

To clarify the procedure for using the flow chart, a numerical example is proposed below. Knowing that:

Q = 15.197 m^3/s, b = 10 m, D = 5 m, $S_0 = 10^{-4}$, $\varepsilon = 10^{-3}$ m, $\nu = 10^{-6}$ m^2/s, calculate the normal depth y_n.

1. The relative conductivity Q^* is given by Eq. (12) as follows:

$$Q^* = \frac{Q}{\sqrt{gS_0 b^5}} = \frac{15.197}{\sqrt{9.81 \times 10^{-4} \times 10^5}} = 1.53434619$$

2. Using Eq. (15), the Z parameter is then:

$$Z = \log Q^* = \log(1.53434619) = 0.18592336$$

3. The relative bottom width is as follows:

4. $\beta = b/D = 10/5 = 2$.

5. Considering Eq. (14) along with Table 1 yields $\overline{\xi}$ as follows:

$$\overline{\xi} = a_6 Z^6 + a_5 Z^5 + a_4 Z^4 + a_3 Z^3 + a_2 Z^2 + a_1 Z + a_0$$
$$= \frac{293}{8029} \times 0.18592336^{\,6} + \frac{355}{3168} \times 0.18592336^{\,5} + \frac{935}{5183} \times 0.18592336^{\,4}$$
$$+ \frac{187}{701} \times 0.18592336^{\,3} + \frac{435}{1046} \times 0.18592336^{\,2} + \frac{358}{687} \times 0.18592336 + \frac{974}{2813}$$
$$= 0.45946738$$

6. The parameters $\sigma(\overline{\xi})$ and $\varphi(\overline{\xi})$ are given by Eqs. (7) and (8), respectively, as

$$\sigma(\overline{\xi}) = \cos^{-1}(1 - 2\overline{\xi}) = \cos^{-1}(1 - 2 \times 0.45946738) = 1.48964204$$

$$\varphi(\overline{\xi}) = 1 - \frac{2(1 - 2\overline{\xi})\sqrt{\overline{\xi}(1 - \overline{\xi})}}{\cos^{-1}(1 - 2\overline{\xi})}$$
$$= 1 - \frac{2(1 - 2 \times 0.45946738)\sqrt{0.45946738 \times (1 - 0.45946738)}}{\cos^{-1}(1 - 2 \times 0.45946738)} = 0.94575983$$

7. According to Eq. (6), the water area of the rough model $\overline{A}$ is such that:

$$\overline{A} = b^2 \left[\frac{\sigma(\overline{\xi})\varphi(\overline{\xi})}{4\beta^2} + \frac{\overline{\xi}}{\beta} \right] = 10^2 \left[\frac{1.48964204 \times 0.94575983}{4 \times 2^2} + \frac{0.45946738}{2} \right]$$
$$= 31.7786415 m^2$$

Therefore, using Eq. (10), the wetted perimeter $\overline{P}$ of the rough model is:

$$\overline{P} = b \left[\frac{\sigma(\overline{\xi})}{\beta} + 1 \right] = 10 \times \left[\frac{1.48964204}{2} + 1 \right] = 17.4482102 m$$

Knowing $\overline{A}$ and $\overline{P}$, the hydraulic diameter $\overline{D_h}$ is as follows:

$$\overline{D_h} = 4\overline{A}/\overline{P} = 4 \times 31.7786415/17.4482102 = 7.28524958 m$$

Finally, the Reynolds number $\overline{R}$ is determined from Eq. (19) such that:

$$\overline{R} = \frac{4Q}{\overline{P}\nu} = \frac{4 \times 15{,}197}{17.4482102 \times 10^{-6}} = 3483910.349$$

8. According to Eq. (18), the dimensionless correction factor ψ of the linear dimension is explicitly computed as follows:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D_h}}{4.75} + \frac{8.5}{\overline{R}}\right)\right]^{-2/5}$$

$$= 1.35 \times \left[-\log\left(\frac{10^{-3}/7.28524958}{4.75} + \frac{8.5}{3483910.349}\right)\right]^{-2/5} = 0.73942962$$

9. Let us give the rough model the following linear dimension, in agreement with the fundamental Eq. (17):

$$\overline{b} = b/\psi = 10/0.73942962 = 13.5239375\,m$$

Note that when assigning to the rough model the new linear dimension b/ψ, the filling rate in the current channel is equal to that in the rough mode, i.e., $\xi = \overline{\xi}$.

Then, the corresponding value of the relative conductivity Q^* is, according to Eq. (12):

$$Q^* = \frac{Q}{\sqrt{gi\overline{b}^5}} = \frac{15.197}{\sqrt{9.81 \times 10^{-4} \times 13.5239375^5}} = 0.72138228$$

10. According to Eq. (15), the new value of the Z parameter is:

$$Z = \log Q^* = \log(0.72138228) = -0.14183453$$

11. Using the same values for the coefficients a0 to a6 considered in step 5, Eq. (14) allows computing the filling rate in the current channel as:

$$\xi = \overline{\xi} = a_6 Z^6 + a_5 Z^5 + a_4 Z^4 + a_3 Z^3 + a_2 Z^2 + a_1 Z + a_0$$

$$= \frac{293}{8029} \times (-0.14183453)^6 + \frac{355}{3168} \times (-0.14183453)^5 +$$

$$\frac{935}{5183} \times (-0.14183453)^4 + \frac{187}{701} \times (-0.14183453)^3 + \frac{435}{1046} \times (-0.14183453)^2$$

$$+ \frac{358}{687} \times (-0.14183453) + \frac{974}{2813} = 0.28001049$$

According to Eq. (16), one may deduce the value of the nondimensional normal depth in the current channel as follows:

$$\eta = \xi / \beta = 0.28001049/2 = 0.14000524$$

12. Finally, the normal depth sought is then:

$$y_n = b\eta = 10 \times 0.14000524 = 1.40005243\,m \approx 1.4\,m$$

13. The purpose of this step is to validate the previous calculation. For this, let us calculate the longitudinal slope of the channel using the Darcy-Weisbach relationship expressed by Eq. (3). If the calculations previously carried out are correct, the slope that will be thus calculated should be equal to the slope given in the problem statement.

The rough model method demonstrates that the friction factor f and the nondimensional correction factor ψ are related by the following relationship (Achour, 2007):

$$f = \psi^5 / 16$$

Thus:

$$f = 0.73942962^5 / 16 = 0.01381542$$

According to Eqs. (7) and (8), the parameters $\sigma(\xi)$ and $\varphi(\xi)$ are, respectively, as follows:

$$\sigma(\xi) = \cos^{-1}(1 - 2\xi) = \cos^{-1}(1 - 2 \times 0.28001049) = 1.11522101$$

$$\varphi(\xi) = 1 - \frac{2(1-2\xi)\sqrt{\xi(1-\xi)}}{\cos^{-1}(1-2\xi)}$$

$$= 1 - \frac{2(1 - 2 \times 0.28001049)\sqrt{0.28001049 \times (1 - 0.28001049)}}{\cos^{-1}(1 - 2 \times 0.28001049)} = 0.64571623$$

The wetted perimeter P in the current channel is given by Eq. (10) as follows:

$$P = b\left[\frac{\sigma(\xi)}{\beta} + 1\right] = 10 \times \left[\frac{1.11522101}{2} + 1\right] = 15.576105\,m$$

Using Eq. (6), the water area A is obtained as:

$$A = b^2\left[\frac{\sigma(\xi)\varphi(\xi)}{4\beta^2} + \frac{\xi}{\beta}\right] = 10^2\left[\frac{1.11522101 \times 0.64571623}{4 \times 2^2} + \frac{0.28001049}{2}\right]$$
$$= 18.5012512\, m^2$$

From the known values of both A and P, the hydraulic diameter D_h is then:

$$D_h = 4A/P = 4 \times 18.5012512/15.576105 = 4.7511881 m$$

Finally, applying Eq. (3) yields the following:

$$S_0 = \frac{f}{D_h}\frac{Q^2}{2gA^2} = \frac{0.01381542}{4.7511881} \times \frac{15.197^2}{2 \times 9.81 \times 18.5012512^2} = 0.000099995 \approx 10^{-4}$$

It is thus clearly demonstrated that the calculated longitudinal slope of the channel is equal to the slope given in the problem statement, which allows us to conclude that the recommended method is reliable.

CONCLUSIONS

Explicit hydraulic relationships have been proposed to calculate the normal depth in rectangular open channels with circular sides by applying the rough model method. Both the current channel and its homologous rough reference model have the same shape. The geometric and hydraulic characteristics of the rough model are known thanks to explicit equations such as the Darcy-Weisbach relationship.

The recommended method takes into account three rational basic relationships of hydraulics, namely, the Colebrook-White equation, Darcy-Weisbach relationship and Reynolds number expression.

From the known linear dimension of the model, it was possible to deduce the linear dimension b of the current channel by using a dimensionless coefficient, denoted ψ, known as the correction factor of the linear dimension. This is closely related to the characteristics of the rough model, and its computation is explicit [Eq. (18)].

Knowing the parameters b and ψ, a change of dimension was made on the rough model by assigning to it the new dimension b/ψ. This judicious manipulation led to equal filling rates in the model and in the current channel, i.e. $\xi = \overline{\xi}$ explicitly calculated using Eq. (14).

The relative normal depth was thus easily deduced by applying Eq. (16), which directly allowed the calculation of the normal depth sought.

It is important to note that the RMM is based on measurable parameters in practice, such as absolute roughness or kinematic viscosity. In addition, this method does not take into account flow resistance coefficients such as those of Chézy or Manning because these coefficients depend on the normal depth sought. It is therefore impossible to assess them before calculating the normal depth of the flow.

A practical example was suggested to highlight the simplicity, efficacy and validity of the advocated method.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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