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# DESIGN OF HORSESHOE-SHAPED TUNNELS USING THE ROUGH MODEL METHOD (RMM)

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## ABSTRACT

The uniform flow in a horseshoe-shaped tunnel is often encountered in many practical cases. This pipe can be used for water drainage in sewerage and construction. For the purpose of designing this type of channel with the presumption of uniform flow, it is necessary to refer to Chezy and Manning relationships. In general, Chezy's and Manning's coefficients are given as data of the problem and are mostly considered constants regardless of the normal flow depth. This is an approximation of the fact that the flow resistance should vary with depth or hydraulic radius. In this study, the pipe is designed with a variable flow resistance coefficient, depending on the fill rate of the pipe. The Chezy coefficient is no longer data of the problem but a variable to be determined. The determination of Chezy's coefficient is made possible by the rough model method (RMM). The proposed sizing method is valid in the entire turbulent flow domain, encompassing smooth, transitional, and rough turbulent flow regimes in a wide practical range.

**Keywords:** Uniform flow, Rough model method, Reynolds, Chezy's coefficient, Turbulent flow.

## INTRODUCTION

Free surface flow is common in engineering, where fluids such as water flow with a free boundary exposed to the atmosphere (Violeau et al., 2016). It is prevalent in open channels, rivers, and natural water systems, and engineers analyze it to design structures such as dams and irrigation channels. The behavior of free surface flow is influenced by factors such as channel geometry, slope, roughness, and flow rate, and mathematical

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models such as Manning's equation are used for predictions (Al Hindasi and Abushandi, 2023).

Water channels vary in size based on their purpose and location. Various elements, such as flow rate, velocity needs, topography, gradient, sediment transport, and environmental factors, impact the size of the channel. Engineers consider these factors and site-specific conditions to determine appropriate dimensions, following local regulations and design guidelines.

The calculation of flow in circular or noncircular-shaped pipes is frequently encountered in the practice of hydraulic engineers **(**Kim et al., 2016)**.** Using a horseshoe-shaped channel in water projects offers advantages such as greater hydraulic efficiency, reduced energy losses, and preventing sediment accumulation and blockages (Achour, 2014). It can accommodate larger flow rates while maintaining stability, making it suitable for high-volume water conveyance projects. These benefits and careful consideration of flow rate, velocity, and environmental conditions make the horseshoe-shaped channel a favorable choice for various water projects.

Currently, and for more than two centuries, hydraulic calculations are based on the two famous formulas of Chezy and Manning, which have proven their effectiveness in the field of engineering because, from a conceptual and practical point of view, they are the most well-founded formulas and the simplest to use (Loukam et al., 2020; Zegait and Achour, 2016)**.** These two formulas express the average velocity of the flow as a function of the hydraulic radius, the longitudinal slope of the channel, and the coefficient of resistance to the flow, which is absolutely considered a constant.

To size the horseshoe-shaped pipe of height *D*, equal to the diameter of the circle that generated it (meaning the linear dimension "a" in the functional relation $\varphi\left(a,\eta,i,\varepsilon,\nu\right)=0$ (Achour et al., 2002). The existing studies related to the sizing of such pipes are not numerous. They propose either a graphical resolution or iterative solutions (Chow, 1973), which are often based on the equations of Chézy (1769) and Manning (1809), which are all based on a constant flow resistance coefficient, regardless of the normal depth of the flow. This is an approximation of the fact that the flow resistance should vary with depth or hydraulic radius. Others give, with explicit approximate relationships, solutions for large pipes filled to 75% (Swamee and Swamee, 2008). Currently, no theoretical approach gives solutions incorporating any value of fill rate $\eta$ between 0 and 1. The rough model method (RMM) has recently demonstrated successful application in sizing circular or noncircular shape channels (Achour, 2007).

In contrast to conventional approaches, this advocated method relies on practical data for absolute roughness, which accurately characterizes the condition of the channel's inner wall. The implementation of the Darcy-Weisbach relationship began with a referential rough model, where the friction factor was arbitrarily chosen. This initial step allowed for explicitly establishing a relationship between the aspect ratio and the relative conductivity. By utilizing the known aspect ratio of the rough model, the nondimensional diameter and the diameter of the actual channel under study were deduced, considering a dimensionless correction factor. A practical example we proposed effectively demonstrated the reliability, simplicity, and efficiency of the RMM.

In this study, the pipe is designed for a variable flow resistance coefficient, depending on the filling rate of the pipe. The coefficient *C* of Chezy is no longer a given of the problem but a variable to be determined.

## HYDRAULIC CHARACTERISTICS

Three distinct case studies can be individually examined in the channel depicted in Fig. 1, following the standard depth $y_n$. As a result, the hydraulic properties, notably including the wetted perimeter *P*, top width *e* wetted cross-sectional area *A*, and hydraulic radius $R_h$, can be expressed in terms of the fill rate represented by the ratio $\eta = y_n/D$.

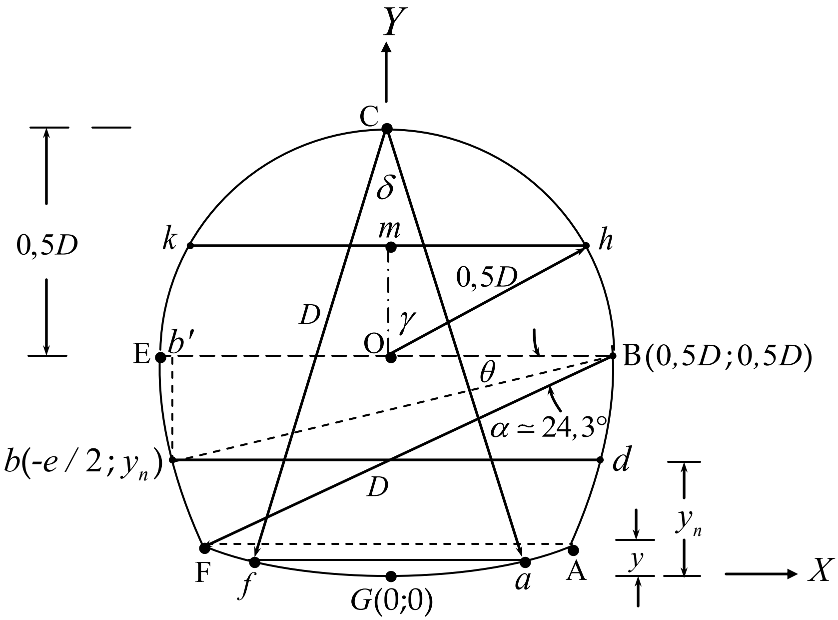


**Figure 1: Horseshoe-shaped channel profile**

It can be demonstrated geometrically that $\eta$ belongs to three domains for which the hydraulic characteristics are as follows:

**$\eta \leq 0.08856$**

$$e = 2D\sqrt{\eta(2-\eta)} \tag{1}$$

$$P = 2D\sigma(\eta) \tag{2}$$

$$\sigma(\eta) = \cos^{-1}(1-\eta) \tag{3}$$

$$A = D^2\sigma(\eta)\varphi(\eta) \quad (4)$$

$$\varphi(\eta) = 1 - \frac{(1-\eta)\sqrt{\eta(2-\eta)}}{\cos^{-1}(1-\eta)} \quad (5)$$

$$R_h = \frac{D}{2}\varphi(\eta) \quad (6)$$

**0.08856≤ *η* ≤ 0.5**

$$e = 2D\left[\sqrt{1-\left(\frac{1}{2}-\eta\right)^2} - \frac{1}{2}\right] \quad (7)$$

$$P = D\,\vartheta(\eta) \quad (8)$$

$$\vartheta(\eta) = 1.69612416 - 2\sin^{-1}\left(\frac{1}{2}-\eta\right) \quad (9)$$

$$A = D^2\zeta(\eta) \quad (10)$$

$$\zeta(\eta) = 0.93662425 - \sin^{-1}\left(\frac{1}{2}-\eta\right) - \left(\frac{1}{2}-\eta\right)\sqrt{1-\left(\frac{1}{2}-\eta\right)^2} - \eta \quad (11)$$

$$R_h = D\frac{\zeta(\eta)}{\vartheta(\eta)} \quad (12)$$

**0.5 ≤ *η* ≤ 1**

$$e = 2D\sqrt{\eta(1-\eta)} \quad (13)$$

$$\tau(\eta) = 3.26692049 - \cos^{-1}(2\eta - 1) \quad (14)$$

$$P = D\,\tau(\eta) \quad (15)$$

$$A = D^2\lambda(\eta) \quad (16)$$

$$\lambda(\eta) = 0.82932 - \frac{1}{4}\cos^{-1}(2\eta-1) + \left(\eta - \frac{1}{2}\right)\sqrt{\eta(1-\eta)} \quad (17)$$

$$R_h = D\frac{\lambda(\eta)}{\tau(\eta)} \quad (18)$$

**BASIC EQUATIONS**

The relationships on which the study is based are simple, well-known hydraulic equations, namely, the Darcy-Weisbach equation (Darcy, 1854), the Colebrook-White equation (Colebrook,1939), and the Reynolds number formula. The energy slope of a conduit or channel is given by the Darcy-Weisbach relationship as follows:

$$i = \frac{f}{D_h}\frac{Q^2}{2gA^2} \tag{19}$$

where $Q$ is the discharge, $g$ is the acceleration due to gravity, $A$ is the wetted area, $Dh$ is the hydraulic diameter, and f is the friction factor given by the well-known Colebrook-White formula:

$$\frac{1}{\sqrt{f}} = -\sqrt{2}\log\left(\frac{\varepsilon / D_h}{3.7} + \frac{2.51}{R\sqrt{f}}\right) \tag{20}$$

where e is the absolute roughness and $R$ is the Reynolds number, which can be expressed as follows:

$$R = \frac{4Q}{P\nu} \tag{21}$$

where $\nu$ is the kinematic viscosity

**REFERENTIAL ROUGH MODEL**

The symbol distinguishes the rough model's geometric and hydraulic characteristics" $\overline{\ }$ ". Fig. 2 compares the geometric and hydraulic characteristics of the current tunnel with those of its rough model. This model is a pipe characterized by a relative roughness $\overline{\varepsilon}/\overline{D} = 0.037$ arbitrarily chosen. The flow is assumed to be turbulent and rough such that the friction coefficient is equal to $\overline{f} = 1/16$ according to the Colebrook-White equation for a Reynolds number. $R = \overline{R} \to \infty$

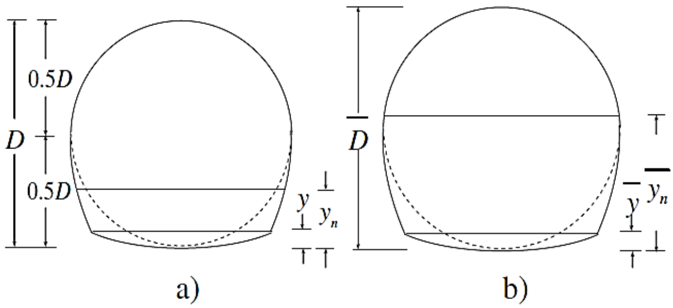


**Figure 2: Definition sketch of normal depth in a horseshoe-shaped channel. a) Current tunnel. b) Rough model**

Furthermore, the rough model is characterized by a diameter $\overline{D}$, flowing a flow $\overline{Q}$ of a liquid of kinematic viscosity $\overline{\nu}$ corresponding to a filling rate $\overline{\eta}$ under a longitudinal slope $\overline{i}$.

## SIZING OF HORSESHOE-SHAPED TUNNEL USING THE RMM

When the diameter *D* of the pipe is not a given of the problem, the known parameters of the problem are the volume flow rate *Q*, the filling rate $\eta$ of the pipe, the longitudinal slope *i*, the absolute roughness η $\varepsilon$ , and the kinematic viscosity $\nu$ of the flowing liquid. For these parameters alone, the RMM (Achour, 2007) allows the determination of the diameter *D* by the fundamental equation of the RMM, which is applicable to any form of geometric profile of pipes and channels (Achour and Bedjaoui, 2006):

$$D = \psi \overline{D} \tag{22}$$

where $\overline{D}$ represents the diameter of the reference rough model and $\psi$ is a dimensionless correction factor of linear dimension.

### Calculation of the diameter $\overline{D}$ using the RMM

According to the RMM, the flow in the rough reference model is characterized by a friction coefficient $\overline{f} = 1/16$ (Achour, 2007), which translates into a Chezy resistance coefficient:

$$\overline{C} = \sqrt{8g / \overline{f}} = 8\sqrt{2g} = \text{constante} \tag{23}$$

To determine the diameter 'D' of the pipe, let us assume the following conditions:

$$\overline{D} \neq D \;, \qquad \overline{Q} = Q \;, \qquad \overline{i} = i \;, \qquad \overline{\eta} = \eta, \qquad \overline{\nu} = \nu$$

Let us define the relative conductivity $Q^*$

$$Q^* = \frac{Q}{\sqrt{C^2 D^5 i}} \tag{24}$$

Equation (24) is then written in dimensionless terms as follows:

**η ≤ 0.08856**

$$Q^* = \frac{\sigma(\eta)[\varphi(\eta)]^{3/2}}{\sqrt{2}} \tag{25}$$

**0.08856≤ η ≤ 0.5**

$$Q^* = \frac{[\zeta(\eta)]^{3/2}}{[\vartheta(\eta)]^{1/2}} \tag{26}$$

**0.5 ≤ η ≤ 1**

$$Q^* = \frac{[\lambda(\eta)]^{3/2}}{[\tau(\eta)]^{1/2}} \tag{27}$$

According to equation (24) and taking into account conditions $\left(\overline{Q} = Q \;, \overline{i} = i\right)$, the relative conductivity $\overline{Q}^*$ of the reference rough model would be such that:

$$\overline{Q}^* = \frac{Q}{\sqrt{\overline{C}^2 \, \overline{D}^5 i}} \tag{28}$$

Or, taking into account equation (23):

$$\overline{Q}^* = \frac{Q}{\sqrt{128 \, g \, \overline{D}^5 i}} \tag{29}$$

The relative conductivity $\overline{Q}^*$ is governed by equations (25), (26), and (27) depending on the range of values of the filling ratio $\eta$. Thus, for:

**η ≤ 0.08856**

$$\overline{Q}^{*}=\frac{\sigma(\eta)\left[\varphi(\eta)\right]^{3/2}}{\sqrt{2}} \tag{30}$$

From equations (29) and (30), it can be deduced that the diameter $\overline{D}$ of the reference rough model is as follows:

$$\overline{D}=\frac{0.43527}{\left[\sigma(\eta)\right]^{0.4}\left[\varphi(\eta)\right]^{0.6}}\left(\frac{Q}{\sqrt{g\,i}}\right)^{0,4} \tag{31}$$

**0.08856≤ η ≤ 0.5**

$$\overline{Q}^{*}=\frac{\left[\zeta(\eta)\right]^{3/2}}{\left[\vartheta(\eta)\right]^{1/2}} \tag{32}$$

Equations (29) and (32) allow us to write the following:

$$\overline{D}=\frac{\left[\vartheta(\eta)\right]^{0.2}}{2.639\left[\zeta(\eta)\right]^{0.6}}\left(\frac{Q}{\sqrt{g\,i}}\right)^{0.4} \tag{33}$$

**0.5 ≤ η ≤ 1**

$$\overline{Q}^{*}=\frac{\left[\lambda(\eta)\right]^{3/2}}{\left[\tau(\eta)\right]^{1/2}} \tag{34}$$

One can deduce from equations (29) and (34) that:

$$\overline{D}=\frac{\left[\tau(\eta)\right]^{0.2}}{2.639\left[\lambda(\eta)\right]^{0.6}}\left(\frac{Q}{\sqrt{g\,i}}\right)^{0.4} \tag{35}$$

**Nondimensional correction factor of linear dimension**

The dimensionless parameter $\psi$ as $0\le\psi\le 1$ is defined by the following equation (Achour and Bedjaoui 2006):

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon}{19\overline{R_h}} + \frac{8.5}{\overline{R}}\right)\right]^{-2/5} \tag{36}$$

where $\overline{R_h}$ the hydraulic radius of the reference rough model is given by equations (6), (12), and (18) depending on the range of values of the filling ratio, while the Reynolds number $\overline{R}$ is given by the following equation (Achour and Bedjaoui, 2006):

$$\overline{R} = 32\sqrt{2}\,\frac{\sqrt{g\,i\,\overline{R}_h^3}}{\nu} \tag{37}$$

Currently, there is no general relationship that can evaluate the hydraulic radius $R_h$ by itself for a horseshoe pipe in the full state, corresponding to the filling ratio $\eta$= 1; the hydraulic radius $R_{h,p}$ is governed by equation (18), and it is easy to show that $R_{h,p}$ is, by virtue of equations (14), (17) and (18), as follows:

$$R_{h,p} \cong 0.254D \tag{38}$$

Therefore, the Reynolds number $R_p$ in the full state of the reference rough model is, according to equation (37):

$$\overline{R_p} \cong 5.7882\,\frac{\sqrt{g\,i\,\overline{D}^3}}{\nu} \tag{39}$$

Using equations (6), (12), and (18) for $\overline{R_h}$ and (39) for $\overline{R_p}$, equation (37) gives:

**η ≤ 0.08856**

$$\overline{R} = 2.764\left[\varphi(\eta)\right]^{3/2}\overline{R_p} \tag{40}$$

Equation (36) becomes:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{9.5\,\varphi(\eta)} + \frac{3.075}{\left[\varphi(\eta)\right]^{3/2}\overline{R_p}}\right)\right]^{-2/5} \tag{41}$$

Then, equation (19) gives:

$$D = 1.35\overline{D}\left[-\log\left(\frac{\varepsilon/\overline{D}}{9.5\,\varphi(\eta)} + \frac{3.075}{\left[\varphi(\eta)\right]^{3/2}\overline{R_p}}\right)\right]^{-2/5} \tag{42}$$

**0.08856≤ η ≤ 0.5**

$$\overline{R} = 7.8185\left[\frac{\zeta(\eta)}{\vartheta(\eta)}\right]^{3/2}\overline{R_p} \tag{43}$$

Equation (36) becomes:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19\left[\zeta(\eta)/\vartheta(\eta)\right]} + \frac{1.087}{\left[\zeta(\eta)/\vartheta(\eta)\right]^{3/2}\overline{R_p}}\right)\right]^{-2/5} \tag{44}$$

$$D = 1.35\overline{D}\left[-\log\left(\frac{\varepsilon/\overline{D}}{19\left[\zeta(\eta)/\vartheta(\eta)\right]} + \frac{1.087}{\left[\zeta(\eta)/\vartheta(\eta)\right]^{3/2}\overline{R_p}}\right)\right]^{-2/5} \tag{45}$$

**0.5 ≤ η ≤ 1**

$$\overline{R} = 7.8185\left[\frac{\lambda(\eta)}{\tau(\eta)}\right]^{3/2}\overline{R_p} \tag{46}$$

Equation (36) becomes:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19\left[\lambda(\eta)/\tau(\eta)\right]} + \frac{1.087}{\left[\lambda(\eta)/\tau(\eta)\right]^{3/2}\overline{R_p}}\right)\right]^{-2/5} \tag{47}$$

$$D = 1.35\overline{D}\left[-\log\left(\frac{\varepsilon/\overline{D}}{19\left[\lambda(\eta)/\tau(\eta)\right]} + \frac{1.087}{\left[\lambda(\eta)/\tau(\eta)\right]^{3/2}\overline{R_p}}\right)\right]^{-2/5} \tag{48}$$

## COMPUTATION STEPS OF SIZING

To determine the diameter (D) of a horseshoe-shaped tunnel, the following data must be given: $Q$, $\eta$, $\varepsilon$, $i$, and $\nu$. Note first that these data are measurable in practice, and second, the flow resistance coefficient, such as Chezy's or Manning's roughness coefficient, is not required. To calculate the diameter (D) of the horseshoe-shaped tunnel, it is recommended to follow these steps:

1. Knowing the value of the filling rate $\eta$, one of the equations (31), (33), or (36) allows us to evaluate the diameter $\overline{D}$ of the reference rough pattern.
2. The known parameters are introduced in equation (39) for calculating the Reynolds number $\overline{R_p}$ in the full state.

3. The coefficient $\psi$ can then be evaluated by one of the equations (41), (44) or (47).
4. Finally, the diameter can be evaluated by equation (22).

**PRACTICAL EXAMPLE**

The horseshoe-shaped pipe represented by Fig. 1 is the seat of a uniform flow. It flows a volume flow $Q$=2.25m$^3$/s of a liquid of kinematic viscosity $\nu$=10$^{-6}$m$^2$/s under a longitudinal slope $i$=3.10$^{-4}$. The filling rate is $\eta$=0.75 and the absolute roughness is $\varepsilon = 0$.

Calculate the value of the diameter $D$ of the horseshoe-shaped tunnel.

**Solution**

Since $0.5 \leq \eta \leq 1$, the diameter $\overline{D}$ of the reference rough model is given by equation (35). The functions $\tau(\eta)$ $\lambda(\eta)$ are given by equations (14) and (17), respectively, and take values:

$$\tau(\eta) = 3.26692049 - \cos^{-1}(2\eta - 1) = 3.26692049 - \cos^{-1}(2(0.75) - 1)$$
$$= 2.219722924$$

$$\lambda(\eta) = 0.82932 - \frac{1}{4}\cos^{-1}(2\eta - 1) + \left(\eta - \frac{1}{2}\right)\sqrt{\eta(1-\eta)}$$

$$= 0.82932 - \frac{1}{4}\cos^{-1}(2(0.75) - 1) + \left(0.75 - \frac{1}{2}\right)\sqrt{0.75(1-0.75)} = 0.675777118$$

According to equation (35), the diameter $\overline{D}$ is:

$$\overline{D} = \frac{[\tau(\eta)]^{0,2}}{2.639[\lambda(\eta)]^{0,6}}\left(\frac{Q}{\sqrt{gi}}\right)^{0,4} = \frac{[2.219722924]^{0,2}}{2.639[0.675777118]^{0,6}}\left(\frac{2.55}{\sqrt{9.81 \times 0.0003}}\right)^{0,4}$$
$$= 2.622987939$$

The *Reynolds* number $\overline{R_p}$ in the full state is, according to relation (39):

$$\overline{R_p} \cong 5.7882\frac{\sqrt{gi\overline{D}^3}}{\nu} = 5.7882 \times \frac{\sqrt{9.81 \times 0.0003 \times 2.622987939^3}}{10^{-6}}$$
$$= 1333930.186$$

The factor $\psi$ is, therefore, according to relation (47):

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19[\lambda(\eta)/\tau(\eta)]} + \frac{1,087}{[\lambda(\eta)/\tau(\eta)]^{3/2}\overline{R_p}}\right)\right]^{-2/5}$$

$$\psi = 1.35\left[-\log\left(\frac{1.087}{(0.675777118/2.219722924)^{3/2}1333930.186}\right)\right]^{-2/5}$$
$$= 0.71250894$$

The diameter $D$ can be evaluated by the fundamental relationship of the RMM, applicable to any form of geometric profile of pipes and channels: $D = \psi\overline{D}$

$$D = \psi\overline{D} = 0.71250894 \times 2.622987939 = 1.868902355 \cong 1.87m$$

Let us check our calculations by determining the flow volume Q by applying the general formula (Achour and Bedjaoui, 2006):

$$Q = -4\sqrt{2g}A\sqrt{R_h i}\log\left(\frac{\varepsilon}{14.8R_h} + \frac{10.04}{R}\right)$$

To do this, let us first evaluate the wetted cross-sectional area $A$, the hydraulic radius $R_h$, and the *Reynolds* number $R$ for the diameter $D$ that we calculated in the first step.

The area of the wetted section $A$ is given by equation (16), i.e.:

$$A = D^2\lambda(\eta) = 1.868902355^2 \times 0.675777118 = 2.360352m^2$$

The hydraulic radius is given by equation (12) as follows:

$$R_h = D\frac{\lambda(\eta)}{\tau(\eta)} = 1.86890236 \times \frac{0.675777118}{2.219722924} = 0.56897257m$$

The *Reynolds* number $R$ is governed by equation (37), that is:

$$R = 32\sqrt{2}\frac{\sqrt{giR_h^3}}{\nu} = 32 \times \sqrt{2} \times \frac{\sqrt{9.81 \times 0.0003 \times 0.56897257^3}}{10^{-6}} = 1053651.76$$

Thus, according to the general relationship, the volume flow $Q$ is equal to:

$$Q = -4\sqrt{2g}A\sqrt{R_h i}\log\left(\frac{\varepsilon}{14.8R_h} + \frac{10.04}{R}\right)$$
$$= -4 \times \sqrt{2 \times 9.81} \times 2.360352 \times \sqrt{0.56897257 \times 0.0003} \times \log\left(\frac{10.04}{1053651.76}\right)$$
$$= 2.5513 \quad m^3/s$$

The deviation between the volume flow $Q$ that we have just calculated and the one given in the problem statement is less than 0.051%.

## CONCLUSION

The practical application of the rough model method (RMM) has yielded successful results in sizing a horseshoe-shaped channel. In contrast to current methods, this approach utilizes practical data to determine the absolute roughness, which describes the condition of the inner wall of the channel.

The Darcy-Weisbach relationship was initially employed on a reference rough model, with the friction factor arbitrarily chosen. This process facilitated the establishment of an explicit relationship between the aspect ratio and the relative conductivity. By utilizing the known aspect ratio of the rough model, the nondimensional diameter and, subsequently the diameter in the studied channel were derived through a nondimensional correction factor.

We presented a practical example demonstrating the reliability, simplicity, and efficiency of the RMM. These findings highlight the effectiveness of this method for accurately sizing a horseshoe-shaped channel.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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