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A NEW VISION OF THE CRITICAL FLOW IN A PARABOLIC CHANNEL (PART 1)

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ABSTRACT

The objective of this study is to analyze the behavior of critical flow in a parabolic channel as a function of all the parameters that influence the flow, such as the slope of the channel S_0, the absolute roughness ε and the kinematic viscosity ν. To do this, we applied two rational relations, namely, the relationship of the critical flow condition and the general formula of the discharge. The combination of these two relations results in an implicit relation consisting of five dimensionless terms that are the dimensionless critical depth $\zeta_c = \sqrt{y_c/B}$, where y_c is the critical depth and B is the linear dimension of the channel, the dimensionless normal depth $\zeta_n = \sqrt{y_n/B}$, where y_n is the normal depth, the relative roughness ε/B, the longitudinal slope S_0, and the modified Reynolds number R_e^*. This implicit relationship was applied to a parabolic channel with a linear dimension B = 1 m in the whole domain of turbulent flow. The detailed study of the rational equations governing the critical and normal flows leads to intriguing results in addition to the establishment of other fundamental relations and significant graphs.

Keywords: Critical depth, Critical flow, Discharge, Normal depth, Parabolic channel, Slope.

INTRODUCTION

As discharge and flow depth are uniquely correlated for a given canal geometry, critical flow conditions play a significant role in open channels (Hager, 1985). According to Chow (1959), a critical flow condition means that specific energy is a minimum for constant discharge or discharge reaches its maximum for constant specific energy. Moreover, it is the flow condition in which the Froude number equals unity. This property is used to determine the critical depth in open channels, also called the critical criterion. The flow is said to be subcritical or calm flow if the Froude Number is less than 1 and supercritical or shooting flow if the Froude Number is greater than 1(Hager , 2010). The critical depth is the depth of flow in which the specific energy for a given discharge corresponds to the minimal value (Achour and Amara , 2020a). This depth is essential for identifying whether a flow is subcritical or supercritical and for classifying varied flows (Achour and Nebbar , 2015). For triangular, rectangular, and parabolic channels, there is an analytical solution to directly determine the critical depth (Chow, 1959 ; Wong and Zhou , 2004; Achour and Khattaoui , 2008), for other types of geometries, the equations that govern the critical depth are implicit; therefore, one must use chart methods with limited precision, trial-and-error, iteration procedures, or other methods (Liu et al., 2012). In the past, graphical methods for circular and trapezoidal channels have been suggested (Chow ,1959; Henderson , 1966; French , 1987). In recent years, numerous researchers have made an effort to create explicit equations for estimating critical depth in open channels with various geometric profiles, such as circular conduits, trapezoidal channels, rounded-bottom sections, egg-shaped channels, and semielliptical channels (Swamee , 1993; Swamee and Rathie , 2005; Liu et al., 2012; Li et al., 2012; Vatankhah and Easa, 2011;Cheng et al., 2018; Shang et al., 2019).

In the majority of studies, the critical depth in different geometric shapes is determined by resorting to the relationship that governs the critical flow. However, in this relation, the influence of the slope of the channel, the kinematic viscosity of the liquid, and the roughness of the channel walls are not taken into account. In this context, some studies addressing the critical flow in conduits and channels have been published, taking into account the effects of all flow and channel parameters (Achour and Amara, 2020a ; 2020b; Hachemi - Rachedi et al., 2021).

The present study is interested in the study of critical flow in the parabolic channel. In practice, the geometry of natural rivers is frequently quite well approximated by the parabolic channel (Vatankhah, 2013).

This study aims to examine the variation in the critical flow and depth in this type of channel as a function of all flow and channel parameters, such as the longitudinal slope S_0, the absolute roughness ε and the kinematic viscosity ν. To obtain this objective, we used two rational relations, namely, the relation of the critical flow condition in which the Froude number equals one and the general formula of the discharge proposed by Achour and Bedjaoui (2006). An implicit relationship of five dimensionless terms results by deleting the discharge Q between these two relations $\psi(\zeta_c, \zeta_n, S_0, \varepsilon/B, R_e^*) = 0$ through a modified Reynolds number R_e^*, and the effect of kinematic viscosity ν is shown. In the domain of turbulent flow (smooth, rough and transition regimes), this implicit relationship was employed in a parabolic channel characterized by a linear dimension B=1 m to show the relative critical depth change according to each flow. The derived relation enabled the creation of graphs showing how the relative critical depth ζ_c varied versus the relative normal depth ζ_n for different slope values S_0. These graphs show unexpected characteristics of the critical flow in the parabolic channel.

By putting $\zeta_n = \zeta_c$ in the deducted relation, the general equation that governs the critical flow in a parabolic-shaped channel can be deduced. This relationship is implicit and composed of four dimensionless parameters, namely, $\psi(\zeta_c, S_0, \varepsilon/B, R_e^*) = 0$. To show how the critical depth is influenced by slope S_0, a smooth parabolic channel with a linear dimension $B = 1$ m is used as an example. The graphs of the change in the relative critical depth ζ_c versus the various values of the slope S_0 provided a clear view of the flow's behavior and led to some intriguing conclusions.

The study's validation of suggested relationships will be accomplished through the examination of the equation for specific energy.

METHODOLOGY

Geometric properties

According to Figure 1, the parabolic channel is characterized by three linear dimensions: the depth flow y and the geometrical elements T_m and Y_m of the parabolic channel.

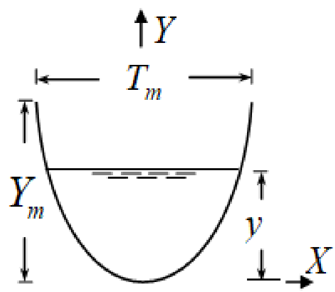

Figure 1: Cross section of a parabolic channel.

According to Achour and Khattaoui (2008), the geometric characteristics of a parabolic channel are given by the following relations:

The water area A is:

$$A = \frac{2}{3} B^2 \zeta^3 \quad (1)$$

where $\zeta = \sqrt{y/B}$ is the relative depth or the aspect ratio, and the linear dimension B is defined by:

$$B = \frac{T_m^2}{Y_m} \quad (2)$$

The wetted perimeter P is:

$$P = \frac{B}{8}\left[4\zeta\sqrt{1+16\zeta^2} + \ln\left(4\zeta + \sqrt{1+16\zeta^2}\right)\right] \quad (3)$$

The hydraulic radius $R_h = A/P$ is then:

$$R_h = B\sigma(\zeta) \tag{4}$$

where:

$$\sigma(\zeta) = \frac{16}{3}\frac{\zeta^3}{4\zeta\sqrt{1+16\zeta^2}+ln(4\zeta+\sqrt{1+16\zeta^2})} \tag{5}$$

The top width T is:

$$T = B\zeta \tag{6}$$

Fundamental relationships

Critical flow

The following gives the criterion for critical flow conditions in an open channel (Swamee, 1993):

$$\frac{Q^2 T_c}{g A_c^3} = 1 \tag{7}$$

where the subscript "c" indicates the state of the critical flow, Q is the discharge, T_c is the critical top width, g is the acceleration due to gravity and A_c is the critical water area.

Normal flow

For uniform flow conditions, Achour and Bedjaoui (2006) suggested a general equation of the discharge Q, which takes into consideration all the factors affecting the flow and channel. It is described by the equation below:

$$Q = -4\sqrt{2g}A_n\sqrt{R_{h,n}S_0}\,\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R_e}\right) \tag{8}$$

where the subscript "n" indicates the uniform flow conditions, A_n is the normal water area, S_0 is the slope of the channel, ε is the absolute roughness of the channel internal wall, $R_{h,n}$ is the hydraulic radius and R_e is the Reynolds number, which may be defined by:

$$R_e = 32\sqrt{2}\frac{\sqrt{gR_{h,n}{}^3 S_0}}{\nu} \tag{9}$$

where ν is the kinematic viscosity. Inserting Eq. (4) into Eq. (9) results in:

$$R_e = 32\sqrt{2}\left[\sigma(\zeta_n)\right]^{3/2}\frac{\sqrt{gS_0B^3}}{\nu} \tag{10}$$

Eq. (9) can be rewritten as follows:

$$R_e = R_e^*\left[\sigma(\zeta_n)\right]^{3/2} \tag{11}$$

where R_e^* is a modified Reynolds number and is written by:

$$R_e^* = 32\sqrt{2}\,\frac{\sqrt{gS_0B^3}}{\nu} \tag{12}$$

Bearing in mind that $R_{h,n} = A_n/P_n$, Eq. (8) can be rewritten as:

$$Q = -4\sqrt{2g}\,\frac{A_n^{3/2}}{P_n^{1/2}}\sqrt{S_0}\,\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R_e}\right) \tag{13}$$

where P_n is the wetted perimeter.

Relation between critical and normal depths

The fundamental relationship linking the properties of the critical and normal flows in a parabolic channel is obtained by deleting Q between equations (7) and (13). It reads:

$$\frac{A_c^{3/2}}{T_c^{1/2}} = -4\sqrt{2}\,\frac{A_n^{3/2}}{P_n^{1/2}}\sqrt{S_0}\,\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R_e}\right) \tag{14}$$

Using Equation (14) on the parabolic channel yields:

$$\frac{B^3\zeta_c^{9/2}}{B^{1/2}\zeta_c^{1/2}} = -4\sqrt{2}\,\frac{B^3\zeta_n^{9/2}}{\frac{B^{1/2}}{\sqrt{8}}\left[4\zeta_n\sqrt{1+16\zeta_n^2} + \ln\left(4\zeta_n + \sqrt{1+16\zeta_n^2}\right)\right]^{1/2}}\sqrt{S_0}\,\log\left(\frac{\varepsilon/B}{14.8\sigma(\zeta_n)} + \frac{10.04}{R_e^*\left[\sigma(\zeta_n)\right]^{3/2}}\right) \tag{15}$$

After simplifications, Eq. (15) is expressed as follows:

$$\zeta_c^{8/2} = -16\,\frac{\zeta_n^{9/2}}{\left[4\zeta_n\sqrt{1+16\zeta_n^2} + \ln\left(4\zeta_n + \sqrt{1+16\zeta_n^2}\right)\right]^{1/2}}\sqrt{S_0}\,\log\left(\frac{\varepsilon/B}{14.8\sigma(\zeta_n)} + \frac{10.04}{R_e^*\left[\sigma(\zeta_n)\right]^{3/2}}\right) \tag{16}$$

R_e^* is defined by Eq. (12).

According to Eq.(5), $\sigma(\zeta_n)$ is determined by:

$$\sigma(\zeta_n) = \frac{16}{3}\,\frac{\zeta_n^3}{4\zeta_n\sqrt{1+16\zeta_n^2} + \ln(4\zeta_n + \sqrt{1+16\zeta_n^2})} \tag{17}$$

The general relationship of critical flow

The critical flow in a parabolic channel is determined by the following general relation, which is obtained by changing the subscript "n" in Equation (16) to the subscript "c":

$$\frac{\left[4\zeta_c\sqrt{1+16\zeta_c^2}+\ln\left(4\zeta_c+\sqrt{1+16\zeta_c^2}\right)\right]^{1/2}}{\zeta_c^{1/2}}=-16\sqrt{S_0}\log\left(\frac{\varepsilon/B}{14.8\sigma(\zeta_c)}+\frac{10.04}{R_e^*\left[\sigma(\zeta_c)\right]^{3/2}}\right) \quad (18)$$

According to Equation (5), $\sigma(\zeta_c)$ is expressed as follows:

$$\sigma(\zeta_c)=\frac{16}{3}\frac{\zeta_c^3}{4\zeta_c\sqrt{1+16\zeta_c^2}+\ln(4\zeta_c+\sqrt{1+16\zeta_c^2})} \quad (19)$$

R_e^* is always given by Eq.(12).

Relationship of the specific energy

The specific energy is written for any shape of the channel as:

$$E_S=y+\frac{Q^2}{2gA^2} \quad (20)$$

When Eq. (20) is applied to the parabolic channel, the following result is obtained:

$$E_S^*=\zeta^2+\frac{9Q^{*2}}{8\zeta^6} \quad (21)$$

where $E_S^* = E_S/B$ represents the relative specific energy, $\zeta = \sqrt{y/B}$ and Q is the relative discharge expressed as $Q^* = Q/\sqrt{gB^5}$.

For the parabolic channel, Eq.(13) can be used to compute the relative discharge Q^*, as a result:

$$Q^*=-16\times\left(\frac{2}{3}\right)^{3/2}\times\frac{\zeta^{9/2}}{\left[4\zeta\sqrt{1+16\zeta^2}+\ln\left(4\zeta+\sqrt{1+16\zeta^2}\right)\right]^{1/2}}\sqrt{S_0}\log\left(\frac{\varepsilon/B}{14.8\sigma(\zeta)}+\frac{10.04}{R_e^*\left[\sigma(\zeta)\right]^{3/2}}\right) \quad (22)$$

RESULTS AND DISCUSSION

Variation in the critical depth versus the normal depth

Special case of a smooth parabolic channel

To observe the variation in the relative critical depth (ζ_c) as a function of the relative normal depth (ζ_n) while changing the slope of channel *S₀*, let us use a smooth parabolic channel ($\varepsilon \to 0$) with a linear dimension B = 1 m and the kinematic viscosity $\nu = 10^{-6} m^2/s$ of the flowing liquid. According to the implicit Eq. (16), the computations were performed by varying ζ_n for a fixed value of the slope of channel S_0. Figure 2 illustrates the computation results.

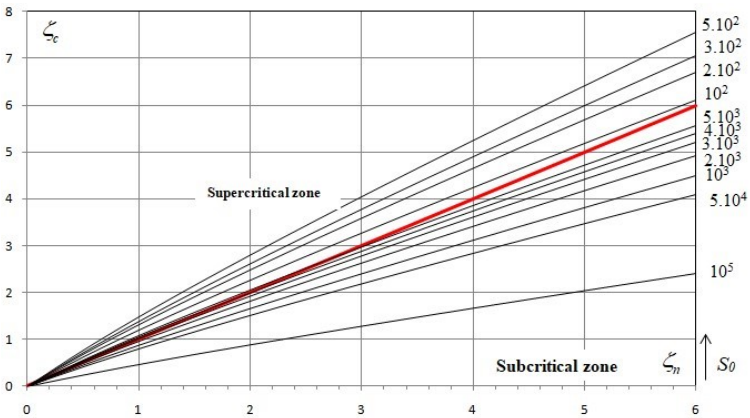

Figure 2: Variation in ζ_c versus ζ_n in a smooth parabolic channel for various slopes S_0. Red line: ( ) first bisector corresponding to $\zeta_c = \zeta_n$.

Figure 2 shows the supercritical zone of the flow, which is above the first bisector, and the subcritical zone, which is positioned below the first bisector. This latter corresponds to $\zeta_c = \zeta_n$, i.e., it contains all the points where the flow is critical. As shown in Figure 2, some of the curves do not intersect the first bisector. This indicates that the channel cannot generate any critical state for certain values of *B* and S_0, and its regime remains subcritical. As an illustration, we employ the situation of the slope $S_0 = 0.0005$, preserving the smooth parabolic channel with a linear dimension *B* = 1 m. The basic Eq. (16) for this situation permits the charting of Figure 3.

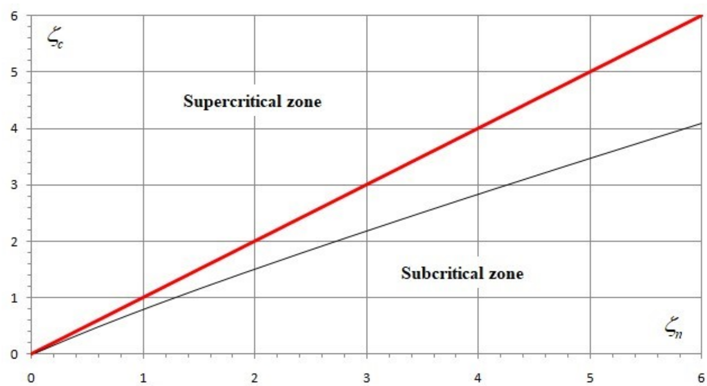

Figure 3: Variation curve of ζ_c depending on ζ_n for a slope $S_0 = 0.0005$.

After carefully examining Figure 1 and Eq. (16), it was concluded that there is a slope limit S_0, so that the curve is tangent to the first bisector at one point. According to the results of the study, the smallest slope that can generate one critical state of the flow for the smooth parabolic channel of a linear dimension B = 1 m is S_0 = 0.0023624406 corresponding to $\zeta_c = \zeta_n \approx 0.451$ or $y_c \approx 0.203m$. Figure 4 provides an illustration of this case.

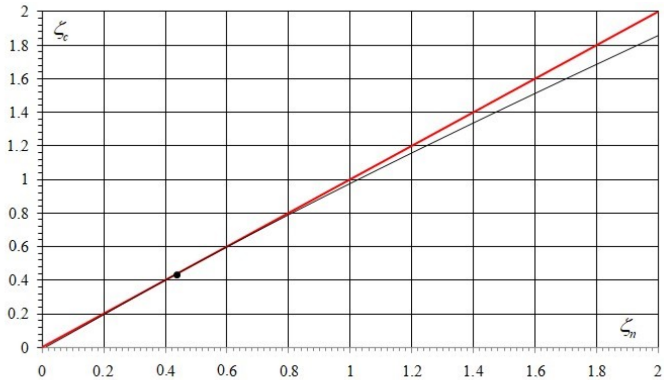

Figure 4: Variation curve of ζ_c depending on ζ_n in a smooth parabolic channel for a slope S_0 = 0.0023624406, (•) single critical flow state, $\zeta_c = \zeta_n \approx 0.451$.

It was shown that for the smooth parabolic channel, all slopes S_0 lower than the slope limit S_0 = 0.0023624406 do not generate any critical state of the flow, as shown in Figure 3. However, slopes that are greater than this value cause two critical flow conditions, each with a different flow rate. The first critical state appears at shallow depths, while the second is observed at greater depths. Let us use the slope S_0 = 0.003 as an example in this case, always keeping the considered channel. Figure 5 graphically illustrates the results of the calculation made using Eq. (16). Figure 5 (a) illustrates the first point of intersection at a low relative depth $\zeta_c = \zeta_n = 0.17764529 \approx 0.178$, and Figure 5 (b) represents the second point of intersection observed at a higher relative depth $\zeta_c = \zeta_n = 1.09998103 \approx 1.1$.

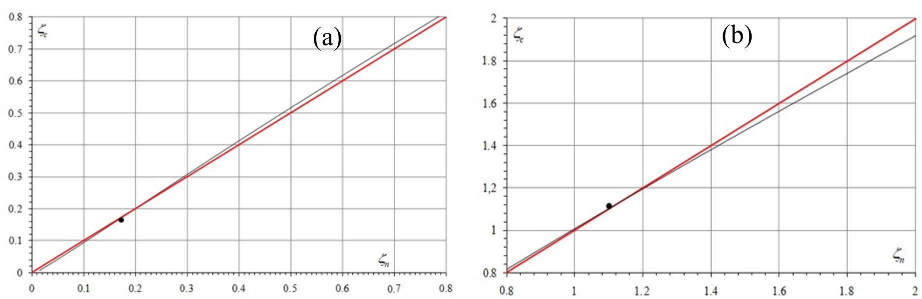

Figure 5 : Variation in ζ_c versus ζ_n for a slope S_0 = 0.003, (•) single critical flow state. (a) First critical state: $\zeta_c = \zeta_n = 0.17764529 \approx 0.178$. (b) Second critical state: $\zeta_c = \zeta_n = 1.09998103 \approx 1.1$.

Special case of the transition regime

The purpose of this part is to observe the variation in the critical depth (ζ_c) in the turbulent transition regime with respect to normal depth (ζ_n) for various slope S_0 values of the channel. To attain this objective, we use Equation (16) and apply it to the parabolic channel with linear dimension B = 1 m and kinematic viscosity $\nu = 10^{-6} m^2/s$. Let us take the absolute roughness $\varepsilon = 0.001 m$ as an example. Figure 6 illustrates the variation between the critical depth and normal depth in a transition regime according to the calculations performed.

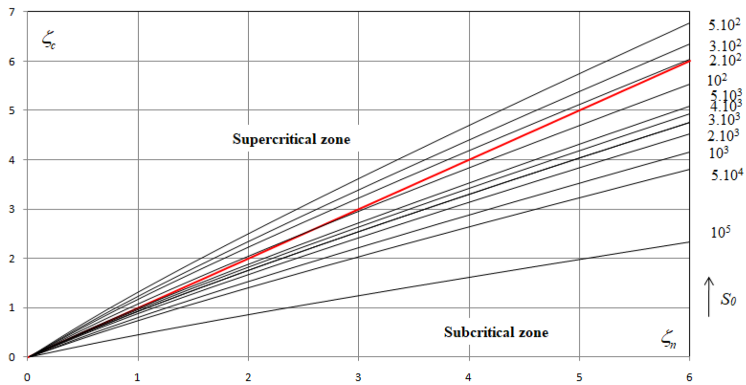

Figure 6: Variation curve of ζ_c depending on ζ_n in the transition regime for various slopes S_0. Red line: ( ) first bisector corresponding to $\zeta_c = \zeta_n$.

Figure 6 shows that there are curves in the subcritical zone that do not have any point intersection with the red line, which indicates that there is no critical state. On the other hand, there are some curves that meet the first bisector at two points, indicating that two critical depths can exist in the channel at various flow rates. The first critical state can be seen at low depths; although it is practically imperceptible in the illustration, this critical state of the flow truly exists theoretically. At higher depths, the second critical state is observed and can be viewed in Figure 6.

By comparing the channel data in the transition regime with the smooth channel data, Figure 6 indicates that the relative critical depth (ζ_c) values in the turbulent transition domain are lower than those observed in the channel with a smooth surface, as shown in Figure 2. One can deduce that the critical depth is dependent on both the geometry of the channel and flow parameters, such as the linear dimension *B,* the slope of the channel S_0, the absolute roughness ε, which characterizes the inner wall of the channel, and the kinematic viscosity ν of the flowing water.

According to the analysis of Equation (16) and Figure 6, Figure 7 shows that for the turbulent transition regime in the parabolic channel ($\varepsilon = 0.001$m) with a linear dimension *B* = 1 m, the smallest slope that produces a single flow critical state is the slope S_0 =

0.004473377, which corresponds to $\zeta_c = \zeta_n \approx 0.4235$. One can observe that the value of the smallest slope for the turbulent transition regime [Figure 7] is greater than the value obtained for the turbulent smooth regime [Figure 4], which demonstrates that the absolute roughness of the channel affects the occurrence of critical flow.

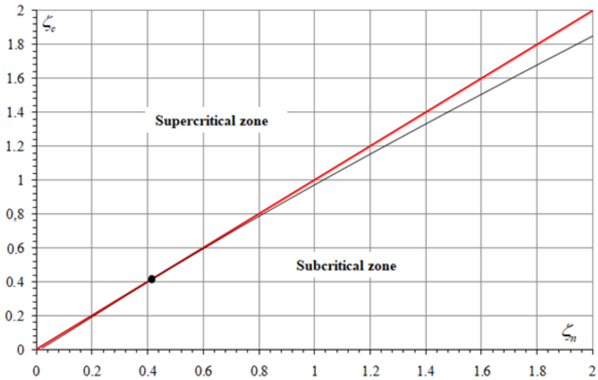

Figure 7 : Variation curve of ζ_c depending on ζ_n in the transition regime for a slope S_0 = 0.004473377, (•) single critical flow state, $\zeta_c = \zeta_n \approx 0.4235$.

Special case of a rough parabolic channel

In this section, the curves of the variation between ζ_c and ζ_n based on different values of slope S_0 are plotted by using Eq.(16) for a rough parabolic channel ($R_e^* \to \infty$) with a linear dimension B = 1 m as well as the absolute roughness characterizing the inner wall of the channel $\varepsilon = 0.001m$ as an example.

Figure 8 illustrates the variation between the relative critical depth and normal depth in a rough parabolic channel for various slopes S_0 of the channel. Figure 8 refers to the same observations obtained in Figures 2 and 6. As seen in Figure 8, the critical depth values for the rough channel are close to those shown in Figure 6 for the transitional channel.

In the case of a rough parabolic channel ($R_e^* \to \infty$) with a linear dimension B = 1 m and an absolute roughness $\varepsilon = 0.001m$. Figure 9 illustrates the slope limit that results in only one critical state in this channel, which corresponds to S_0 = 0.0044091014 and $\zeta_c = \zeta_n \approx 0.4201$. We find that this value of the slope limit is slightly lower than the value obtained in the transition regime [Figure 7]. This indicates that the effect of viscosity on the occurrence of critical flow is insignificant compared to the absolute roughness.

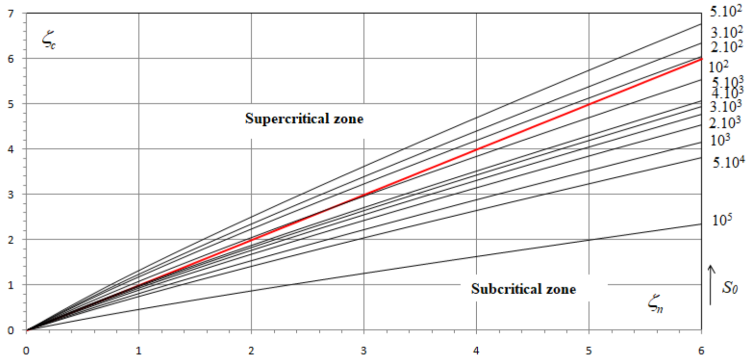

Figure 8: Variation in ζ_c according to ζ_n in a rough parabolic channel for various slopes S_0. Red line :( ) first bisector corresponding to $\zeta_c = \zeta_n$.

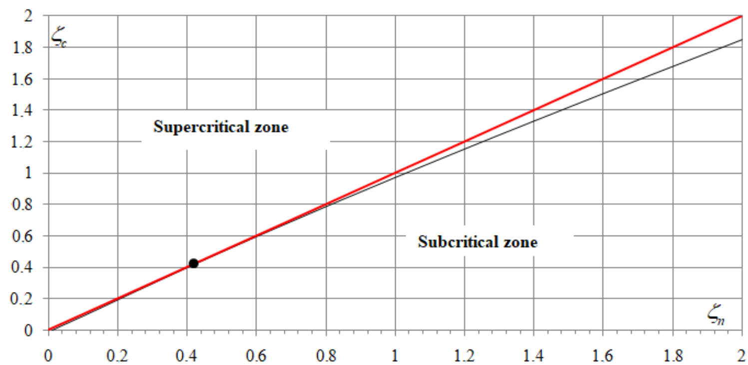

Figure 9 : Variation curve of ζ_c depending on ζ_n in a rough parabolic channel for a slope S_0 = 0.0044091014, (•) single critical flow state, $\zeta_c = \zeta_n \approx 0.4201$.

Validation

In this part, we will use the relationship of the specific energy to verify the obtained results of the two critical flow states, which are presented in Figure 5 for the smooth parabolic channel. Using Eq.(22) leads to the following two relative discharge values: $Q^* = 0.000542098$ and $Q^* = 0.803565812$, which correspond to $\zeta = 0.178$ and $\zeta = 1.1$, respectively. For a smooth parabolic channel ($\varepsilon \to 0$) with a linear dimension B = 1 m and a slope S_0 = 0.003, Equation (21) allowed the drawing of the two diagrams shown in Figure 10. This illustrates the variation in the relative critical ζ versus E^*.

In Figure 10, one can observe that for the two relative discharges calculated above, the relative specific energy is minimal and that the corresponding relative depths are $\zeta =$

0.178 and $\zeta = 1.1$, in fact, those shown in Figures 5(a) and 5(b). This confirms the validity of the proposed relationships.

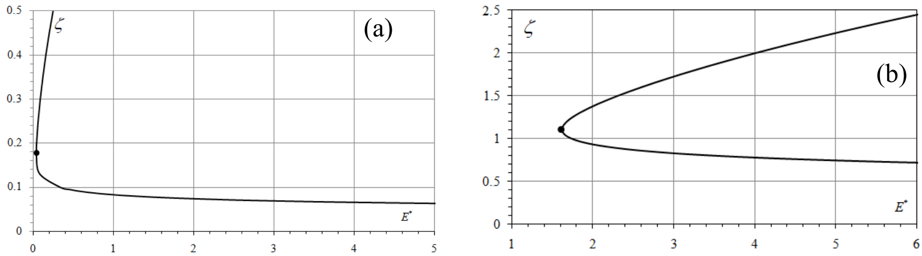

Figure 10: Curve of relative specific energy E^* for the smooth parabolic channel.

(a): $Q^* = 0.000542098$, **(•)** E^*minimum ≈ 0.0421, $\zeta = 0.178$.

(b): $Q^* = 0.803565812$, **(•)** E^*minimum ≈ 1.6201, $\zeta = 1.1$.

Critical State of Flow

The aim of this section is to identify the flow behavior in the channel under study by observing the variation in the relative critical depth ζ_c as a function of the slope of channel S_0. To illustrate this, let us use the same smooth parabolic channel ($\varepsilon \rightarrow 0$) with a linear dimension B = 1 m. In this situation, the computations were performed using the implicit equation (18). The results we obtained enabled us to create the graph shown in Figure 11.

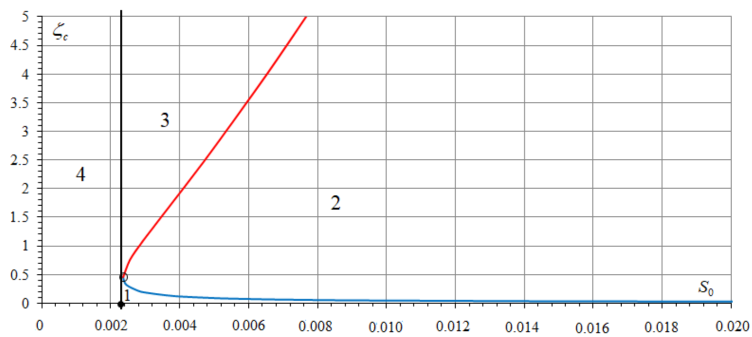

Figure 11: Variation in the relative critical depth ζ_c versus the slope S_0. (•) The smallest slope S_0 = 0.0023624406 generates only one critical state of the flow, (o) the relative critical depth $\zeta_c \approx 0.451$.

The behavior of the flow in the smooth parabolic channel is clearly shown in Figure 11. Looking at Figure 11, one can notice two curves for the flow's critical state because the slopes are higher than the smallest slope S_0 = 0.0023624406, and as a result, two critical states of flow are generated. The first curve, shown in blue, corresponds to the first critical state that forms at shallow depths, while the second curve in red represents the second critical state at larger depths. It shows four zones of the flow regime, which can be interpreted as follows:

Zone 1: This zone is a subcritical flow region. The flow occurs in this zone regardless of the slope S_0.

Zone 2: In this zone, the flow is supercritical. The flow in zone 1, which was previously subcritical, changes to supercritical flow in zone 2 when it passes the critical state at the intersection with the red curve.

Zone 3: The flow is subcritical in this zone. The flow, previously supercritical in zone 2, changes back to being critical on the blue curve before returning to being subcritical in zone 3.

Zone 4: This area of the flow is subcritical. Since the slopes are weak and less than the slope limit S_0 = 0.0023624406, this region does not generate any critical flow state and remains in the subcritical domain.

CONCLUSION

The main objective of this research was to determine how the critical depth in a parabolic channel change depending on all the parameters that influence the flow, including the slope of the channel, the absolute roughness that characterizes the inner wall of the channel, and the kinematic viscosity of the flowing water. The implicit relation between the critical and normal depths [Eq.(16)] was used in this study to achieve this purpose. The examination of this relationship revealed that there is a slope limit S_0 corresponding to the smallest slope resulting in one critical condition for the flow.

For the smooth parabolic channel with linear dimension B = 1 m, it seems that the slope S_0 = 0.0023624406 indicates the minimum slope S_0 that provides a single critical state of the flow corresponding to relative critical depth $\zeta_c \approx 0.451$ [Figure 4]. For the turbulent transition regime in a parabolic channel with the linear dimension B = 1 m, the kinematic viscosity $\nu = 10^{-6} m^2/s$, and the absolute roughness $\varepsilon = 0.001m$ is taken as an example. The computations showed that S_0 = 0.004473377 denotes the smallest slope of this channel [Figure 7], which corresponds to $\zeta_c \approx 0.4235$. For the rough parabolic channel ($R_e^* \to \infty$), keeping the linear dimension of the channel at B = 1 m and the absolute roughness $\varepsilon = 0.001m$, the smallest slope is S_0 = 0.0044091014 corresponding to $\zeta_c \approx 0.4201$ [Figure 9]. By comparing the values of the slope limit for the three cases, we observe that the smallest slope for the transition regime is the greatest value. This leads to the conclusion that the occurrence of the critical flow is influenced by the channel geometry and all the parameters flow previously mentioned.

The general relationship governing the critical flow in a parabolic channel [Eq.(18)] was derived from the previous relationship by putting $\zeta_c = \zeta_n$. This relationship allowed for the elicitation of the graph [Figure 11] showing the behavior of the flow in the smooth parabolic channel with linear dimension B = 1 m. The graph clearly demonstrates changes in flow regimes, going from subcritical to critical, then from critical to supercritical, and ultimately returning to subcritical. Additionally, it can be seen that there are no critical flow states for all slopes lower than the smallest slope (S_0 = 0.0023624406); instead, the flow stays in the subcritical zone, regardless of discharge, whereas slopes greater than this value cause two critical flow states. The first critical state occurs at shallower depths, while the second state appears at greater depths.

Additional investigation on the behavior of the critical flow in a parabolic channel is advised to complete this study. In the study's second part, the authors will attempt to elucidate this by focusing on the effects of diameter and absolute roughness.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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