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LAX-FRIEDRICHS NUMERICAL SCHEME FOR SIMULATING THE FAILURE WAVE OF A DAM IN THE PRESENCE OF OBSTACLES

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ABSTRACT

Numerical methods are very useful for predicting water levels, velocities and flow rates in hydraulic systems. The Saint-Venant system of equations, which is a hyperbolic partial differential system, is widely used in the modelling of a flood wave due to a dam failure. In this study, two explicit finite difference Lax-Friedrichs and modified Lax-Friedrichs numerical schemes are used to simulate the failure wave of a dam. The calibration of the obtained results is done in relation to experimental measurements and numerical results existing in the literature specialized in this field of research. The experimental set-up consists of a water reservoir that simulates the reservoir of a dam, followed downstream by a horizontal dry bottom section, a triangular bottom sill with steep slopes and a small tide of water at rest after the triangular obstacle which ends with a vertical diaphragm. The simulation obtained results with the new formulation of the Lax-Friedrichs numerical scheme showed good agreement with the simulated, experimental and numerical results of other researchers.

Keywords: Obstacle, Experimental data, Free surface flow, Numerical schemes, Reservoir, Numerical simulation, Flood wave, Lax-Friedrichs scheme.

INTRODUCTION

Flooding due to tsunamis, prolonged bad weather, landslides, dam failures are increasingly catastrophic (Zokagoa and Soulaïmani, 2010; Pilotti et al., 2011; Peng and Zhang, 2012; Kostecki and Banasiak, 2020). Flood protection measures generally consist of actions on the watercourse through the channelling of flows, the calibration of the minor plane, but also in developments such as the construction of protective dams, flood control dams or drainage channels (Zokagoa and Soulaïmani, 2010). The development of flood maps identifying worst-case scenario flood zones are used to draw up an emergency evacuation plan (Saikia and Sarma, 2006a; Ikni et al., 2021). The implementation of such protection measures requires the intervention of researchers in the description of the physics of the flooding phenomenon and the dynamics of flows (Berreksi et al., 2022). In hydrodynamics in particular, the prediction of flood zones must be increasingly accurate through the development of an adequate numerical simulation model (Chen et al., 2005; You et al., 2012). The main objective of this work is to simulate the dam break wave in the presence of an obstacle with the Lax-Friedrichs scheme proposed by Sarma and Saikia (2006) using the 1D Saint-Venant equations. In this case, the flow term in the Lax-Friedrichs scheme is transformed into a new formulation. An artificial viscosity is added to the calculated solution. The experimental and simulation results were used to validate the simulation results with the new formulation of the Lax-Friedrichs scheme.

SAINT-VENANT EQUATIONS

The flows characterising flooding due to dam failure can be modelled through the Saint-Venant equations. These are derived from Navier Stokes equations system by applying the assumption of a hydrostatic pressure. The one-dimensional Saint-Venant system consists of two equations, conservation of mass and quantity of motion. The conservative form of these equations in terms of the flow variables (h, uh) in a channel with a uniform cross-section has the following form (Hsu and Yeh, 2002; Soleymani et al., 2015):

$$\frac{\partial V}{\partial t} + \frac{\partial F}{\partial x} = S \tag{1}$$

In the latter relationship, V represents the vector of conservative variables, $F(V)$ the flux vector and S the source vector. These vectors are given by:

$$V = \begin{pmatrix} h \\ uh \end{pmatrix}; \quad F = \begin{pmatrix} hu \\ hu^2 + g\dfrac{h^2}{2} \end{pmatrix}; \quad D = \begin{pmatrix} 0 \\ gh\left(S_{ox} - S_{fx}\right) \end{pmatrix}; \quad S_{ox} = sin\left(\alpha_x\right); \quad S_{fx} = \frac{u|u|n^2}{h^{4/3}} \tag{2}$$

where $q = uh$ is the specific discharge, h the water height, g the gravity acceleration, S_{ox} the energy slope, S_{fx} the bed slope and n the Manning coefficient of friction.

NUMERICAL RESOLUTION

The Saint-Venant partial differential equations are hyperbolic and non-linear; they can be solved numerically except only in simplified cases. The numerical solution of these equations for the case of a complex domain is far from being an easy task. The numerical schemes used should be able to simulate rapid changes in hydraulic characteristics with accuracy. The Lax-Friedrichs scheme is a simple and robust scheme in the calculations and is easy to use. This scheme consists of a single step (Sarma and Saikia, 2006; Saikia and Sarma, 2006b).

$$\begin{cases} V_i^{n+1} = (1-\theta)V_i^n + \theta \dfrac{V_{i+1}^n + V_{i-1}^n}{2} - \dfrac{\Delta t}{2\Delta x}\left[F_{i+1}^n - F_{i-1}^n\right] + \Delta t S_i^n \\ 0 \le \theta \le 1 \end{cases} \tag{3}$$

where, θ represents a coefficient of the numerical viscosity of the Lax-Friedrichs scheme.

The new formulation of the Lax-Friedrichs scheme after decomposition of the flux term is:

$$V_i^{n+1} = (1-\theta)V_i^n + \theta \frac{V_{i+1}^n + V_{i-1}^n}{2} - \frac{\Delta t}{\Delta x}\left[(1-\theta)(F_i^n - F_{i-1}^n) + \theta(F_{i+1}^n - F_i^n)\right] \tag{4}$$
$$+ \Delta t((1-\theta)S_i + 0.5 * \theta(S_{i+1}^n + S_{i-1}^n))$$

The fractional step method is used to solve the Saint-Venant equations system in order to make a comparative study with equation (4). It consists of the following two stages (Strang, 1968; Zoppou and Roberts, 2000):

$$\begin{cases} V_i^P = (1-\theta)V_i^n + \theta \dfrac{V_{i+1}^n + V_{i-1}^n}{2} - \Delta t\left((1-\theta)\dfrac{F_i^n - F_{i-1}^n}{\Delta x} + \theta \dfrac{F_{i+1}^n - F_i^n}{\Delta x}\right) \\ V_i^{n+1} = (1-\theta)V_i^P + \theta \dfrac{V_{i+1}^P + V_{i-1}^P}{2} + \Delta t\left((1-\theta)S_i^P + \theta \dfrac{S_{i+1}^P + S_{i-1}^P}{2}\right) \\ 0 \le \theta \le 1 \end{cases} \tag{5}$$

BOUNDARY CONDITIONS

The methods used to deal with the boundary conditions are linear extrapolation and the reflection method. Let V_2^{n+1} to V_{N-1}^{n+1} values calculated at time t^{n+1} using the numerical scheme, the values V_1^{n+1} and V_N^{n+1} will be calculated as follows:

Dam failure on a horizontal plane

Free conditions (Das and Bagheri 2015; Ikni et al., 2018):

$$\begin{cases} V_1^{n+1} = 2V_2^{n+1} - V_3^{n+1} \\ V_N^{n+1} = 2V_{N-1}^{n+1} - V_{N-2}^{n+1} \end{cases} \tag{6}$$

Dam failure on a horizontal plane in the presence of an obstacle

Reflection conditions (Gu et al., 2017) :

$$\begin{cases} h_1^{n+1} = h_2^{n+1} \\ q_1^{n+1} = -q_2^{n+1} \\ h_N^{n+1} = h_{N-1}^{n+1} \\ q_N^{n+1} = -q_{N-1}^{n+1} \end{cases} \tag{7}$$

DAM BREAK PROBLEM

The present tests relate to the validation of the previously developed numerical model. For this purpose, four validation tests are considered. The first and second tests simulate an idealized dam failure (complete and instantaneous rupture) on a horizontal channel without friction, the third test simulates this phenomenon on a horizontal channel with friction, and finally the fourth test simulates the dam failure problem in the presence of an obstacle.

Dam failure on a dry frictionless horizontal plane

This test concerns a flow generated by the failure of a dam located at x=L/2, where L=1m is the total length of the horizontal channel. The dam is characterised by water levels of h =1m upstream and h =0.00001m downstream of the position x=L/2 (Figure 1). A structured mesh of 1000 nodes is used for the numerical simulations. A complete and instantaneous collapse of the dam is assumed. This case is considered in order to test the robustness of the two numerical schemes used. Figure 1 shows respectively the analytical and numerical simulation results at t=0.08 seconds after the dam failure. It can be seen from Figure 1 that the numerical simulation results converge with Stoker's analytical solution (Stoker, 1957) for both versions of the Lax-Fiedrichs and modified Lax-Friedrichs schemes (Figures 1a and 1b). The obtained results show that the new formulation of the flux term gives better simulations for the modified Lax-Friedrichs scheme (Figures 1a and b).

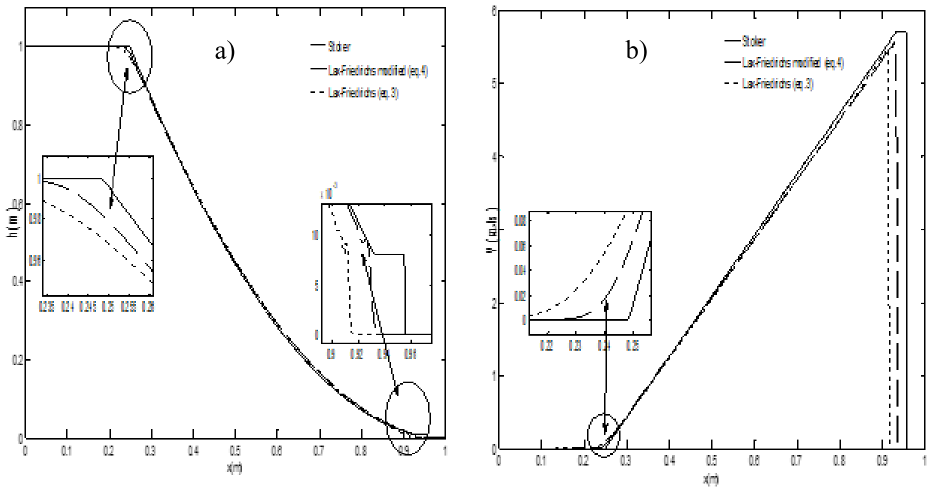

Figure 1: Comparison of analytical and simulated results for the depth ratio $h_{downstream}/h_{upstream}$= 0.00001 and t=0.08s : a) water depth h (m); b) flow velocity

The three-dimensional view of the water level variation with time is shown in Figure 2. This figure shows the movement of the wavefront in the channel as a function of time.

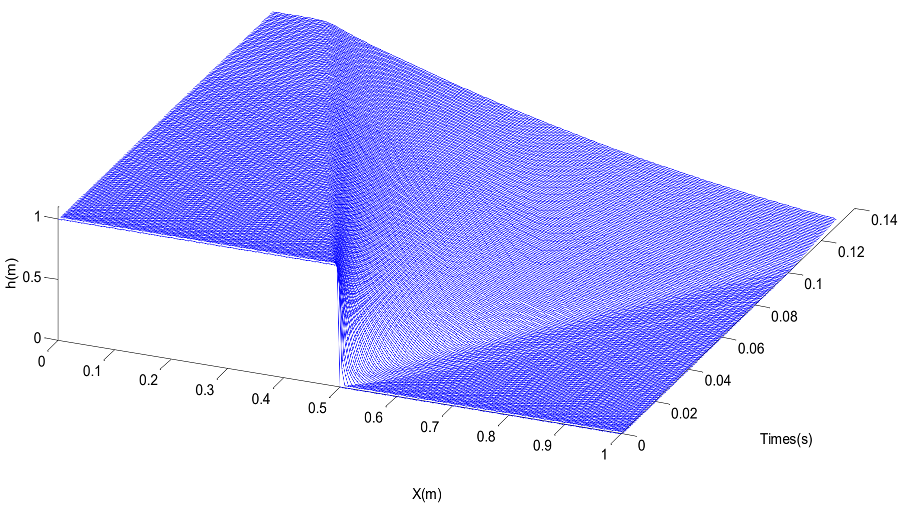

Figure 2: Three-dimensional view of the variation of the water level with time Comparison of the present model with two schemes Roe and HLL used by Peng in 2012

In this section, a dam failure test on a wet horizontal plane is considered. In this problem, the channel used by Peng in 2012 is of length L=1000m. For a perfect liquid the effect of friction is neglected.

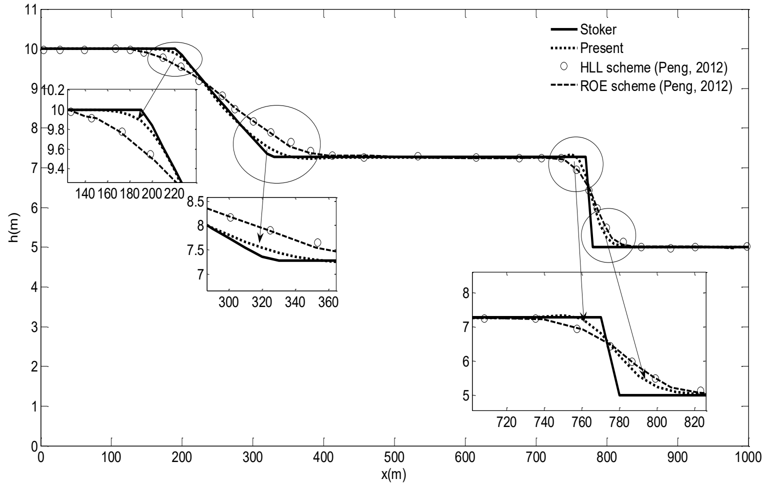

Figure 3: Dam failure on a horizontal plane for depth ratio $h_{downstream}/h_{upstream}$ =0.5

A dam is located in the middle of the channel, and at time t=0 the dam is completely removed and the water is released in two waves, one heading upstream and the other downstream. The obtained results with the present model (eq. 4) and with the two schemes Roe and HLL used by Peng in 2012 are shown in Figure 3. This figure shows that the present model gives the best calibration.

Dam failure on a horizontal plane with friction

Schoklitsch (1917) studied the failure flow of a dam on a dry bottom. The channel is 0.096 m wide, 0.08 m high and 20 m long. The dam is placed in the middle of the channel. The water level upstream of the dam is 0.074m and downstream of the dam the channel is dry. The Manning roughness coefficient used in this experiment is 0.009 s/m1/3 (Lai and Khan, 2012). In 2012, Lai and Khan used the finite element method to simulate the measured water profiles. The element size Δx and time step Δt used are Δx=0.1 m and Δt=0.0001s respectively. For the application used in this test, the element size Δx remains constant and the time step Δt will be calculated with the Courant-Friedrichs-Lewy (CFL) condition (Peng, 2012; Das and Bagheri, 2015; Ikni et al., 2018). Figure 4 shows the water surface profiles measured by Schoklitsch (1917), simulated by Lai and Khan (2012) and calculated with equations (4) and (5) at t=9.40 seconds after dam failure. This figure shows a good agreement between the experimental water surface profile and the obtained profiles. These show that the wavefront motion is well simulated by the current numerical model (eqs.4 and 5) compared to the simulation results of Lai and Khan (2012). These results prove that the two-stage numerical model (eq.5) is more accurate than the onestage numerical model (eq.4). From the curve, the position of the wavefront is well estimated with equation (5). These results prove that the technique used in this work is capable of modelling the progressive wave after dam failure on an initially dry plane with friction.

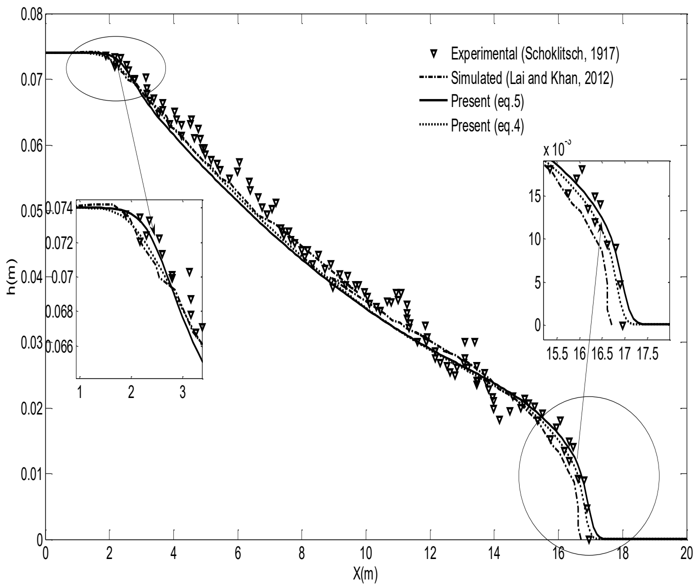

Figure 4: Water level along the canal after dam failure at T=9.4s.

Dam failure in the presence of an obstacle on a wet plane

In this section, the experimental results of the dam break wave in the presence of a triangular obstacle and those calculated by Gu et al. (2017) are used to make a comparative study with the obtained results with the current numerical model (Eq.5). Fig. 5 shows the description of the dam failure experience in the presence of an obstacle. The dam is located 15.5 m from the upstream entrance of the 38 m long canal. Downstream of the dam site, a triangular obstacle 6 m wide and 0.4 m high is installed.

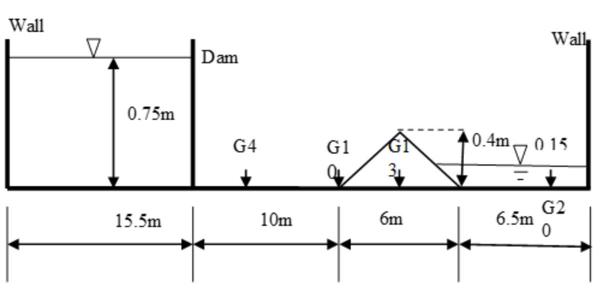

Figure 5: Dam failure in the presence of a triangular obstacle on a wet plane

The water depth upstream of the dam is 0.75 m and a further 0.15 m is retained behind the triangular obstacle, creating a wet limit on the downstream end of the channel which is closed (Figure 5). Four experimental water level measurement points are located in G4 (4 m), G10 (10 m), G13 (13 m) and G20 (20 m) downstream of the dam respectively (Gu *et al.*, 2017). The channel width is 1.75 m and the channel roughness coefficient used is $n=0.0125\ s.m^{-1/3}$. In 2017, Gu et al. applied The Smoothed Particle Hydrodynamics (SPH) method to simulate this phenomenon using the two-dimensional water flow equations with SWE-SPHysics software. The discretization step Δx is fixed at 0.025 m and the CFL number's used in this experiment is 0.4. The experimental and calculated water levels of Gu et al. (2017) at locations G4, G10, G13 and G20 will be used to validate the results of the new version of the Lax-Friedrichs scheme (Eq.4). Figure 6 shows the experimental water depths calculated from Gu et al. (2017) and simulated with the new model (Eq.5). The same discretization step (Δx =0.025 m) is used with a CFL equal to 0.85.

Fig. 6 illustrates that the technique in question simulates the experimental results well.

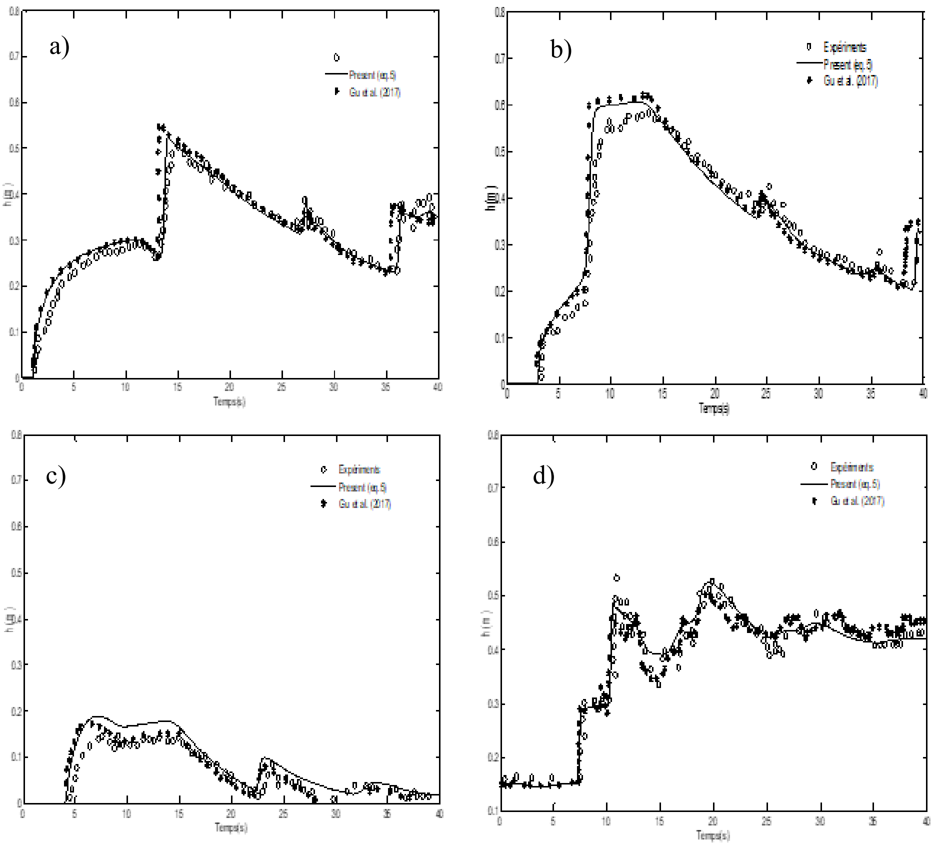

Figure 6: Variation of water depth with time at stations: a) G4, b) G10, c) G13 and d) G20

Fig. 7 also shows the profile of the free surface along the channel at different times. Once this wave reaches the obstacle, part of it is reflected and moves upstream while the other part moves up the obstacle (Figure 7a). When this second part goes up the obstacle, it will meet the wall after the obstacle. This second part of the wave will be reflected again and go upstream of the wall (Figure 7 b).

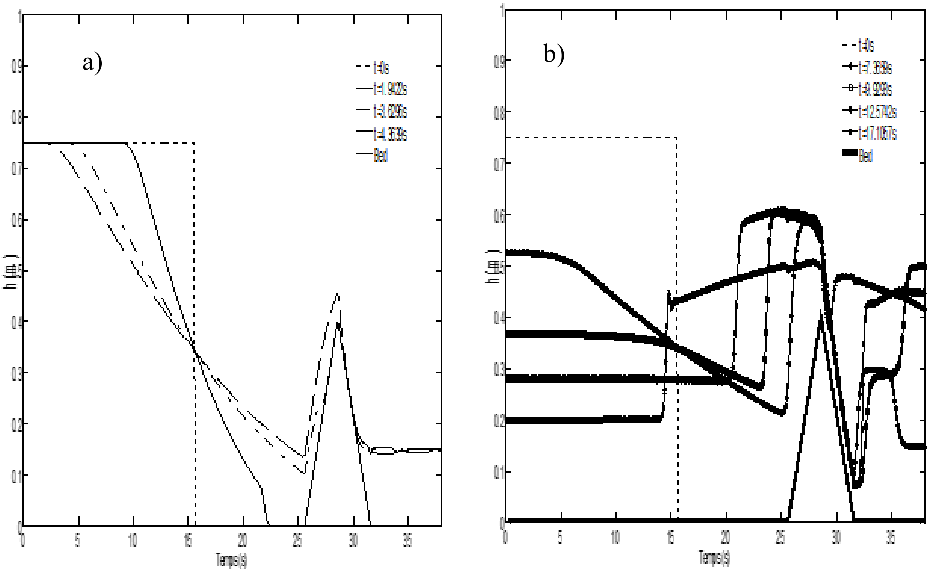

Figure 7: Water depth along the canal at different times

CONCLUSION

In the course of this work, the numerical modelling of two free surface flow problems was investigated. The common feature of the problems addressed is the mathematical formulation which is based on the Saint-Venant equations. The work was devoted to the problem of dam failure in two configurations. The first is dedicated to the study of the propagation of the dam break wave on a dry and frictionless horizontal plane. In the second configuration the study concerns the flow due to dam failure in the presence of a frictional obstacle.

The new formulation of the Lax-Friedrichs scheme after decomposition of the flow term is used to simulate dam failure without and with an obstacle. The numerical accuracy is fully demonstrated by using the new formulation of the Lax-Friedrichs scheme for the simulation of this phenomenon. From the numerical simulations discussed above, it can be concluded that the proposed new formulation is powerful for simulating the one-dimensional dam failure problem in any flow regime even in the presence of an obstacle.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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