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A SEMI-ANALYTICAL METHOD FOR PARAMETRIC ANALYSIS OF HYDROSEISMIC FORCES ON DAMS

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ABSTRACT

Analytical expressions for the determination of hydro-seismic forces acting on a rigid dam with irregular upstream face geometry in presence of a compressible viscous fluid are derived through a linear combination of the natural modes of water in the reservoir based on a boundary method making use of complete sets of complex T-functions.

The formulas obtained for parametric analysis of both shear forces and overturning moments are computationally effective and useful for the preliminary design of dams. They show clearly the separate and combined effects of compressibility and viscosity of water. They also have the advantage of being able to cover a wide range of excitation frequencies even beyond the cut-off frequencies of the natural modes of the reservoir. Key results obtained using the proposed analytical expressions of the hydrodynamic forces are validated using numerical and experimental solutions published for some particular cases available in the specialized literature.

Keywords: Hydro-seismic forces, Dams, Irregular upstream-face, Compressible viscous fluid, Earthquakes.

INTRODUCTION

Analytical expressions of hydrodynamic forces on dams are rare and available only under simple geometry of the water dam interface. (Westergaard, 1933) was the pioneer to have derived an analytical expression to evaluate the hydrodynamic pressures applied to a rigid dam with vertical upstream face under a horizontal harmonic ground motion. Using the electric analog method, (Zangar, 1953) studied experimentally the hydrodynamic effect of horizontal earthquake action on a rigid dam having upstream face with either constant or compound slopes in the presence of an incompressible fluid. (Chopra, 1967) published an analytical solution for vertical rigid dams under horizontal and vertical earthquake ground motions taking into account the effect of compressibility of the fluid in the reservoir. (Tsai, 1992) developed a semi-analytical solution for hydrodynamic pressure distribution on rigid dams with arbitrary upstream face considering water compressibility.

Moreover, several authors have used the numerical methods essentially based on the F.E.M, to include the effects of compressibility of the fluid (Hall and Chopra, 1982), the flexibility of the dam (Tiliouine and Seghir, 1998) and pressure wave absorption by sediments at the bottom of the reservoir (Fenves and Chopra, 1985). There are also, semi-analytical methods, which remain valid and are an important input for the preliminary dam design (Avilés, 1998), (Tadjadit and Tiliouine, 2013). In this paper, analytical expressions for the determination and parametric analysis of hydro-seismic forces acting on a rigid dam with irregular upstream face geometry in presence of a compressible viscous fluid are derived through a linear combination of the natural modes of water in the reservoir based on a boundary method making use of complete sets of complex T-functions. Key results obtained using the proposed analytical expressions are validated using numerical and experimental solutions published for some particular cases available in the specialized literature.

BACKGROUND

Assumptions

Consider a rigid dam with partially inclined upstream face impounded by a reservoir of infinite length and rigid bottom subjected to horizontal earthquake short durations. In figure 1, the motion of the dam-reservoir system is two-dimensional and the water in the reservoir is considered linearly compressible, viscous and irrotational. H is the depth of the water in the reservoir; C is the fraction of height H and θ the angle formed by the inclined portion of the upstream face with the vertical. Since the dam undergoes a displacement of rigid body, consequently the set of points belonging to the fluid-structure interface are assumed to have, at each time, the same acceleration as the base of the dam.

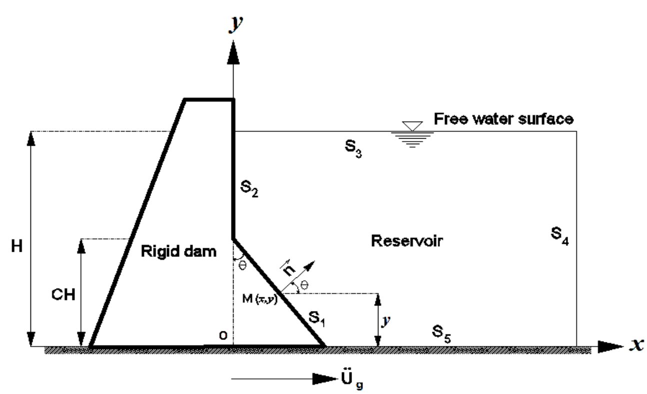

Figure 1: Rigid dam subjected to a horizontal ground motion

On figure 1, the reservoir is delimited by four contours defined as:

$S_1 \cup S_2$: Contour delimiting the upstream face of the dam; S_3: Contour defining the free surface of water; S_4: Contour defining the boundary of truncation of the reservoir and S_5: Contour defining the reservoir bottom. CH is defined as the height of the inclined portion of the upstream face and $\vec{n}$ is the outward normal direction to the dam-water interface.

Formulation of governing equation

The hydrodynamic pressure in excess of the hydrostatic pressure in the reservoir is governed by the equation of the compression waves given as follows:

$$\nabla^2 p = (1/c^2)(\partial^2 p/\partial t^2) \tag{1}$$

Where

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 \tag{2}$$

Corresponds to the two-dimensional Laplace operator in Cartesian coordinates with:

$$c = \sqrt{\lambda/\rho} \tag{3}$$

In equation (3), c represent the speed of sound waves in water, λ the Lame's modulus and ρ the mass density of water. Since we are assuming small deformations and considering the combined effects of compressibility and viscosity of the fluid in the reservoir, the linear visco-elastic Kelvin-Voigt model was adopted to represent the internal dissipation. Lame's modulus is then expressed by a complex valued function depending on the angular frequency of excitation w and it is given by Eq. (4).

$$\lambda^c = \lambda\,(1 + i2\eta\xi) \tag{4}$$

Where $\eta = wH/c$ is the dimensionless frequency; w is the angular frequency of the excitation and ξ the fraction of the critical damping of water. It is assumed that the dam vibrates as a rigid body with the same horizontal ground acceleration given as follows:

$$\ddot{U}_g(t) = e^{iwt} \tag{5}$$

The hydrodynamic pressure in the reservoir can be given in the frequency domain as:

$$p = P(x, y, w)\, e^{iwt} \tag{6}$$

Substituting Eq. (6) in Eq. (1) provides the Helmholtz differential equation of compression waves in the water, where $K = w/c$, corresponds to the compression wave number.

$$\partial^2 P/\partial x^2 + \partial^2 P/\partial y^2 + K^2 P = 0 \tag{7}$$

Boundary conditions

1. On the upstream part of the dam enclosed by the contour $S_1 \cup S_2$, it is assumed that the hydrodynamic pressures gradient in the direction normal to the upstream face of the dam and the inertial forces generated in the mass of the water are in a state of equilibrium, which allows us to write:

$$\partial P\,(x, y, w)/\partial n|_{S_1 \cup S_2} = -\rho \ddot{U}_n \quad (8)$$

$\ddot{U}_n$ is the normal component of the horizontal ground acceleration.

2. At the free surface of water, we assume that:

$$P(x, H, w)|_{S_3} = 0 \quad (9)$$

3. At the limit of truncation S_4, supposed far enough from the dam upstream face (when L ≥ 3H; L is the length of the reservoir), we assume that

$$P\,(\infty, y, w)|_{S_4} = 0 \quad (10)$$

4. At the reservoir bottom, the gradient of associated hydrodynamic pressures is also zero:

$$\partial P\,(x, 0, w)/\partial y|_{S_5} = 0 \quad (11)$$

Analytical expression for distribution of the total shear forces

The hydrodynamic pressure $P(x, y, w)$ is given by the following relationship:

$$P = \sum_{i=1}^{+\infty} A_i\, T_i(x, y, w) = C_s \gamma H C_p \quad (12)$$

with

$$T_i(x, y, w) = e^{-\mu_i x} \cos \lambda_i y \quad (13)$$

Where, $T_i(x, y, w)$ define the natural water modes of vibration in the reservoir propagating horizontally, CS = $\ddot{U}_n/g$, g is the acceleration of gravity, γ is the unit weight of water and C_p the pressure coefficient. Here, C_p is approximated by a series of complex functions of real variable y belonging on the compact interval I= [0, H] as follows:

$$C_p(y)\big|_{S_1 \cup S_2} = (1/C_s \gamma H)\, \sum_{i=1}^{+\infty} A_i\, e^{-\mu_i x} \cos \lambda_i y \quad (14)$$

$$\textit{Where} \quad \lambda_i = (2i-1)\pi/2H \qquad \textit{and} \qquad \mu_i = \sqrt{\lambda_i^{\,2} - K^2} \quad (15)$$

A_i, correspond to the unknown coefficients with i = 1, 2, …, ∞. They are obtained after solving a system of linear equations given by the relation below (Eq. (16)) using a numerical calculation program (MATLAB).

$$[F_{ji}]\{A_i\} = \{G_j\} \quad \forall\, j, i = 1, 2, \ldots, \infty \quad (16)$$

The elements of the Hermitian matrix $[F_{ji}]$ and the column vector $\{G_j\}$ are calculated as defined in (Avilés, 1998).

The distribution of the horizontal component of the total shear forces along $S_1 \cup S_2$ is given as follow:

$$F_h(y) = \int_S C_s \gamma H C_p \; ds \tag{17}$$

ds represent infinitely small segment of the $S_1 \cup S_2$ boundary. Substituting Eq. (14) into Eq. (17) yields:

$$F_h(y) = \int_y^H \sum_{i=1}^{+\infty} A_i \, e^{-\mu_i x} \cos \lambda_i y \, dy \tag{18}$$

When we interchanges between the operator (∫) and the operator (Σ), the Eq. (18) takes the following form:

$$F_h(y) = \sum_{i=1}^{+\infty} A_i \int_y^H e^{-\mu_i x} \cos \lambda_i y \, dy \tag{19}$$

After successive integrations, we finally obtain:

$$for\; y \in [0, CH] \qquad F_h(y) = \sum_{i=1}^{+\infty} A_i \begin{Bmatrix} -e^{-\gamma_i (CH-y)} F_i(y) \\ +F_i(CH) \\ + \frac{[\sin \lambda_i H - \sin \lambda_i CH]}{\lambda_i} \end{Bmatrix} \tag{20}$$

$$And\; for\; y \in [CH, H] \qquad F_h(y) = \sum_{i=1}^{+\infty} A_i \{[\sin \lambda_i H - \sin \lambda_i y] / \lambda_i\} \tag{21}$$

$$With \quad F_i(y) = (\lambda_i \sin \lambda_i \, y + \mu_i \tan\theta \cos \lambda_i y) / \alpha_i^+,$$

$$F_i(CH) = (\lambda_i \sin \lambda_i \, CH + \mu_i \tan\theta \cos \lambda_i CH) / \alpha_i^+,$$

$$\gamma_i = \mu_i \tan\theta, \quad \alpha_i^+ = \lambda_i^2 + \gamma_i^2 \;\; and \;\; \alpha_i^- = \lambda_i^2 - \gamma_i^2 \tag{22}$$

Analytical expression for distribution of the overturning moments

The distribution of the total overturning moments about the Z-axis at any elevation *y* is defined as:

$$M_z(y) = \int_S F_h(y) \, ds \tag{23}$$

After successive integrations, Eq. (23) becomes:

for $y \in [0, \mathrm{CH}]$

$$M_Z(y) = \sum_{i=1}^{+\infty} A_i \left\{ \begin{array}{c} e^{-\gamma_i(CH-y)} M_i(y) - M_i(CH) + \\ (CH-y) \begin{bmatrix} (\lambda_i sin\, \lambda_i CH + \gamma_i cos\, \lambda_i CH)/\alpha_i^{+} \\ +(sin\, \lambda_i H - sin\, \lambda_i CH)/\lambda_i \end{bmatrix} \\ + \frac{[\lambda_i H(1-C) sin\, \lambda_i H - cos\, \lambda_i CH]}{\lambda_i^2} \end{array} \right\} \quad (24)$$

And for y ∈ [CH, H]

$$M_z(y) = \sum_{i=1}^{+\infty} A_i \left\{ [\lambda_i (H-y) sin\, \lambda_i H - cos\, \lambda_i y]/\lambda_i^2 \right\} \quad (25)$$

$$With: \; M_i(y) = [m_i\, sin\, \lambda_i y + n_i cos\lambda_i y]/\alpha_i^{+2},$$

$$M_i(CH) = [m_i\, sin\, \lambda_i CH + n_i cos\lambda_i CH]/\alpha_i^{+2},$$

$$m_i = 2\lambda_i \gamma_i \;\; and \;\; n_i = -\alpha_i^{-}. \quad (26)$$

RESULTS AND DISCUSSION

In order to implement the proposed analytical formulas, a computer program in Matlab language (Tadjadit, 2013) was modified to incorporate, in the frequency-domain, the effects of compressibility and viscosity of water in the reservoir. Results obtained for total shear forces and overturning moments are expressed, respectively in terms of dimensionless coefficients $C_F = |F_h|/F_{st}$ and $C_M = |M_z|/M_{st}$, in which $|F_h|$ and $|M_z|$ are the modulus of the complex frequency responses of F_h and M_z. $F_{st} = \rho g H^2/2$ and $M_{st} = \rho g H^3/6$ are respectively, the total hydrostatic force and the corresponding overturning moment at the base of the dam. They are, respectively expressed, in Newton and Newton-meter per unit of width of the dam.

The results obtained were presented for a wide range of values of the dimensionless frequency η and different damping ratios ξ of the water. The characteristic parameters of the dam-reservoir system are $\rho = 1000\ kg/m^3$ and $C_s = 0.1$. In figures 2 and 3, both real and imaginary parts of C_F are presented for the dimensionless frequencies $\eta = 0$, 1.5, 3 and 6. We can easily see the effect of the excitation frequency on the response of the system. When η exceeds $\pi/2$ ($w > w_1$, where w_1 represent the first fundamental frequency of the reservoir), the response is complex valued with the imaginary part representing the loss of energy in waves moving away from the dam.

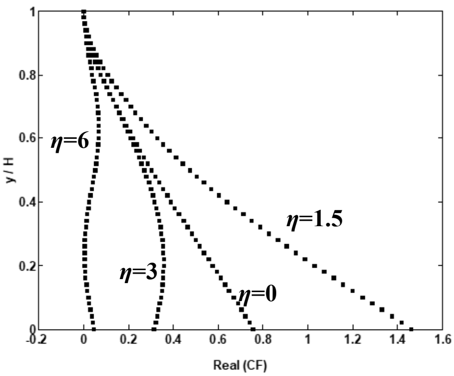

Figure 2: Real parts of total shear forces on dam with $\theta = 37.6°$ $C = 0.75$ and $\xi = 0$ %

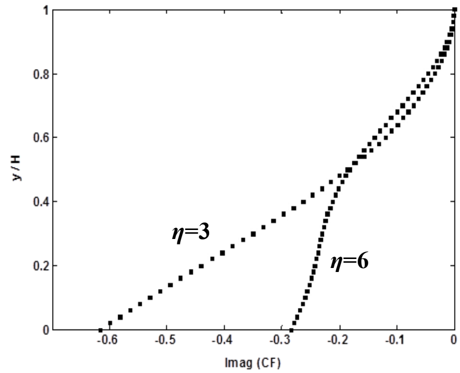

Figure 3: Imaginary parts of total shear forces on dam with $\theta = 37.6°$ $C = 0.75$ and $\xi = 0$ %

The same reasoning can be adopted for the distribution of the total overturning moments (Figures 4 and 5).

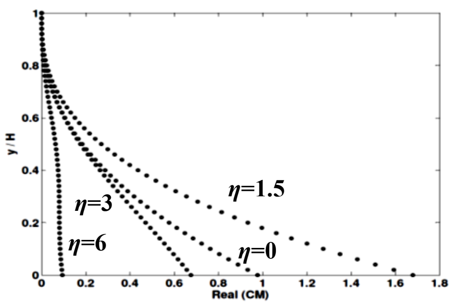

Figure 4: Real parts of total overturning moments on dam: $\theta = 37.6°, C = 0.75, \xi = 0$ %

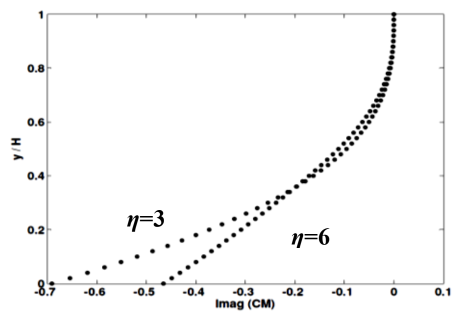

Figure 5: Imaginary parts of total overturning moments on dam: $\theta = 37.6°, C = 0.75, \xi = 0$ %

Now, to evaluate the combined effects of compressibility and viscosity of water, another example is given for, respectively the cases of a vertical and partially inclined dams. Figures 6 and 7 shows the variation of the dimensionless coefficient C_F with the frequency ratio w/w_1 for damping ratios $\xi = 1\%$ and $\xi = 5\%$. It is seen that the effect of water viscosity can be considered negligible insofar as the excitation frequency is not very close to that of the fundamental modes of the reservoir.

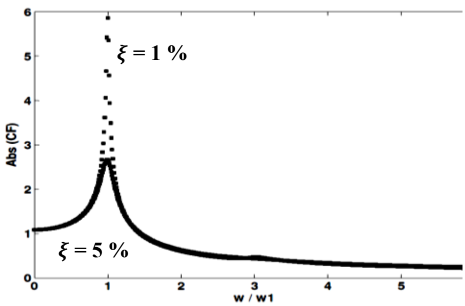

Figure. 6 Total shear forces on dam with a vertical upstream face for $\xi = 1\,\%$ and $\xi = 5\,\%$

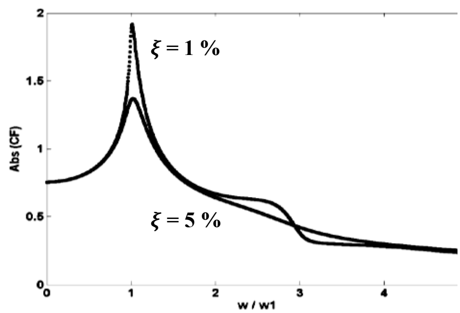

Figure. 7 Total shear forces on a partially inclined dam $\theta = 37.6°$, $C = 0.75$, $\xi = 1\,\%$ and $\xi = 5\,\%$

CONCLUSION

Analytical expressions for the determination of hydro-seismic forces acting on a rigid dam with irregular upstream face geometry in presence of a compressible viscous fluid are derived through a linear combination of the natural modes of water in the reservoir based on a boundary method making use of complete sets of complex T-functions. They show clearly the separate and combined effects of compressibility and viscosity of water. The study was then extended to the case of a rigid dam with irregular geometry in presence of compressibility and viscosity of the water. When compressibility effect is neglected, the percent errors, in the present study, are found to be in the order of 14–17 % for shear forces and less than 13 % for overturning moments. In general, the effect of viscosity of the water may be neglected insofar as the frequency of the seismic excitation is not very close to that of the natural modes of vibration of the reservoir. However, at the resonance frequency, the generalized seismic forces are controlled essentially by the damping ratio of the water in the reservoir. The formulas obtained for distributions of both shear forces and overturning moments are simple, computationally effective and useful for the preliminary design of dams. They also have the advantage of being able to cover a wide range of excitation frequencies even beyond the cut-off frequencies of the natural water modes of the reservoir.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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APPENDIX A

SPECIAL CASES FOR THE DISTRIBUTIONS OF $\boldsymbol{F}_{\mathbf{h}}(\mathbf{y})$ AND $\boldsymbol{M}_{\mathbf{Z}}(\mathbf{y})$

Rigid dam with sloping upstream face

$$F_h(y) = \sum_{i=1}^{+\infty} A_i \left[-e^{-\gamma_i(H-y)} F_i(y) + F_i(H)\right] \quad \text{(A1)}$$

With $\quad F_i(y) = \frac{[\lambda_i \sin\lambda_i\, y + \mu_i \tan\theta\, \cos\lambda_i y]}{\alpha_i^+}$

$$F_i(H) = [\lambda_i \sin\lambda_i\, H + \mu_i \tan\theta\, \cos\lambda_i H]/\alpha_i^+$$

$$M_Z(y) = \sum_{i=1}^{+\infty} A_i \begin{bmatrix} e^{-\gamma_i(H-y)} M_i(y) - M_i(H) \\ +[\lambda_i (H-y) \sin\lambda_i H]/\alpha_i^+ \end{bmatrix} \quad \text{(A2)}$$

With $\quad M_i(y) = \frac{[m_i \sin\lambda_i y + n_i \cos\lambda_i y]}{\alpha_i^{+2}}$

and $\quad M_i(H) = m_i \sin\lambda_i H/\alpha_i^{+2}$

Rigid dam with vertical upstream face

In this case, we have: $\alpha_i^+ = \alpha_i^- = \lambda_i^2$ and $\gamma_i = 0$

$$F_h(y) = \sum_{i=1}^{+\infty} \frac{A_i}{\lambda_i} [\sin\lambda_i H - \sin\lambda_i y], \quad \text{(A3)}$$

$$M_Z(y) = \sum_{i=1}^{+\infty} \frac{A_i}{\lambda_i^2} [-\cos\lambda_i y + \lambda_i (H-y) \sin\lambda_i H] \quad \text{(A4)}$$

Maximum values of the total shear forces and overturning moments

The maximum values of the total shear forces and the associated overturning moments are given at the base of the dam (y = 0) as follows:

$$F_h(0) = \sum_{i=1}^{+\infty} A_i \begin{Bmatrix} -\mu_i \tan\theta\, e^{-\gamma_i CH}/\alpha_i^+ + F_i(CH) + \\ [\sin\lambda_i H - \sin\lambda_i CH]/\lambda_i \end{Bmatrix} \quad \text{(A5)}$$

$$M_Z(0) = \sum_{i=1}^{+\infty} A_i \begin{Bmatrix} e^{-\gamma_i CH} M_i(0) - M_i(CH) + \\ CH \begin{bmatrix} (\lambda_i \sin\lambda_i CH + \gamma_i \cos\lambda_i CH)/\alpha_i^+ \\ +(\sin\lambda_i H - \sin\lambda_i CH)/\lambda_i \end{bmatrix} \\ + \frac{[\lambda_i H(1-C)\sin\lambda_i H - \cos\lambda_i CH]}{\lambda_i^2} \end{Bmatrix} \quad \text{(A6)}$$

With $\quad M_i(0) = n_i/\alpha_i^{+2}$