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PERFORMANCE EVALUATION OF CSARAO-1 HYBRID ALGORITHM ON ESTIMATING SPATIALLY DISTRIBUTED HYDRAULIC CONDUCTIVITY TENSORS IN A STEADY-STATE GROUNDWATER FLOW MODEL

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ABSTRACT

The present work compares the performance of three recent metaheuristic algorithms which are crow search algorithm, Rao-1 algorithm and the CSARao-1 hybrid algorithm in identifying spatially distributed hydraulic conductivities in a steady-state groundwater flow model. The employed algorithms were benchmarked on a hypothetical problem, and the identification process was conducted by combining each algorithm with the corresponding finite element model. Two inverse problems were analyzed; the first was based on errorless hydraulic head measurements, and the second on noisy measurements. The CSARao-1 hybrid algorithm was found to be the most efficient in terms of accuracy, robustness and speed of convergence.

Keywords: Metaheuristics, Finite element, Performance, Optimization, Hydraulic conductivity.

INTRODUCTION

Steady flow of water through confined aquifers is governed by an elliptic partial differential equation (PDE) that contains spatially distributed physical parameters termed hydraulic conductivities that should be accurately identified to construct a reliable flow model. Their identification consists of resolving an inverse problem which is generally performed by linking an optimization algorithm to a numerical model that solves the corresponding PDE over the flow domain. The purpose of this linkage is to estimate the hydraulic conductivities of the flow domain by minimizing an objective function that measures the discrepancies between the measured (observed) state variables (i.e hydraulic head) and those computed using the numerical model. The objective functions encountered in such optimization problems are non-convex, highly nonlinear and complexes, therefore, the best choice is to avoid the use of deterministic optimizers and to employ metaheuristics as optimization algorithms for the qualities they offer such as derivation free mechanism, local optima avoidance, flexibility (Mirjalili et al., 2014). The field of optimization has been continuously flooded with new metaheuristic algorithms ever since the introduction of the ''no free lunch theorem'' (Wolpert and Macready, 1997). This theorem asserts that there is no universally efficient algorithm capable of solving all optimization problems. Over the past decade, an average of 38 algorithms per year have emerged (Rajwar et al., 2023), and most of them draw inspiration from nature. This trend is driven by the recognition that different optimization problems necessitate distinct techniques, as certain algorithms are more suitable for specific types of optimization than others. Metaheuristic algorithms are in majority population-based and have two main characteristics: exploration (diversification) and exploitation (intensification). Exploration is responsible in exploring the search space in its totality in order to find promising regions that may contain the global optimum, while exploitation intensifies the search in the neighborhood of the founded promising regions. Apart from introducing new algorithms, the hybridization of metaheuristics could create an even more efficient algorithm in terms of accuracy, speed of convergence and robustness. The present work evaluates for the first time the performance of three recent optimizers on identifying spatially distributed hydraulic conductivity tensors of a finite-element-steady-state groundwater flow model, which are: the crow search algorithm (CSA) (Askarzadeh, 2016), Rao-1 algorithm (Rao, 2020) and the CSARao-1 hybrid algorithm (Tadj et al., 2021). Note that the latter algorithm has been developed to make the two formers work in a synergic way to achieve better performance. The remainder of the paper has the following structure: Section 2 briefly presents the proposed CSARao-1 hybrid algorithm. Section 3 describes the finite-element-steady-state groundwater flow model. Section 4 is dedicated to the results and discussion. Conclusion of the present study is provided in Section 5.

THE CSARAO-1 HYBRID ALGORITHM

Like all population-based algorithms, CSARao-1 hybrid algorithm (Tadj et al., 2021) iteratively guides a population of candidate solutions (crows) towards the global optimum. It hybridizes two recent optimization algorithms which are CSA (Askarzadeh, 2016) and Rao-1 algorithm (Rao, 2020), and was proposed to address the shortcomings of the CSA that are slow convergence and poor robustness (Han et al., 2020). The weakness of the CSA can be summarized in two points: (1) the main CSA's search mechanism does not invoke the best solution found so far (available in the memory matrix at current iteration) in the calculation of the next solution which weakness its ability in exploiting promising regions in the search space, and (2) the random search mechanism when called generates a random new solution in the search space which slows down the algorithm's convergence. Hence, CSARao-1 hybrid algorithm replaces the CSA's random search mechanism by the Rao-1 search mechanism. This new search mechanism is exploitative and is free of any tuning parameters. Instead of generating a random solution, it exploits the knowledge available in the memory matrix which enhances the new hybrid algorithm's local search; and this is the essence of the proposed CSARao-1 hybrid algorithm. Fig. 1 shows the pseudo-code of the proposed CSARao-1 algorithm, where N is the population size, d is the number of parameters being optimized, fl is the flight length, AP is the awareness probability, $iter$ denotes the current iteration, $iter_{max}$ is the maximum number of iteration, rand [1,0] refers to random numbers, x (N,d) is the matrix of candidate solutions, and m (N,d) denotes memory matrix. CSARao-1 hybrid algorithm has proved its superiority over other competitive optimization techniques in terms of accuracy, speed of convergence and robustness in analyzing transient time-drawdown data in leaky and confined aquifers (Tadj et al., 2021), and in this note we extend its applicability in identifying spatially distributed physical parameters of an elliptic PDE.

```
Input: N, d, fl, AP, search space's bounds
Generate a random initial population of crows
Evaluate the objective function of each crow
Initialize the memory of each crow
DO iter = 1, iter_max
    DO i = 1, N
        Randomly select a crow k ∈ [1, N] to follow
        Do j=1, d
            IF (rand_k[0,1] ≥ AP ) THEN
                x_{i,j,iter+1} = x_{i,j,iter} + rand_i[0,1] × fl × (m_{k,j,iter} − x_{i,j,iter})
            ELSE
                x_{i,j,iter+1} = x_{i,j,iter} + rand_j[0,1] × (m_{best,j,iter} − m_{worst,j,iter})
            END IF
        END DO
    END DO
Verify the feasibility of the new candidate solutions
Evaluate the objective function of each candidate solution
Update the Memory matrix
END DO
```

Figure 1: FORTRAN style pseudo-code of the CSARao-1 hybrid algorithm (Tadj et al., 2021).

FINITE-ELEMENT-STEADY-STATE GROUNDWATER FLOW MODEL

A steady-state two-dimensional flow through a heterogeneous and anisotropic confined aquifer (Fig. 2a), governed by the following elliptic PDE is considered is this study.

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right) = Q_w.\,\delta(x_w, y_w) \tag{1}$$

where h [L] represents the hydraulic head; the terms $k_x(x,y)$ and $k_y(x,y)$ [L^2/T] are the diagonal elements of the hydraulic conductivity tensor in x direction and in y-direction, respectively; Q_w [L^3/T] represents steady source/sink points; δ is Kronecker delta function which equals to 1 in the presence of injection/pumping well at x_w, y_w [L], otherwise, it is taken as zero. The flow domain consists of two zones having constant hydraulic conductivities: k_{1x}, k_{1y} for zone 1 , and k_{2x} , k_{2y} for zone 2. The mathematical model is solved by the finite elements method that consists of replacing the continuous system by an equivalent discrete one. In this work, the flow domain was meshed using four nodes quadrilateral elements (Fig 2b). The finite element formulation of the governing PDE was obtained using Galerkine's weighted residual method (Smith et al., 2014; Hutton, 2008). The problem treated in this work is hypothetic in order to eliminate the sources of uncertainty due to measurements of the hydraulic head.

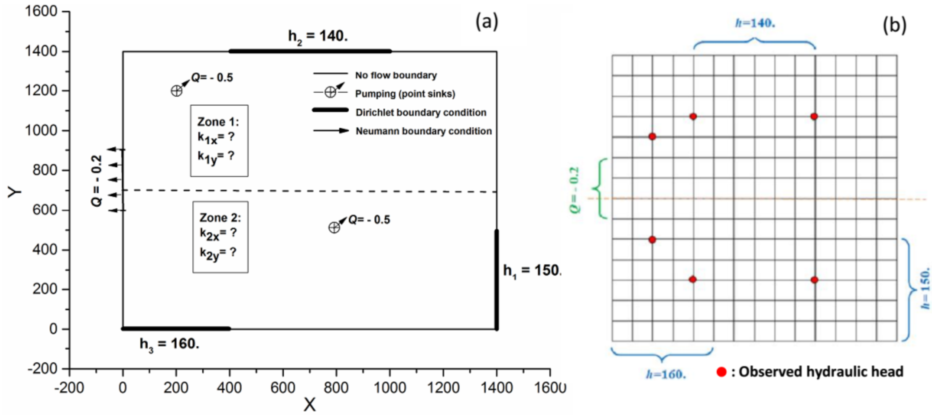

Figure 2: Hypothetical problem: (a) flow domain (b) mesh and location of observed hydraulic heads.

RESULTS AND DISCUSSION

A direct finite element simulation with a set of hydraulic conductivities provided the inversion measurements, which are the **errorless** observed hydraulic heads, and their use allows us to test the ability of the selected metaheuristic algorithms to identify the physical parameters that were used for their calculation. The errorless hydraulic heads were then perturbed with a Gaussian noise of mean μ=0 and standard deviation $\sigma = \pm 0.05$, to simulate the measurement errors that can be made in the field and to check the stability of the identification process. The simulation-optimization linkage (Figure 3) consists of incorporating the finite-element simulation model into the optimization scheme. While the simulation model calculates the hydraulic heads, the optimization algorithm is responsible for the accuracy of the hydraulic conductivities to be identified. The objective function to be minimized is the sum of squared errors (*SSE*) between the observed hydraulic heads and those computed by the considered numerical model. Since the employed algorithms are stochastic, each one was independently executed 30 times, and the standard deviation (*SD*) was used as an index to assess the algorithms robustness; the most robust algorithm is the one that provides the same results at every run and thus has the lower *SD* values. In this work, the population size *N* was set equal to 60. The lower and upper bounds of the search space are 10E-10 and 0.1, respectively. The flight length *fl* and the awareness probability *AP* were set equal to 2 and 0.1, respectively, and the maximum number of iterations was fixed to 500. The results are presented in terms of mean values. The reported values were rounded to few significant digits; the whole precision cannot be presented, this is common to avoid reporting too many digits, but the most accurate values were written in bold.

It can be seen from tables 1 and 2 that the hydraulic conductivities were successfully identified by the three employed algorithms even for a limited number of observation points (6 points). As we can see, the relative errors rise when a white noise is added to hydraulic head measurements. With a standard deviation *SD*=0, CSARao-1 hybrid algorithm was the most robust, followed by CSA. The identified conductivities by the three algorithms using noisy measurements were inserted in the direct simulation model to plot the identified hydraulic head. As Fig. 4 shows, the hydraulic head maps provided by the inverse problems match well with the reference hydraulic head map.

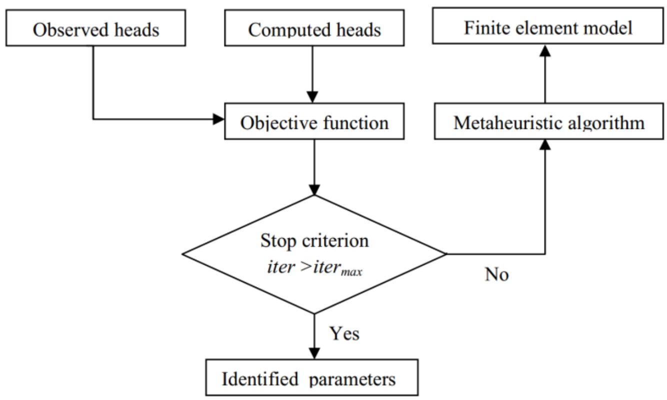

Figure 3: Simulation–optimization linkage, (b): Reference hydraulic map and identified hydraulic maps using noisy measurements.

Table 1: Identified hydraulic conductivities (errorless measurements*)*

Algorithms	**Exact Parameters**	**Identified parameters**	**Relative errors (%)**	***SD***
CSA	k1x = 0.02	k1x = 0.0200	0.04	3.37E-13
	k1y = 0.05	k1y = 0.0500	0.08	
	k2x = 0.06	k2x = 0.0601	0.10	
	k2y = 0.01	k2y = 0.0100	0.01	
Rao-1	k1x = 0.02	k1x = 0.0200	0.04	1.26E-10
	k1y = 0.05	k1y = 0.0500	0.08	
	k2x = 0.06	k2x = 0.0601	0.10	
	k2y = 0.01	k2y = 0.0100	0.01	
CSARao-1	k1x = 0.02	k1x = 0.0200	0.04	0.
	k1y = 0.05	k1y = 0.0500	0.08	
	k2x = 0.06	k2x = 0.0601	0.10	
	k2y = 0.01	k2y = 0.0100	0.01	

The convergence curves are presented in Fig. 5. It can be seen that CSARao-1 hybrid algorithm converged faster towards the global optimums; and this is because of its enhanced exploration-exploitation balance. CSA ranks second in terms of speed of convergence.

Table 2: Identified hydraulic conductivities (noisy measurements*)*

Algorithms	Exact Parameters	Identified parameters	Relative errors (%)	*SD*
CSA	k1x = 0.02	k1x = 0.0202	0.84	1.32E-18
	k1y = 0.05	k1y = 0.0509	1.89	
	k2x = 0.06	k2x = 0.0596	0.74	
	k2y = 0.01	k2y = 0.0098	2.47	
Rao-1	k1x = 0.02	k1x = 0.0202	0.84	9.53E-11
	k1y = 0.05	k1y = 0.0509	1.89	
	k2x = 0.06	k2x = 0.0596	0.74	
	k2y = 0.01	k2y = 0.0098	2.47	
CSARao-1	k1x = 0.02	k1x = 0.0202	0.84	0.
	k1y = 0.05	k1y = 0.0509	1.89	
	k2x = 0.06	k2x = 0.0596	0.74	
	k2y = 0.01	k2y = 0.0098	2.47	

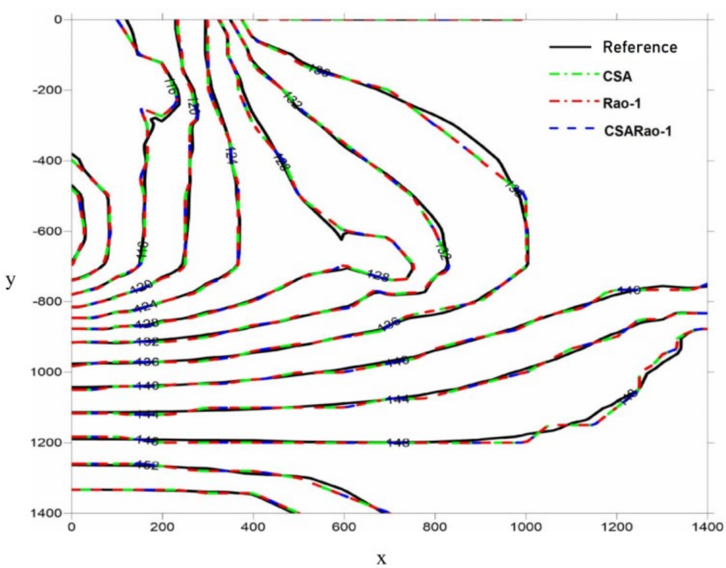

Figure 4: Reference hydraulic map and identified hydraulic maps using noisy measurements.

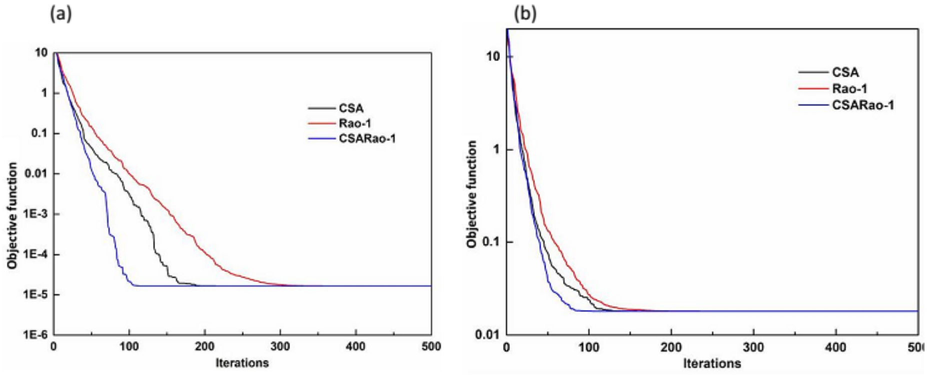

Figure 5: Convergence curves: (a): errorless measurements, (b): noisy measurements.

CONCLUSION

This study assessed the performance of three recent metaheuristic algorithms in estimating coefficients of an elliptic PDE that governs steady flows through porous media. Three optimization frameworks were developed by linking each algorithm to the corresponding finite element model, and the purpose of these linkages was to minimize the misfit between observed and computed hydraulic heads. The results obtained by these frameworks were compared with each other in terms of accuracy, speed of convergence and robustness. Overall, the CSARao-1 hybrid algorithm exhibited better accuracy, speed of convergence and robustness. CSA ranked second followed by Rao-1 algorithm. Finally, the application of CSARao-1 hybrid algorithm may be extended to further real-world engineering problems.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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