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HYDRAULIC JUMPS IN A STRAIGHT RECTANGULAR COMPOUND CHANNEL: THEORETICAL APPROACH AND EXPERIMENTAL STUDY

RESSAUTS HYDRAULIQUES DANS UN CANAL COMPOSÉ RECTANGULAIRE DROIT: APPROCHE THÉORIQUE ET ÉTUDE EXPÉRIMENTALE

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ABSTRACT

In this paper, theoretical developments, regarding the establishment of dimensionless relationships for sequent depths ratio and relative energy loss of hydraulic jumps are achieved in a straight rectangular compound channel. These relationships were given with and without consideration of a volume force *Fx*, which is assimilated by analogy to Borda-Carnot's expression. The Experiment was carried out with three different values of the width ratio τ_y. For each τ_y ratio, several values of inflow Froude number were considered according to the five inflow ratio depths' values τ_z. The experiments proved the validity of the proposed theoretical relationships. The study showed the need to consider the force *Fx* when the ratio τ_y reaches the value of 0.5. It reveals also the practical usefulness of the compound channel in terms of energy dissipation capability compared to the rectangular channel.

Key words: compound channel; depth ratio; energy dissipation; hydraulic jump; volume force; width ratio.

RÉSUMÉ

Dans cet article, les relations adimensionnelles régissant les rapports des hauteurs conjuguées et les pertes relatives d'énergie ont été établies pour le ressaut hydraulique dans un canal composé rectangulaire droit. Ces relations sont présentées, en tenant compte ou non, de la force de volume *Fx* dont l'expression a été déduite par analogie à la relation de Borda-Carnot. L'expérimentation a été menée en considérant trois valeurs différentes de la largeur relative τ_y. Pour chaque valeur de τ_y, plusieurs valeurs du nombre de Froude incident ont été considérées en fonction des cinq rapports de la profondeur initiale relative τ_z. Les expériences ont démontré la validité des relations proposées. L'étude a montré la nécessité de prendre en considération la force *Fx* lorsque le rapport τ_y atteint la valeur 0,5. Il a été également mis en évidence l'intérêt pratique du canal composé rectangulaire en termes de capacité de dissipation d'énergie, comparé au canal rectangulaire.

Mots clés: canal composé, dissipation d'energie, force de volume, rapport des hauteurs, rapport des largeurs, ressaut hydraulique.

INTRODUCTION

The estimation of the various features of the flow in the compound channels has been the subject of intense and successful scientific contributions.

There have been many attempts to approach the theme, despite its inherent complexity. The difficulty of the problem is due to the implicit interaction between the floodplains and the main channel, characterizing every particular shape of compound channels.

The methods are based on the theoretical developments, and laboratory experimental validations, where the field conditions are simulated. Different flow aspects are studied with several of channels' shapes, (e.g., energy losses (Proust et al., 2010), critical depth (Liu et al., 2012), momentum transfer (Wang et al., 2007; Farooq et al., 2016), estimation of discharge (Yang et al., 2012), flow resistance (Yang et al., 2007), analytic stage-discharge (Liao and Knight, 2007), shear stress (Liu et al., 2013) and rapidly varied flow (Peltier et al., 2013).

Computational fluid dynamic models have more and more been used for compound channel cases, to predict the flow fields and progress the understanding of turbulent flow behavior, one-, two- and three-dimensional case studies were designed (e.g., (Morvan et al., 2008; Shekari et al., 2014; Babaali et al., 2015).

In order to estimate the characteristics of the flow for various compound channels with different setups, the experimental approaches are also coupled with stochastic modeling (e.g., (Al-Khatib and Gogus, 2014) and artificial intelligence methods (e.g., (Sahu et al., 2011; Azamathulla and Zahiri, 2012).

All the above-mentioned studies have focused on features of the flow in compound channels. According to our knowledge and after a long bibliographical research (e.g., (Rajaratnam, 1995; Beirami and Chamani, 2010; Habibzadeh et al., 2016), the hydraulic jump as an energy dissipator in the compound channels has not been considered yet.

In this context, the single theoretical attempt without experimental validation was given by Khattaoui and Achour (2012), while the present contribution is founded, at the same time, on the theoretical development and also supported by the laboratory experimentation on a physical model, which will be described further.

The main objective of our study is to present theoretical approaches and experimental investigations for the classical hydraulic jump, that occurs in a straight rectangular compound channel, where the Froude number is varied to produce different jump heights.

Theoretical developments regarding the establishment of dimensionless relationships for sequent depths of hydraulic jumps and their efficiencies in terms of energy dissipation will be given with and without taking into account the volume force. The former force is assimilated by analogy to Borda-Carnot's expression according to Achour (2000). It is a volume force, (source of "head loss"), due to the vertical enlargement, necessarily linked to the transfer of the mass and the momentum, between the main channel and the flood plain. The generated force cannot be ignored in the first analysis, unlike the frictional head loss.

Experiments were carried out with three different values; (1/4, 1/3 and 1/2), of the τ_y ratio of width of the main channel to the width of the flood plain. For each τ_y ratio, several values of inflow Froude number were considered according to five inflow ratio depths' values τ_z (0.167, 0.200, 0.253, 0.287 and 0.333); τ_z is the ratio of the first sequent depth to the depth of the flow in the main channel. The analysis of the lengths of hydraulic jumps is not considered in this work.

MATERIALS AND METHODS

Experimental setup

Fig. 1 shows a sketch of hydraulic jump in a straight rectangular compound channel. Fig. 2 and Fig. 3 show an overall view of the experimental installation. The experiments were performed in a 4m long straight rectangular compound channel, the main channel width and depth are 0.1 m, 0.15 m respectively. The flood plain has a 0.5 m total depth and an adjustable width. The main channel was made of sheets metal; it is surmounted by two vertical side walls; one is metallic, and the other is made of transparent plexiglass, allowing the visualization of the flow. In each of the experiments, water is supplied through a closed flume by an axial pump. The flows rates *Q's*, measured by an ultrasonic flowmeter, range between 9.9 l/s and 19 l/s. The incident flow h_1 is generated by a convergent box, to provide different initial flow depths and Froude numbers. The depths h_2 of the flow were measured using a point gauge with an accuracy of ± 0.5 mm, that was placed on rails at the top of the channel. The lengths *Lj* of jumps were evaluated with a graduated ribbon with an accuracy of ± 0.05 m. The experimentation is related to three cases; (1/4, 1/3 and 1/2) of the ratio between the main channel width and the flood plain one, where each ratio is corresponding to five initial depths; h_1 = 2.5 cm; 3.0 cm; 3.8 cm; 4.3 cm and 5 cm. The former parameter ranges are described in Table 1 and their designations in Table 2.

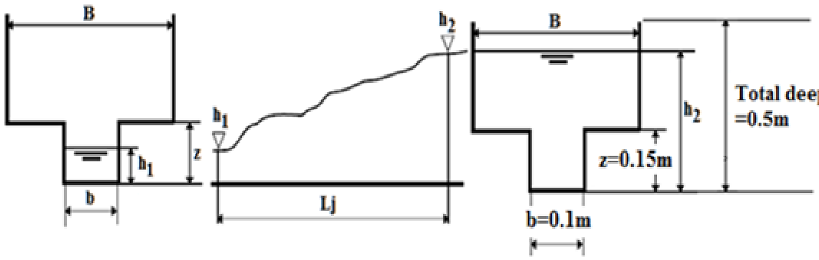

Figure 1: Sketch of the hydraulic jump in a straight rectangular compound channel

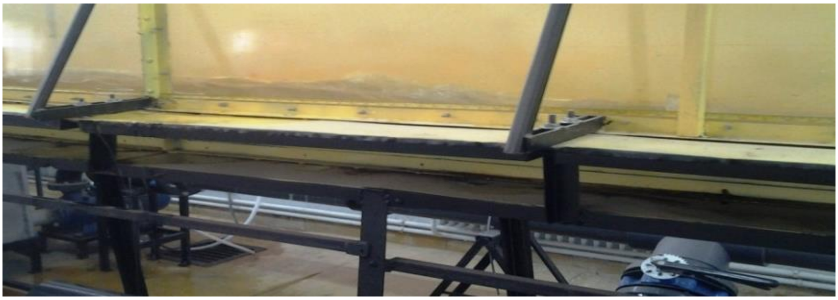

Figure 2: Experimental setup

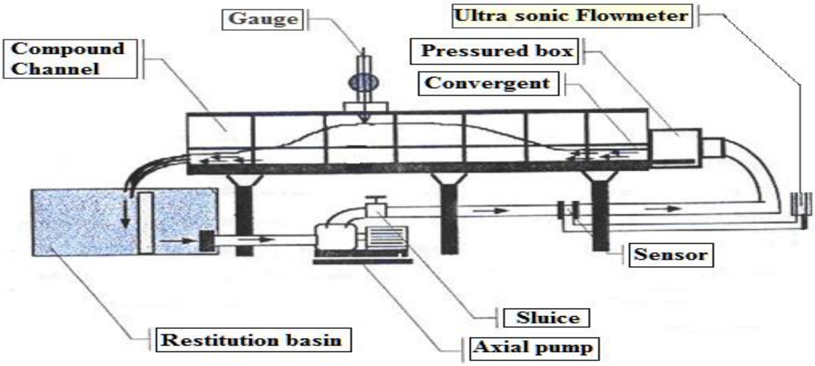

Figure 3: Schematic view of the experimental setup

Table 1: Experimental data range

Parameter	Range
First sequence depth (h_1)	2.5 - 5.0 cm
Second sequence depth (h_2)	24.5- 32.5 cm
Length of jump (Lj)	120 - 220 cm
Discharge (Q)	9.9 - 19.0 l/s
Inflow Froude Number (F_1)	4.28 - 10.02
Width ratio (τ_y)	0.25 - 0.5
Depth ratio (τ_z)	0.167 - 0.333

Table 2: Nomenclature

Notation	Parameter
A_1	Upstream cross-sectional area of flow
A_2	Downstream cross-sectional area of flow
B	Width of flood plain
b	Width of main channel
Q	Discharge
Fx	Volume force
g	Acceleration due to gravity
h_1	First sequent depth of flow
h_2	Second sequent depth of flow
F_1	Inflow Froude number $(Q^2/gb^2h_1^3)^{0.5}$
Lj	Length of jump
Y	Sequent depth ratio ($Y=h_2/h_1$)
τ_y	Width ratio ($\tau_y=b/B$)
τ_z	Depth ratio ($\tau_z=h_1/z$)
η	Relative energy loss

Theoretical approach for the sequent depths

In the analysis of the hydraulic jump phenomenon, the energy loss is noticeable and unknown while it is commonly assumed that the boundary shear stresses are negligible. Therefore, the momentum equation is a good approach. After analyzing the flow using the momentum equation, to find the sequent depths relationship h_1 and h_2 of the hydraulic jump, the energy equation can be used to find a function that defines the loss of energy (Houichi et al., 2013).

The theoretical equation, (the momentum one) which governs the hydraulic jump in a straight rectangular compound channel is first given without taking into account the influence of Fx, and then taking into consideration the proposed relation for Fx.

The first sequent depth remains entirely in the main channel, however the second one proceeds to spill over into the flood plain.

The first case allows writing:

$$\rho Q^2\left(\frac{1}{A_2}-\frac{1}{A_1}\right)=\rho g\left(\bar{h}_1A_1-\bar{h}_2A_2\right)-Fx \qquad (1)$$

With:

$$Fx \approx 0$$

And:

$$A_1 = b\, h_1 \tag{2}$$

$$A_2 = \left(h_2 - z\right)B + bz \tag{3}$$

$$\bar{h}_1 = \frac{h_1}{2} \tag{4}$$

$$\bar{h}_2 = \frac{\frac{B}{2}\left(h_2 - z\right)^2 + \left(h_2 - \frac{z}{2}\right)bz}{\left(h_2 - z\right)B + bz} \tag{5}$$

We can write after developing:

$$F_1^2 = \frac{1 - \frac{\left(Y - \frac{1}{\tau_z}\right)^2}{\tau_y} - \frac{\left(2Y - \frac{1}{\tau_z}\right)}{\tau_z}}{2\left(\frac{\tau_y}{Y - \frac{\left(1 - \tau_y\right)}{\tau_z}} - 1\right)} \tag{6}$$

For the second case and starting from Eq. (1), *Fx* is proportional to (*B*-*b*) and to h_1. According Achour (2000), one case analogous to Borda-Carnot's expression is reasonably given by:

$$Fx = \rho g \frac{h_1}{2}\left(h_2 - z\right)\left(B - b\right) \tag{7}$$

Finally, after several transformations, the Eq. (1) gives:

$$F_1^2 = \frac{1 - \frac{\left(Y - \frac{1}{\tau_z}\right)^2}{\tau_y} - \frac{\left(2Y - \frac{1}{\tau_z}\right)}{\tau_z} - \left(\frac{1}{\tau_y} - 1\right)\left(Y - \frac{1}{\tau_z}\right)}{2\left(\frac{\tau_y}{Y - \frac{\left(1 - \tau_y\right)}{\tau_z}} - 1\right)} \tag{8}$$

If $h_2 = z$ then $Y = \frac{1}{\tau_z}$

The Eqs. (6) and (8) become:

$$F_1 = \frac{\left(\tau_z + 1\right)^{0.5}}{\sqrt{2}\tau_z} \tag{9}$$

Eq. (9) gives the limit value of the inflow Froude number as if the jump is in a rectangular channel. This equation allows also to draw the curves, $Y = f\,(F_1)$, for different values of F_1 according to the limit values given in Table 3.

Table 3: Limit values of the inflow Froude number according τz (Eq. (9))

τ_z	0.167	0.200	0.253	0.287	0.333
F_1	4.583	3.873	3.125	2.798	2.449

The relative energy loss

After mathematical development, in the cases where *Fx* is negligible and not negligible, the relative energy loss is given by Eq. (19). This equation indicates that the relative energy loss in this study case is a dimensionless function, depending on the inflow Froude number F_1, the sequent depth ratio *Y* and of course on τ_y and τ_z.

$$\eta = \frac{\Delta H}{H_1} = \frac{H_1 - H_2}{H_1} = 1 - \frac{H_2}{H_1} \tag{10}$$

Where H_1 and H_2 are the specific energy before and after the jump respectively. They are given by:

$$H_1 = h_1 + \frac{Q^2}{2gA_1^2} \tag{11}$$

Taking into account the expression of the inflow Froude number (Table 2) and Eq. (2), thus Eq. (11) can be written as:

$$H_1 = h_1\left[1 + \frac{F_1^2}{2}\right] \tag{12}$$

And:

$$H_2 = h_2 + \frac{Q^2}{2gA_2^2} \tag{13}$$

Eq. (13) can be rewritten as:

$$H_2 = Yh_1 + \frac{Q^2 h_1}{2g\,\frac{A_2^2}{h_1^2}\,h_1^3} \tag{14}$$

Inserting Eq. (3) into Eq. (14), we may write:

$$H_2 = Yh_1 + \frac{Q^2 h_1}{2g\,\frac{\left[(h_2 - z)B + bz\right]^2}{h_1^2}\,h_1^3} \tag{15}$$

Eq. (15) can be also rewritten as:

$$H_2 = Yh_1 + \frac{Q^2 h_1}{2gb^2 h_1^3\left[(h_2/h_1 - z/h_1)(B/b) + z/h_1\right]^2} \tag{16}$$

Taking into account equations of F_1 and Y (Table 2), thus Eq. (16) can be written as:

$$H_2 = h_1\left[Y + \frac{F_1^2}{2\left\{\frac{Y}{\tau_z} - \frac{1}{\tau_z}\left(\frac{1}{\tau_y} - 1\right)\right\}^2}\right] \tag{17}$$

Considering Eq. (10), the relative energy loss is finally given by:

$$\eta = 1 - \frac{Y + \frac{F_1^2}{2\left(\frac{Y}{\tau_y} - \frac{1}{\tau_z}\left(\frac{1}{\tau_y} - 1\right)\right)^2}}{1 + \frac{F_1^2}{2}} \tag{18}$$

RESULTS AND DISCUSSION

Validation of the sequent depths relationship

The performance of each model (with *Fx* and without *Fx*) is evaluated by using root mean square error (*RMSE*) and the absolute of the maximum relative error (*ERMaxAb*) criteria. *RMSE* is the most commonly used performance criteria in modelling processes and the ideal value is zero.

The *RMSE* is computed based on the number of paired (*n*) theoretical values (V_{th}) and the experimental values (V_{exp}) as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(V_{th} - V_{exp}\right)^2}{n}} \tag{19}$$

Also, *ERMaxAb* given in (%) as:

$$ERMaxAb = Max\left[100\left|\frac{\left(V_{th} - V_{exp}\right)}{V_{th}}\right|\right] \tag{20}$$

The experimental validation of Eqs. (6) and (8), which respectively give the relationship of the sequent depths of the hydraulic jump in a straight rectangular compound channel, with and without *Fx*, is given by the results of the application of the adopted performance criteria defined by the Eqs. (19) and (20). These results are shown in Table 4.

Table 4, shows that the model performances are minimum when τ_y ratio is equal to 0.5, while considering the force *Fx*, given by the approach of Borda-Carnot according to Eq. (7). This finding is justified by the minimum values of *RMSE* and *ERMaxAb* which are 0.3054 and 6.47, respectively. Taking into account this deduction, the following figures show the layout of the theoretical and experimental values of *Y* according to F_1.

Table 4: Performances of each model regarding the values of τ_y (Eqs. (6) and (8))

τ_y	RMSE without *Fx*	RMSE with *Fx*	*ERMaxAb* without *Fx* (%)	*ERMaxAb* with *Fx* (%)
0.25	0.3041	0.5250	6.9	11.11
0.333	0.2794	0.4159	6.6	9.56
0.5	0.3814	0.3054	8.7	6.47

Fig. 4 gives the relationship between *Y* and F_1 for τ_y = 0.25 (without *Fx*), Fig. 5 gives the relationship between *Y* and F_1 for τ_y = 0.333 (without *Fx*) and Fig. 6 gives the relationship between *Y* and F_1 for τ_y = 0.5 (with *Fx*).

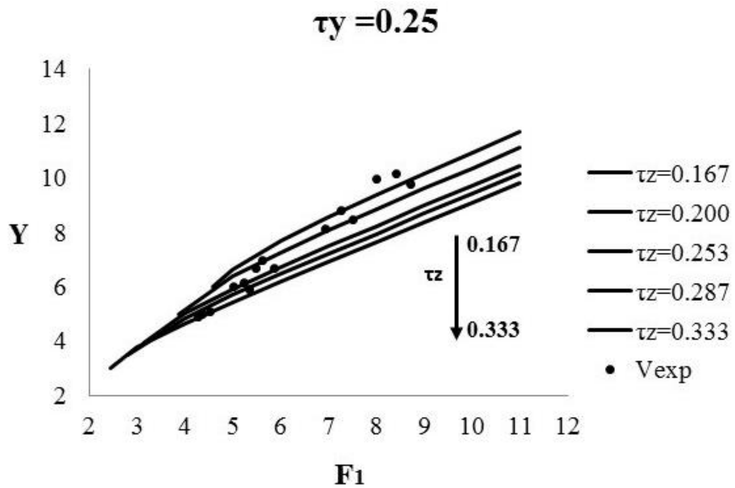

Figure 4: Sequent depth ratio versus the inflow Froude number for τ_y = 0.25 (without *Fx*)

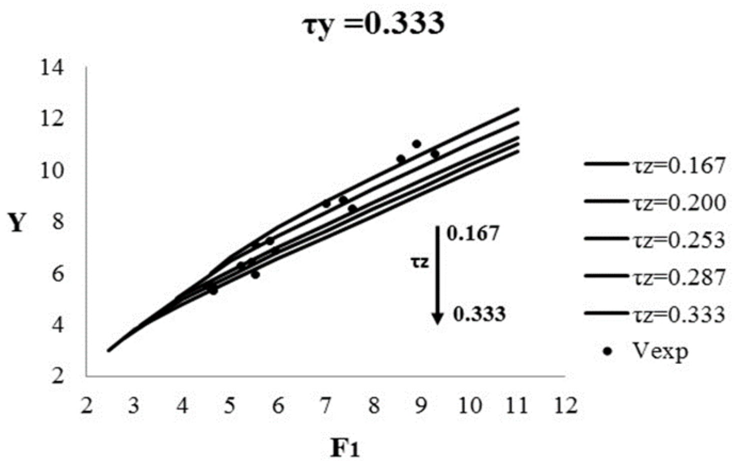

Figure 5: Sequent depth ratio versus the inflow Froude number for τ_y = 0.333 (without *Fx*)

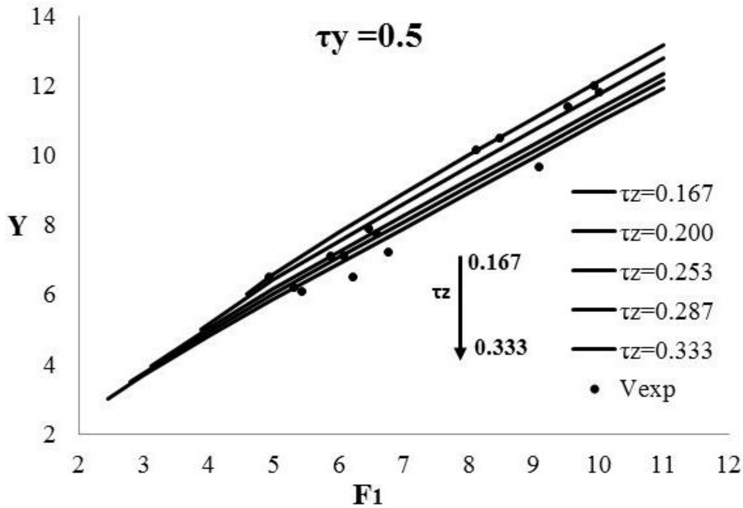

Figure 6: Sequent depth ratio versus the inflow Froude number for τ_y = 0.5 (with Fx)

Validation of the relative energy loss relationship

Eq. (18) allows calculating explicitly the relative energy loss of the hydraulic jump in the straight rectangular compound channel; knowing F_1 and Y for the different rate values τ_y and τ_z. It should be noted that the values of Y are calculated firstly with the Eq. (6) for τ_y = 0.25 and 0.333; secondly with the Eq. (8) for τ_y = 0.5. This consideration is required by the conditions of validating of relations (6) and (8).

The relative energy loss of the hydraulic jump in the straight rectangular compound channel is plotted against the inflow Froude number. In the experimental range of Froude number between 4 and 10, the relative energy loss is in a range of 51.4% and 76.01%.

In the same figure and in the same hydraulic flow conditions, the equivalent relative energy loss in a rectangular channel (η_{REC}), according to Eq. 22 (Chow, 1981), is also given for comparison.

$$\eta_{REC} = 1 - \frac{\left(8F_1^2+1\right)^{1.5} - 4F_1^2 + 1}{8F_1^2\left(2+F_1^2\right)} \tag{21}$$

Moreover, Fig. 7 shows that the compound channel far exceeds the rectangular channel, in terms of energy dissipation, indeed, for the same range F_1 between 4 and 10, the relative energy loss in the rectangular channel is between 42.3% and 72.7%, which means an improved dissipation of 6% on average.

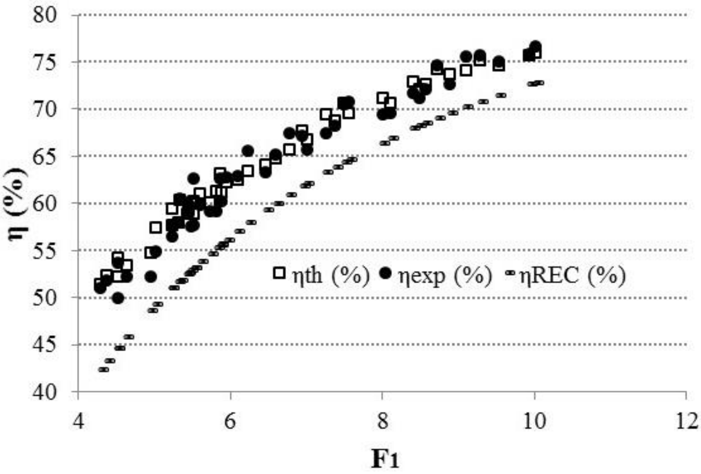

Figure 7: Relative energy loss versus the inflow Froude number; theoretical and experimental values in the straight rectangular compound channel, and in the rectangular one

The following figure, (Fig. 8) shows the evolution of theoretical and experimental relative energy loss. This evolution is surrounded between two envelopes (upper and lower limits) that prove the theoretical relationship thus established is validated experimentally with a maximum error not exceeding 5%, i.e. 4.83% exactly.

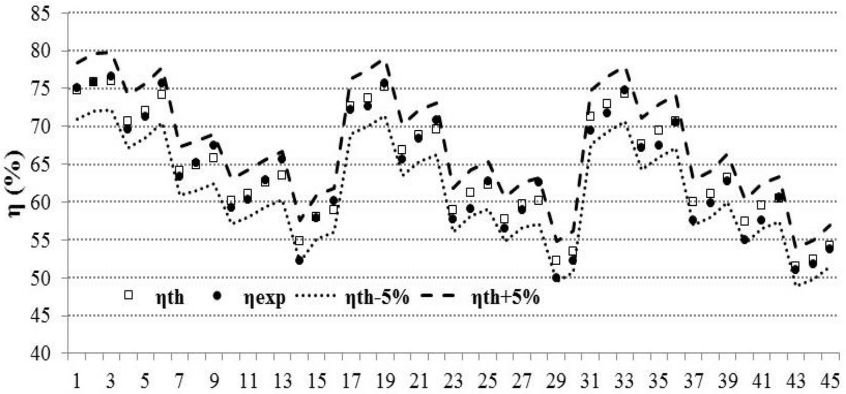

Figure 8: Evolution of the relative energy loss; theoretical and experimental values, between the two limits of ±5%

CONCLUSIONS

The hydraulic jump operating in a straight rectangular compound channel was examined theoretically and experimentally. The relationships for the sequent depths were given with and without consideration of the volume force F_x, which is assimilated by analogy to Borda-Carnot's expression as: $Fx = \rho g \frac{h_1}{2}\left(h_2 - z\right)\left(B - b\right)$. Experiments were carried out with three different values of the ratio τ_y: ratio between the main channel width and the flood plain one. The τ_y values were: (0.25, 0.333 and 0.5). For each τ_y ratio, five values of the inflow ratio depths' τ_z are considered; (0.167, 0.200, 0.253, 0.287 and 0.333); τ_z: ratio between the first sequent depth and of the main channel one.

The experimental investigation proved the validity of the theoretical relationships elaborated with the force *Fx* when the ratio τ_y is equal to 0.25 and 0.333 and without the force *Fx* when the ratio τ_y is equal to 0.5. This classification is based on minimum values of the root mean-square error and the maximum relative error absolute relative error, which are equal to 0.3054 and 6.47 respectively. The experimental validation of the relative energy loss is elaborated and not exceeding 4.83% as maximum error. The study also reveals the usefulness of the compound channel in terms of energy dissipation capability when it is compared with the referential rectangular channel.

For future research, we would like to point out that the relation (7) could be more accurate, it could be possibly replaced by a more realistic formulation, which might be determined by approximation.

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