



ON THE RECENT RISING OF SEA LEVEL FROM SATELLITE ALTIMETRY: TRENDS IN GLOBAL AND OCEANIC SCALES

SUR LA MONTÉE RÉCENTE DU NIVEAU DE LA MER À PARTIR DES DONNÉES DE ISSUES DE L'ALTIMÉTRIE SATELLITAIRE: TENDANCES À L'ECHELLE GLOBALE ET OCÉANIQUE

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ABSTRACT

In this paper, the seasonal and interannual variability of global and oceanic mean sea level variability are studied through a decomposition approach in the framework of the singular spectrum analysis (SSA) of available time series of sea level anomalies that extend back to 1993. The results show that the global mean sea level variability is dominated by an increasing trend. The rate of the global trend as seen by SSA appears to be about 3.36 mm/ year during the period 1993-2015. However, the trends from the different ocean regions show dissimilar patterns. The major contributions to the global sea level rise during 1993–2015 are from the Indian Ocean (3.80 mm/year).

Keywords: Sea level variability; Global and regional trends; Singular spectrum analysis; Seasonal-Trend Decomposition Based on LOESS.

RESUME

La variabilité saisonnière et interannuelle du niveau moyen de la mer à l'échelle globale comme aux échelles océaniques est étudiée par l'utilisation d'une approche de décomposition des séries temporelles disponibles depuis 1993 dans le cadre de l'analyse du spectre singulier (SSA). Les résultats montrent que la

variabilité du niveau moyen global des mers et océans est dominée par une tendance à la hausse. Le taux de la tendance globale observée par SSA semble être de 3.36 mm/an au cours de la période 1993-2015. Cependant, les tendances des différentes régions océaniques montrent des profils différents. Les principales contributions à l'élévation du niveau moyen global au cours de la période 1993-2015 proviennent de l'océan Indien (3.80 mm/an).

Mots clés : Variabilité du niveau de la mer; Tendances mondiale et régionales; Analyse du spectre singulier; régression LOESS.

INTRODUCTION

The "climate change" is generally a question of global phenomena that apply either to the whole earth or to large ensembles: hemispheres, lands, oceans, poles, Etc. The first sign of global warming is obviously the global rise in temperature (of the atmosphere as well as of the oceans). This change leads to particularly serious consequences like the rising of sea-water level by the thermal expansion of water and through the addition of water to the oceans from the melting of mountain glaciers, ice caps, and ice sheets. Actually, the spatial sampling offered by satellite altimetry and its continuity during the last years are major assets to provide an improved vision of the spatial and temporal oceanic variability. The continuity and accuracy of altimetry measurements allow even today to identify an accelerated rise in the mean level on a global scale. Altimeter measurements show that between 1993 and 2003, the mean global sea level amounted to 3.1 ± 0.7 mm / year (AVISO Altimetry, 2017a). Although the altimetry indicates a rise in the global mean sea level, the rise in the level of the oceans is far from uniform. In fact, while in certain ocean regions the sea level has indeed risen (by up to 20 millimetres a year in places), in others it has fallen an equivalent amount (AVISO Altimetry, 2017b). The main "suspects" factors which may be responsible for the nonuniformity in regional trends are: regional changes in thermal expansion, water mass adding to the oceans, and melting of continental ice which leads to the significant geographic variations in the sea level change (Jevrejeva et al., 2006).

This study aims to investigate the main behavior of the ocean surface variability at global and regional scales by use of the Singular Spectrum Analysis (SSA) technique. For this purpose, four sea level time series (1993-2015) have been investigated: Global mean sea level, Indian Ocean, Pacific Ocean and Atlantic Ocean. The result of the SSA processing is a decomposition of these time series into several components, which can be identified as trend, seasonalities and

noise components. The remainder of the paper is structured as follows. Section 2 provides a description of the used datasets; the used SSA approach is described in Section 3. Section 4 offers analysis and empirical findings of the paper by using SSA modeling with concluding remarks on the findings in Section 5.

DATA SETS

Radar altimeters permanently transmit signals to Earth and receive the reflected echo from the sea surface. The satellite orbit has to be accurately tracked, and its position is determined relative to a reference surface (an ellipsoid). The sea surface height (SSH) is calculated by subtracting the measured distance between the satellite-sea surface from the precise orbit of the satellite. The sea surface height anomalies (SSHA), defined as variations of the SSH with respect to a priori mean sea surface, are generally used as a precious and main indicator for development of scientific applications which aims to study the ocean variability (mesoscale circulation, seasonal variation, El Niño,...).

In our research, we investigate four SSHA time series from TOPEX, Jason-1, and Jason-2 altimeters. These time series are defined as the area averaged of SSHA at different scales: global scale, Atlantic Ocean, Indian Ocean and Pacific Ocean. These data sets were obtained from the University of Colorado at Boulder, version 2016 rel4, and are available under the KNMI Climate Explorer website. These datasets cover the period from January 1993 to the end of 2015, with a sampling rate of 1 month. The main input data for these time series processing are the Geophysical Data Records produced by NASA and CNES (TOPEX, Jason-1, Jason-2), which are therefore of the highest quality, notably in orbit determination. All of the standard corrections to the altimeter range were applied to the SSH including removal of ocean tides and an inverted barometer correction (Nerem et al., 2010). Figure 1 presents the four sea level time series (in millimeters): from the top to the bottom and from the left to the right: global sea level, Indian Ocean, Pacific Ocean and Atlantic Ocean.

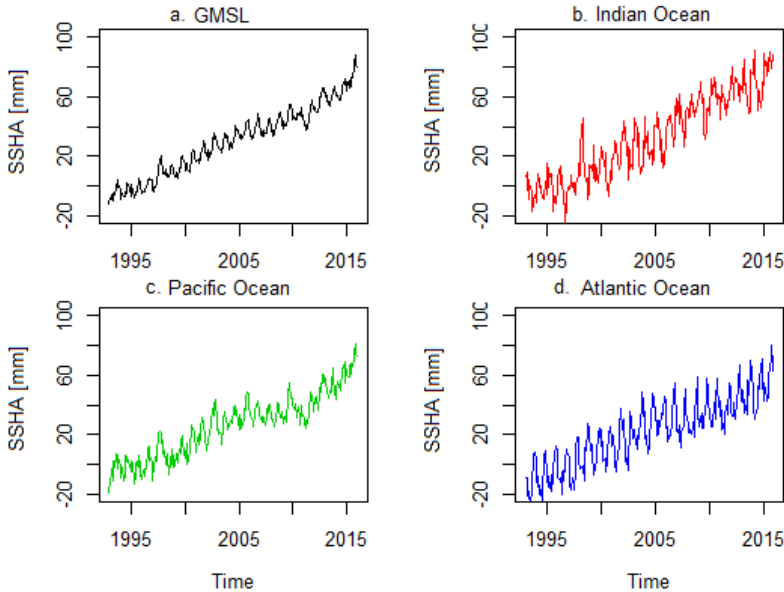


Figure 1 : Monthly mean sea level series from January 1993 to December 2015; a. Global mean seal level, b. Indian ocean, c. Pacific ocean and d. Atlantic ocean.

METHODOLOGY-SOME IMPORTANT REMINDERS ABOUT SINGULAR SPECTRUM ANALYSIS (SSA)

The main idea of SSA is performing a singular value decomposition (SVD) of the trajectory matrix obtained from the original time series with a subsequent reconstruction of the series. As described in (Golyandina et al., 2001); (Hassani, 2007) and (Hassani, 2010), the basic version of SSA consists of five steps, which are performed as follows:

Step 1. Embedding. The first step consists in the construction of the so-called trajectory matrix (\mathcal{J}_{SSA}) by associating a time series $\mathbf{x} = (x_1, \dots, x_N)$ to $(K = N - L + 1)$ column-vectors of dimension L , where L is the window length:

$$\mathcal{J}_{SSA} = \mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_k \\ x_2 & x_3 & x_4 & \dots & x_{k+1} \\ x_3 & x_4 & x_5 & \dots & x_{k+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_L & x_{L+1} & x_{L+2} & \dots & x_N \end{bmatrix} \tag{1}$$

Note that the trajectory matrix \mathbf{X} is a Hankel matrix, which means that all the elements along the diagonal $i + j = \text{const}$ are equal.

Step 2. Singular Value Decomposition. Compute the eigenvalues and eigenvectors of the matrix $S = \mathbf{X}\mathbf{X}^T$. Let $\lambda_1 \geq \dots \geq \lambda_L \geq 0$ be eigenvalues of the matrix S , $d = \max\{j: \lambda_j > 0\}$, $(U_1, V_1); \dots; (U_d, V_d)$ the associated singular vectors left and right. By SVD, the trajectory matrix \mathbf{X} can be written as (Golyandina et al., 2001):

$$\mathbf{X} = \sum_{j=1}^d \mathbf{X}_j, \text{ with } \mathbf{X}_j = \sqrt{\lambda_j} U_j V_j^T \quad (2)$$

The triple $(\sqrt{\lambda_j}, U_j, V_j)$ is called the j^{th} eigentriple.

Step 3. Grouping. This step partitions the X_j set of indices $\{1, \dots, d\}$ into m disjoint subsets I_1, \dots, I_m . For a subset $\{i_1, \dots, i_p\}$, the matrix X_I corresponding to the group I is defined as:

$$X_I = X_{I_1} + \dots + X_{I_m} \quad (3)$$

Step 4. (Reconstruction of the one-dimensional series). At this final step, each matrix of the decomposition (3) is transferred back to the form of the input object \mathbb{x} . Let the *Hankelization* operator $\Pi^{(H)}$ be averaging of the corresponding diagonals of the matrix $\widehat{X}_k = X_{I_k}$ for $i = 1, \dots, m$. Thus, denote $\widehat{X}_k = X_{I_k}$ the reconstructed matrices, $\widetilde{X}_k = \Pi^{(H)} \widehat{X}_k$ the trajectory matrices of the reconstructed data and $\widetilde{\mathbb{x}}_k = \mathcal{T}^{-1} \widetilde{X}_k$ the reconstructed data themselves. Then the resultant decomposition of the initial data has the form:

$$\mathbb{x} = \widetilde{\mathbb{x}}_1 + \dots + \widetilde{\mathbb{x}}_m \quad (4)$$

Empirical analysis

In this section, the monthly mean sea level series, with a length of $N = 276$ are decomposed into trend, harmonic and residual components using the SSA technique. The results of implementation of SSA are obtained by means of the R package Rssa (Golyandina et al., 2015).

The window length L is the only parameter in the embedding stage. Assuming that there is a dominant annual periodicity in each monthly series (see, Figure 1), we take $L = N/2 = 138$. As we mentioned earlier, the third step of SSA demands the grouping to make subgroups of the decomposed trajectory matrix and diagonal averaging to reconstruct the new time series from the subgroups. Figure 2 represents the four matrices corresponding to the four considered series

of absolute values of w correlations, depicted in gray scale from white to black corresponding to the absolute values of correlations from 0 to 1.

Figure 2 shows that for the GMSL series, the block of 12-50 components is “gray”, therefore we can expect that these components are mixed and are produced by noise. Based on this information, we select the first 12 components for the identification of the harmonic and trend components in GMSL series and consider the rest as noise. In the same way, we select the first 11, 15 and 12 components for the Indian Ocean, Pacific Ocean and Atlantic Ocean’s series, respectively.

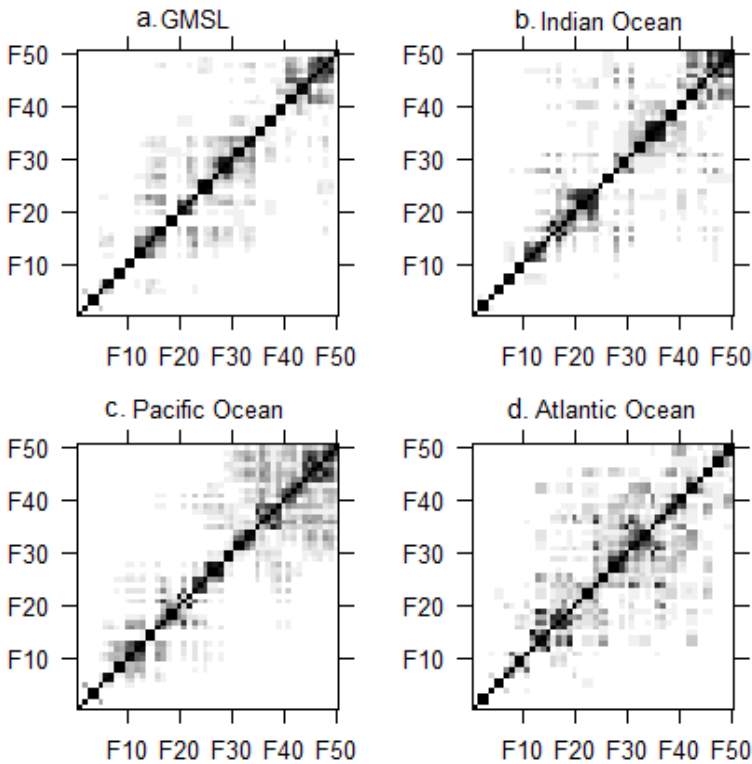


Figure 2 : Matrix of weighted correlations; a. Global mean sea level, b. Indian ocean, c. Pacific ocean and d. Atlantic ocean.

Using these parameters, we analyze separately each sea level time series to determine its trend. Here, we present first in detail the results of global sea level time series analysis, and then, a summary is given for all ocean level time series analysis.

GLOBAL SEA LEVEL TIME SERIES ANALYSIS

In practice, the singular values of the two eigentriples of a harmonic series are often very close to each other, and this fact simplifies the visual identification of the harmonic components. Therefore, explicit plateau in the eigenvalue spectra prompts the ordinal numbers of the paired eigentriples of harmonic components. Another way to identify the harmonic components of the series is to examine the pairwise scatterplots of the singular vectors. Pairwise scatterplots like spiral circles, spiral regular polygons or stars determine periodic components of the time series provided these components are separable from the residual component.

As shown in Figure 3, there are two evident pairs produced by modulated sine-waves, corresponding to two harmonic components: (3-4) and (10-11); since the corresponding 2D-scatterplots of eigenvectors are similar to regular polygons. The periodicities of the two paired harmonic components (3-4 and 10-11) are: 12.035 months (annual signal) and 6.012 months (semi-annual signal), respectively.

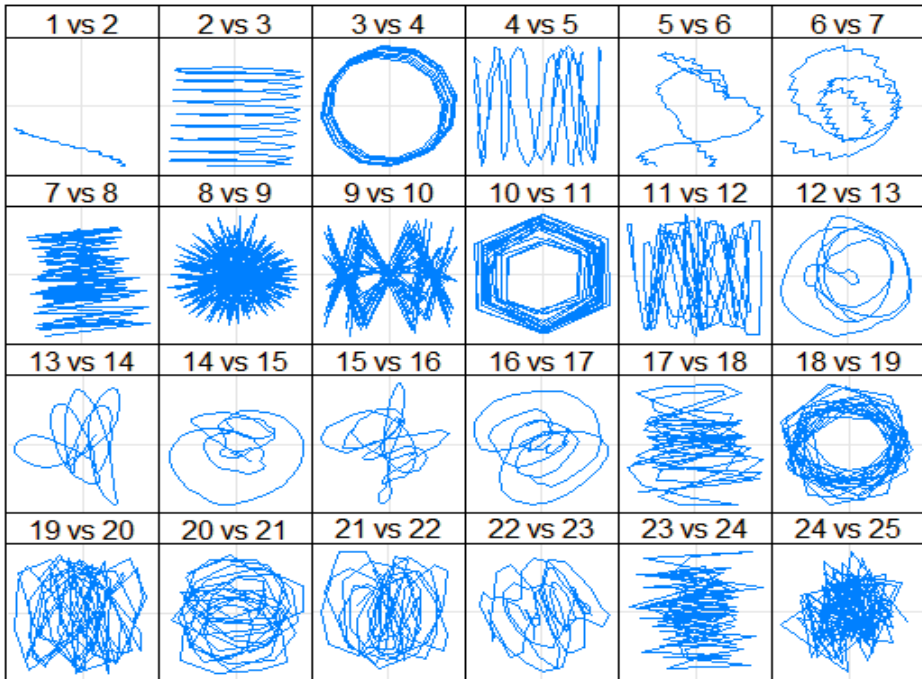


Figure 3 : 2D scatterplots of the GMSL leading paired eigenvectors.

Within the framework of SSA, the trend is defined as a smooth component containing information about time series global change. Figure 4 shows that the eigenvectors: 1, 2, 5, 6, 7 and 12, are slowly-varying and therefore, should be included in the trend group that represents a contribution of 98.46 % of the initial global mean sea level series.

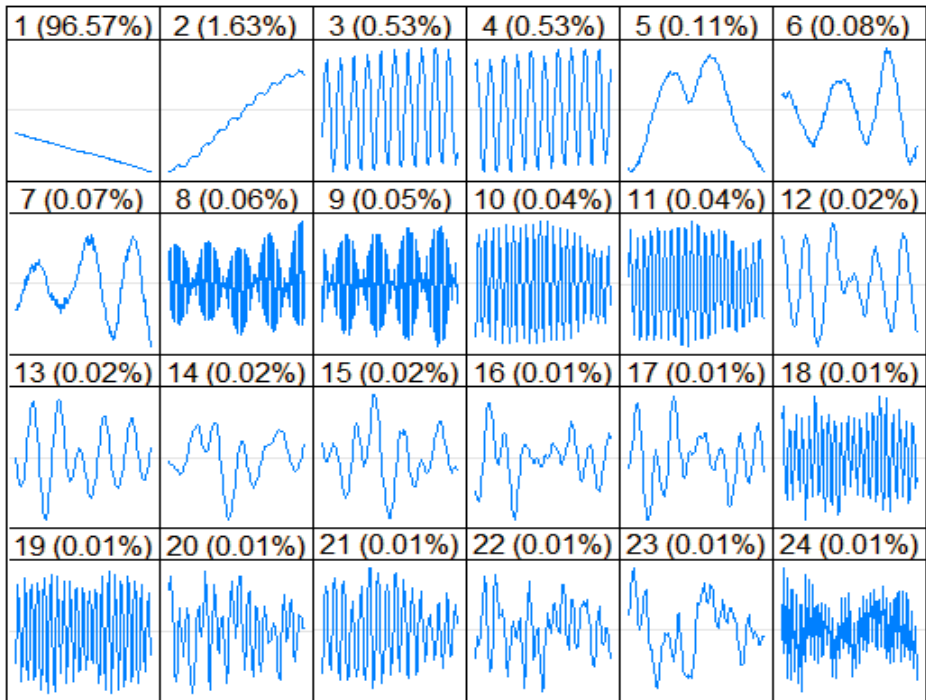


Figure 4 : 1D graphs of the GMSL leading eigenvectors. The variance explained by each eigenvector is shown above each panel.

SUMMARY OF TIME SERIES DECOMPOSITION RESULTS

Table 1 and 2 give the summary of obtained SSA decomposition results for the four sea level time series. The columns of table 1 indicate: the sea level region, the identified pairs of ETs that are related to identified harmonic components, the corresponding periodicities and contribution of the harmonics in the original time series in percentage,

The columns of table 2 indicate: the sea level region, the identified ETs that are related to identified trend component and their total contribution in the original time series in percentage, the trend rate and their standard error in millimeters per year. Note that the trend rate for each time series is estimated for the period from the beginning of 1993 to the end of 2015 by least squares linear regression of the related extracted trend. Note that the realistic error assigned in estimated trend rates is of ± 0.4 mm/year. This quantity is based on comparisons of altimeter heights with tide gauge-based sea level measurements (Mitchum, 2000). For further information on comparison of altimetry based sea level with in-situ measurements, the reader is referred to (Ablain, 2009).

Table 1. Seasonality extraction results for the all sea level time series

Sea level region	Seasonality			
	Pairs of eigentriples	Periodicity (months)	Contribution	Total contribution
Global sea level	(3-4)	12.035	1.06 %	1.14 %
	(10-11)	6.012	0.08 %	
Indian Ocean	(2-3)	12.263	3.48 %	3.48 %
Pacific Ocean	(3-4)	12.091	2.03 %	2.21 %
	(14-15)	6.101	0.18 %	
Atlantic Ocean	(2-3)	12.090	10.01 %	10.61 %
	(5-6)	6.031	0.60 %	

Table 2. Trend extraction results for the all sea level time series

Sea level region	Trend		
	Eigentriples	Total contribution	Rate (mm/year)
Global sea level	1, 2, 5, 6, 7	98.46 %	3.36 \pm 0.03
Indian Ocean	1,4,7,8,11	92.79 %	3.80 \pm 0.03
Pacific Ocean	1, 2, 5, 6, 7, 10, 11, 12, 13	95.98 %	2.83 \pm 0.05
Atlantic Ocean	1, 4, 7, 8, 11	85.60 %	2.81 \pm 0.03

RECONSTRUCTION AND VALIDATION

In order to cross-validate the identified trends and seasonal components in the four considered monthly sea level series using SSA, we performed a second time a decomposition of the original series using the Seasonal-Trend Decomposition Based on LOESS - STL technique. This technique initially proposed in (Cleveland, 1979) and developed in (Cleveland and Delvin, 1988) is an iterative procedure that uses local weighted regression functions, known as LOcally wEighted regrESSion Smoother - LOESS. The simplicity of STL technique allows fast computation, even for very long time series. Figure 5

shows the reconstructed trends and total seasonalities using SSA technique (represented in black line) and the decomposition results using the STL technique (represented in blue line). The decomposition's results obtained both the SSA and STL techniques are very similar.

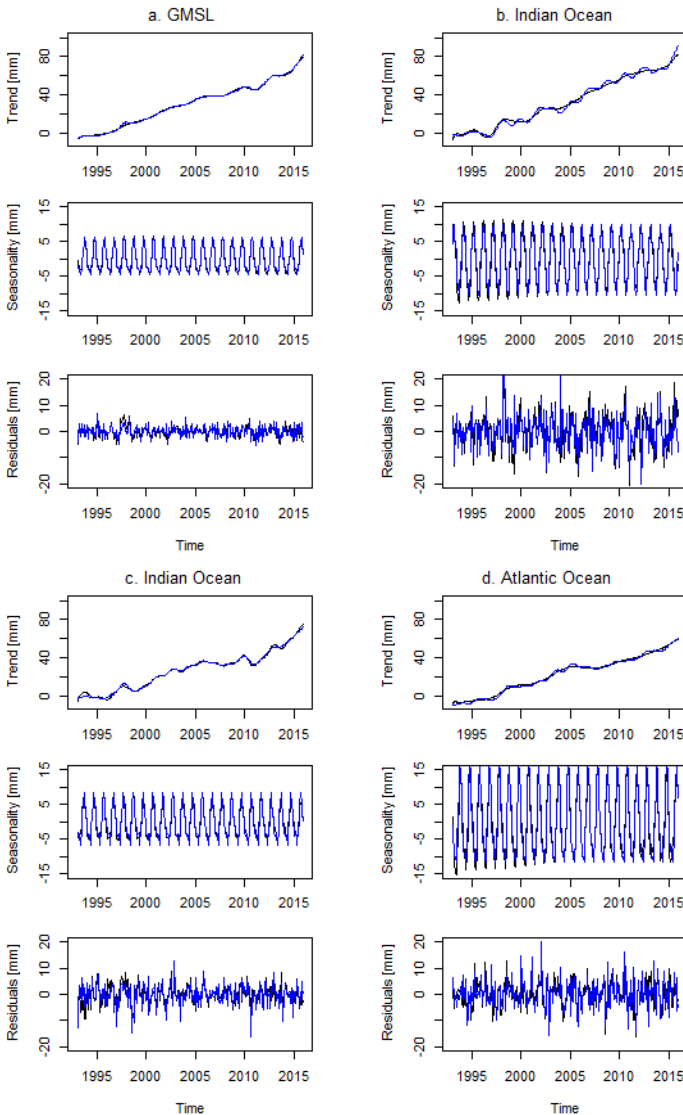


Figure 5 : Trend, seasonality and Residuals; blue line: SSA and dark line: STL; a. Global mean seal level, b. Indian ocean, c. Pacific ocean and d. Atlantic ocean.

CONCLUSION

In this paper, an approach of trend extraction using singular spectrum analysis has been used to extract trends of global and oceanic level time series from TOPEX, Jason-1, and Jason-2 altimeters that extend back to 1993. The obtained results show that the global mean sea level is characterized by its strong trend, which represents about 98.46 % of the original sea level signal. Applying a least squares linear regression analysis to the global sea level trend gives a rate of 3.36 mm/year between 1993 and 2015. Although the global trend indicates a rise in the mean sea level, our results show that sea level trends of oceans are nonuniform: the Indian Ocean, Pacific Ocean and Atlantic Ocean, exhibit a rate of 3.80, 2.83 and 2.81 mm/year since 1993, respectively.

The cross-validation of the identified trends and seasonal components in the four considered monthly sea level series by using the Seasonal-Trend Decomposition Based on LOESS - STL technique, confirm if necessary again that the SSA is an excellent tool to decompose and reconstruct signal, such as the variations in the average sea level.

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