



NEW APPROCH FOR THE NORMAL DEPTH COMPUTATION IN A TRAPEZOIDAL OPEN CHANNEL USING THE ROUGH MODEL METHOD

NOUVELLE APPROCHE AU CALCUL DE LA PROFONDEUR NORMALE DANS UN CANAL TRAPÉZOÏDAL PAR LA MÉTHODE DU MODÈLE RUGUEUX

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ABSTRACT

Normal depth plays a significant role in the design of open channels and in the analysis of the non-uniform flow as well. Currently, there is no analytical method for calculation of the normal depth in open channels, including the trapezoidal profile. Current methods are either iterative or approximate. They also consider, unreasonably, Chezy's coefficient or Manning's roughness coefficient as a given data of the problem, despite the fact that these coefficients depend on the normal depth sought. In this study, a new analytical method is presented for calculating the normal depth in an trapezoidal open channel. The method takes into account, in particular, the effect of the absolute roughness which is a readily measurable parameter in practice. In a first step, the method is applied to a referential rough model in order to establish the relationships that govern its hydraulic characteristics. In a second step, these equations are used to easily deduce the required normal depth by introducing a non-dimensional correction factor. A practical example is considered to better explain the advocated method and to appreciate its simplicity and efficiency.

Keywords: Normal depth, Trapezoidal open channel, Uniform flow, Discharge, Slope, Turbulent flow.

RESUME

La profondeur normale joue un rôle important dans la conception des canaux ouverts et dans l'analyse des écoulements non uniformes. Actuellement, il n'y a pas de méthode analytique pour le calcul de la profondeur normale dans les canaux ouverts, y compris le profil trapézoïdal. Les méthodes actuelles sont itératives ou approximatives. Ils considèrent également, de façon injustifiée, le coefficient de Chézy ou le coefficient de rugosité de Manning comme une donnée du problème, malgré que ces coefficients dépendent de la profondeur normale recherchée. Dans cette étude, une nouvelle méthode analytique est présentée pour le calcul de la profondeur normale dans un canal ouvert de forme trapézoïdale. La méthode prend notamment en compte l'effet de la rugosité absolue qui est un paramètre facilement mesurable en pratique. Dans un premier temps, la méthode est appliquée à un modèle rugueux de référence afin d'établir les relations qui régissent ses caractéristiques hydrauliques. Dans un second temps, ces équations sont utilisées pour déduire facilement la profondeur normale requise en introduisant un facteur de correction non-dimensionnel. Un exemple pratique est considéré pour mieux expliquer la méthode préconisée et apprécier sa simplicité et son efficacité.

Mots Clés : Profondeur normale, Canal ouvert de forme trapézoïdale, Ecoulement uniforme, Débit, Pente, Ecoulement turbulent.

INTRODUCTION

It is known that the normal depth plays a major role in the classification of varied flow and in the design of canals and conduits (Achour, 2015a). In the past, the solutions were graphics for most known channels geometric profiles (Chow, 1973; French, 1986; Henderson, 1966). In recent years, solutions have become iterative or approximate and the trapezoidal open channel does no exception to this rule (Swamee, 1994; Srivastava, 2006; Kouchakzadeh and Vatankhah, 2007). The focus on this channel is purely practical as it is widely used in hydraulic structures (Das, 2007). Calculating the normal depth in a trapezoidal open channel continues to generate real interest among researchers (Vatankhah, 2013), because existing methods are not satisfactory. In current methods of calculation, the major problem lies not in their iterative nature but

rather in the fact that they consider the coefficient of Manning or Chezy as a given data of the problem. It is here that lays the real difficulty because the question is how to impose these coefficients as they depend on the normal depth sought. Even with a lot of experience, it is very difficult or impossible to set the value of these coefficients in advance, before calculating the normal depth. The only practical measurable parameter which is related to the internal state of the channel wall is the absolute roughness, usually referred to ε . It is this parameter that must be, in principle, a given data of the problem instead of Chezy and Manning coefficients. Currently, there is no explicit method that considers this parameter (Achour, 2014a; 2014b; 2014c; Lakehal and Achour, 2014). It is in this context that this study is proposed, based on a new method known as the Rough Model Method (RMM) (Achour, 2013; 2014a; 2014b; 2014c; 2014d; 2014e; 2015b; Lakehal and Achour, 2014; Achour and Bedjaoui, 2012; 2014; Achour and Sehtal, 2014; Achour and Riabi, 2014; Riabi and Achour, 2014; Achour and Khattaoui, 2014). This method does not require Manning's roughness coefficient or Chezy's coefficient. For the calculation of normal depth in trapezoidal open channel, it requires only measurable parameters in practice, namely the discharge Q , the side slope m , the longitudinal slope i , the horizontal linear dimension b of the channel, the absolute roughness ε and the kinematic viscosity ν of the flowing liquid (Achour, 2014c). The method is based on geometric and hydraulic characteristics of a referential rough model whose parameters are well defined (Achour, 2014a). This is made possible through a non-dimensional correction factor of linear dimension (Achour, 2015). Resulting RMM equations are valid in the entire domain of turbulent flow, corresponding to Reynolds number $R \geq 2300$ and relative roughness ε/D_h varying in the wide range $[0; 0.05]$ (Achour, 2014b). An example is presented to better understand the calculation procedure and to appreciate its simplicity and efficiency.

BASIC EQUATIONS

In this study, three simple relationships, commonly used in hydraulics, will be used. These relations are : the Darcy-Weisbach equation (Darcy, 1854), the Colebrook-White equation (Colebrook, 1939) and Reynolds number formula. The longitudinal slope i of the channel is given by Darcy-Weisbach's formula as follows:

$$i = \frac{f}{D_h} \frac{Q^2}{2gA^2} \quad (1)$$

where Q is the discharge, g is the acceleration due to gravity, A is the wetted area, D_h is the hydraulic diameter and f is the friction factor given by the now famous Colebrook-White formula as :

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon / D_h}{3.7} + \frac{2.51}{R\sqrt{f}} \right) \quad (2)$$

where ε is the absolute roughness and R is the Reynolds number which can be expressed as :

$$R = \frac{4Q}{P\nu} \quad (3)$$

where ν is the kinematic viscosity and P is the wetted perimeter.

REFERENTIAL ROUGH MODEL

All geometric and hydraulic characteristics of the rough model are distinguished by the symbol " $\bar{\quad}$ ". Fig. 1 compares the geometric and hydraulic characteristics of the current channel with those of its rough model. The rough model is particularly characterized by $\bar{\varepsilon} / \bar{D}_h = 0,037$ as the arbitrarily assigned relative roughness value, where \bar{D}_h is the hydraulic diameter. The chosen relative roughness value is so large that the prevailed flow regime is fully rough. Thus, the friction factor is $\bar{f} = 1/16$ according to Eq. 2 for $R = \bar{R}$ tending to infinitely large value (Achour, 2013; 2014a; 2014b; 2014c; 2014d; 2014e; 2015a; 2015b; Lakehal and Achour, 2014; Achour and Bedjaoui, 2012; 2014; Achour and Sehtal, 2014; Achour and Riabi, 2014; Riabi and Achour, 2014; Achour and Khattaoui, 2014). The rough model is also characterized by the horizontal linear dimension $\bar{b} = b$, a inverse side slope "m" vertical to 1 horizontal and a longitudinal slope $\bar{i} = i$ (Fig. 1).

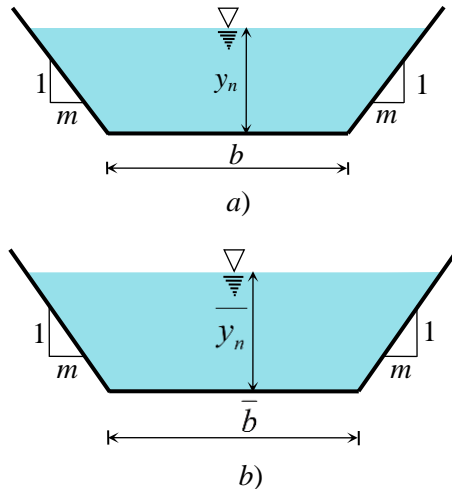


Figure 1 : Schematic representation of normal depth in trapezoidal cross section. a) Current channel. b) Rough model

The discharge is $\bar{Q} = Q$ implying $\bar{y}_n \neq y_n$ and even $\bar{y}_n > y_n$. The aspect ratio, also known as the non-dimensional normal depth, is thus $\bar{\eta} = \bar{y}_n / b \neq \eta = y_n / b$ (Achour, 2014c).

Applying Eq. (1) to the rough model, we can write:

$$i = \frac{\bar{f}}{D_h} \frac{Q^2}{2g\bar{A}^2} \quad (4)$$

Inserting $\bar{D}_h = 4\bar{A} / \bar{P}$ and $\bar{f} = 1/16$ into Eq. (4) and rearranging leads to:

$$i = \frac{1}{128g} \frac{\bar{P}}{\bar{A}^3} Q^2 \quad (5)$$

The wetted perimeter \bar{P} and the water area \bar{A} are expressed respectively as (Achour, 2014c):

$$\bar{P} = b \left(1 + 2\bar{\eta} \sqrt{1 + m^2} \right) \quad (6)$$

$$\bar{A} = b^2 \bar{\eta} \left(1 + m\bar{\eta} \right) \quad (7)$$

Bearing in mind that Eqs. (6) (7), Eq. (5) can be rewritten as:

$$\left[\frac{1 + 2\bar{\eta}\sqrt{1+m^2}}{128(1+m\bar{\eta})^3\bar{\eta}^3} \right] \frac{Q^2}{gib^5} = 1 \tag{8}$$

Let us assume the following relative conductivity:

$$Q^* = \frac{Q}{\sqrt{gib^5}} \tag{9}$$

Thus, Eq. (8) is reduced to:

$$\left[\frac{1 + 2\bar{\eta}\sqrt{1+m^2}}{128(1+m\bar{\eta})^3\bar{\eta}^3} \right] Q^{*2} = 1 \tag{10}$$

All the parameters of Eq. (9) are known, which allows determining the value of the relative conductivity Q^* . What is needed is the computation of the non-dimensional normal depth $\bar{\eta}$ of the rough model using Eq. (10) for the given value of Q^* . However, as one can observe, eq. (10) is implicit towards the non-dimensional normal depth $\bar{\eta}$. The computation involves iterative procedure or graphical method. One way to avoid this is to provide an approximate relationship for Eq. (10). We suggests approximating Eq. (10) by the following handy and explicit polynomial law:

$$\Lambda = a(m)Z^5 + b(m)Z^4 + c(m)Z^3 + d(m)Z^2 + e(m)Z + f(m) \tag{11}$$

Where:

$$\Lambda = \log\left(\frac{\bar{\eta}}{3\pi}\right) \tag{12}$$

From which it can be deduced that:

$$\bar{\eta} = 3\pi \cdot 10^\Lambda \tag{13}$$

The parameter Z is expressed as:

$$Z = \log Q^* \tag{14}$$

But, the parameters of ajustment : $a(m), b(m), c(m), d(m), e(m)$ and $f(m)$ can be remplaced by $\zeta(m)$. This parameter is expressed as:

$$\zeta(m) = \zeta_1 M^5 + \zeta_2 M^4 + \zeta_3 M^3 + \zeta_4 M^2 + \zeta_5 M + \zeta_6 \tag{15}$$

for $a(m)$, $b(m)$, $c(m)$ and $d(m)$,

where:

$$M = \log m \tag{16}$$

or :

$$\zeta(m) = \zeta_1 m^5 + \zeta_2 m^4 + \zeta_3 m^3 + \zeta_4 m^2 + \zeta_5 m + \zeta_6 \tag{17}$$

for $e(m)$ and $f(m)$.

The values of the parameters ζ_1 , ζ_2 , ζ_3 , ζ_4 , ζ_5 and ζ_6 , are reported in Table 1.

Table 1: Values of ζ_1 , ζ_2 , ζ_3 , ζ_4 , ζ_5 and ζ_6 for computation of the parameter $\zeta(m)$ by Eq. (15)

	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5	ζ_6
$\zeta(m)=a(m)$	$\frac{243}{21731}$	$-\frac{32}{18167}$	$-\frac{251}{20946}$	$\frac{89}{11766}$	$-\frac{14}{214477}$	$-\frac{25}{30239}$
$\zeta(m)=b(m)$	$-\frac{247}{4886}$	$\frac{86}{7529}$	$\frac{619}{9861}$	$-\frac{290}{6893}$	$-\frac{131}{23982}$	$\frac{226}{29455}$
$\zeta(m)=c(m)$	$\frac{166}{2487}$	$-\frac{167}{10116}$	$-\frac{124}{1167}$	$\frac{64}{1099}$	$\frac{115}{2768}$	$-\frac{141}{6928}$
$\zeta(m)=d(m)$	$-\frac{61}{6149}$	$-\frac{378}{106445}$	$\frac{183}{4496}$	$\frac{49}{1636}$	$-\frac{745}{10984}$	$-\frac{557}{20987}$
$\zeta(m)=e(m)$	$-\frac{15}{96638}$	$\frac{40}{15613}$	$-\frac{45}{2617}$	$\frac{121}{1956}$	$-\frac{419}{2958}$	$\frac{846}{1151}$
$\zeta(m)=f(m)$	$-\frac{6}{17893}$	$\frac{104}{24163}$	$-\frac{163}{7461}$	$\frac{1425}{24728}$	$-\frac{843}{7865}$	$-\frac{7069}{4355}$

The Eqs. (11), (15) and (17) was established in the wide range $0.1 \leq \bar{\eta} \leq 6$ and $0.1 \leq m \leq 4$. The maximum relative deviation caused by Eqs. (11), (15) and (17) used together is less than 0.88 % only which is more than enough for practical applications. Fig. 2 graphically shows the relative deviation, $\frac{\Delta \bar{\eta}}{\bar{\eta}}$, for different values of m . As can be seen from Fig. 2, the relative deviations are in the majority equal to $\pm 0.1\%$ for $0.1 \leq \bar{\eta} \leq 3$, and they are generally equal to $\pm 0.2\%$ for $3 \leq \bar{\eta} \leq 6$.

NON-DIMENSIONAL CORRECTION FACTOR OF LINEAR DIMENSION

The rough model method states that any linear dimension L of a channel and the linear dimension of its rough model are related by the following equation, applicable to the whole domain of the turbulent flow:

$$L = \psi \bar{L} \tag{18}$$

where ψ is a non-dimensional correction factor of linear dimension, less than unity, which is governed by the following relationship (Achour and Bedjaoui, 2006; 2012):

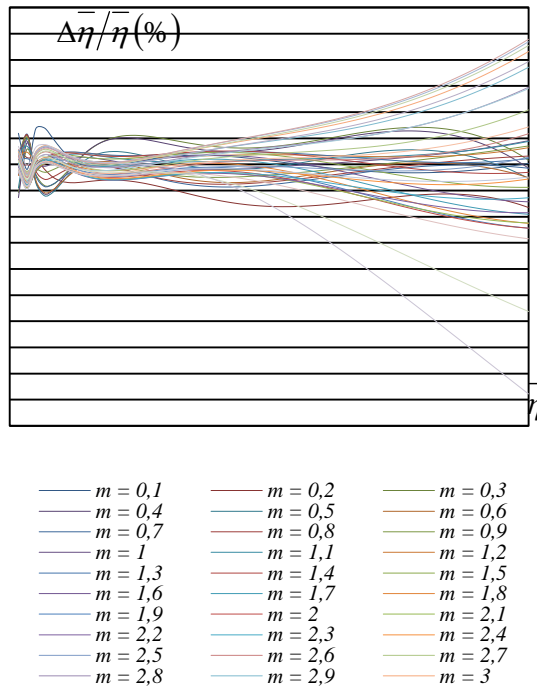


Figure 2: Relative deviation, $\Delta\bar{\eta}/\bar{\eta}$, of different values of m

$$\psi \cong 1.35 \left[-\log \left(\frac{\varepsilon/\overline{D}_h}{4.75} + \frac{8.5}{\overline{R}} \right) \right]^{-2/5} \quad (19)$$

where \overline{R} is the Reynolds number in the rough model given by :

$$\overline{R} = \frac{4Q}{P\nu} \quad (20)$$

COMPUTATION STEPS OF NORMAL DEPTH

To compute the normal depth y_n in a channel of trapezoidal cross section, the following parameters must be given: the discharge Q , the bottom width b , the inverse side slope m , the longitudinal slope i , the absolute roughness ε and the kinematic viscosity ν . Note firstly that these data are measurable in practice and secondly the flow resistance coefficient such as Chezy's coefficient or Manning's roughness coefficient is not imposed. The normal depth y_n can be computed according to the following steps:

1. Compute the relative conductivity Q^* by the use of Eq. (9).
2. Calculate the value of parameter Z using Eq. (14).
3. Compute the parameter M using Eq. (16).
4. Compute the parameters of ajustement $\zeta(m) = a(m), b(m), c(m), d(m), e(m)$ or $f(m)$, using Eq. (15) or Eq. (17).
The parameters $\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5$ and ζ_6 are taken from Table 1.
5. According to Eq. (11), parameter Λ is then worked out.
6. With the calculated value of Λ , compute the aspect ratio $\overline{\eta}$ in the rough model using Eq. (13).
7. As a result, Eqs. (6) and (7) give the wetted perimeter \overline{P} and the water area \overline{A} respectively. This allows deducing the hydraulic diameter $\overline{D}_h = 4\overline{A}/\overline{P}$ and Reynolds number \overline{R} by the use of Eq. (20).
8. Whith the calculated values of \overline{D}_h and \overline{R} , compute the non-dimensional correction factor of linear dimension by applying the explicit Eq. (19).

9. Assign to the rough model the new linear dimension $\bar{b} = b/\psi$ according to Eq. (18) and derive the corresponding value of the relative conductivity Q^* using Eq. (9).
10. Compute Z and Λ according to the steps 2 and 5 respectively. The parameters M and $\zeta(m)$ calculated respectively in steps 3 and 4 do not change.
11. By introducing this value of Λ in Eq. (13), one obtains the aspect ratio $\bar{\eta}$ in the rough model equal to the aspect ratio η in the current channel.
12. The required normal depth is finally as: $y_n = b\eta$.

PRACTICAL EXAMPLE

Compute the normal depth y_n in the trapezoidal channel shown in Fig. 1, for the following data:

$$Q = 3 \text{ m}^3/\text{s}, b = 2 \text{ m}, i = 10^{-4}, m = 2, \varepsilon = 10^{-3} \text{ m}, \nu = 10^{-6} \text{ m}^2/\text{s}.$$

Not that Chezy's coefficient C or Manning's roughness coefficient n is not required.

(For the sake of calculation, the counts will not be rounded off).

1. Using Eq. (9), the relative conductivity Q^* is:

$$Q^* = \frac{Q}{\sqrt{gib^5}} = \frac{3}{\sqrt{9.81 \times 10^{-4} \times 2^5}} = 16.93213654$$

2. Using Eq. (14), the parameter Z is as:

$$Z = \log Q^* = \log 16.93213654 = 1.22871176$$

3. The parameter M was easily calculated using Eq. (16) such that:

$$M = \log m = \log 2 = 0.30103$$

4. According to Eqs. (15), (17) and Table 1, the parameters of ajustement $\zeta(m) = a(m), b(m), c(m), d(m), e(m)$ and $f(m)$ Are:

$$\zeta(m) = a(m) = \zeta_1 M^5 + \zeta_2 M^4 + \zeta_3 M^3 + \zeta_4 M^2 + \zeta_5 M + \zeta_6$$

$$\begin{aligned} \zeta(m) = a(m) &= \frac{243}{21731} \times 0.30103^5 - \frac{32}{18167} \times 0.30103^4 \\ &- \frac{251}{20946} \times 0.30103^3 + \frac{89}{11766} \times 0.30103^2 - \frac{14}{214477} \times 0.30103 \\ &- \frac{25}{30239} = -0.00047465 \\ \zeta(m) = b(m) &= \zeta_1 M^5 + \zeta_2 M^4 + \zeta_3 M^3 + \zeta_4 M^2 + \zeta_5 M + \zeta_6 \\ \zeta(m) = b(m) &= -\frac{247}{4886} \times 0.30103^5 + \frac{86}{7529} \times 0.30103^4 + \frac{619}{9861} \times 0.30103^3 \\ &- \frac{290}{6893} \times 0.30103^2 - \frac{131}{23982} \times 0.30103 + \frac{226}{29455} = 0.00389708 \\ \zeta(m) = c(m) &= \zeta_1 M^5 + \zeta_2 M^4 + \zeta_3 M^3 + \zeta_4 M^2 + \zeta_5 M + \zeta_6 \\ \zeta(m) = c(m) &= \frac{166}{2487} \times 0.30103^5 - \frac{167}{10116} \times 0.30103^4 \\ &- \frac{124}{1167} \times 0.30103^3 + \frac{64}{1099} \times 0.30103^2 + \frac{115}{2768} \times 0.30103 \\ &- \frac{141}{6928} = -0.00543746 \\ \zeta(m) = d(m) &= \zeta_1 M^5 + \zeta_2 M^4 + \zeta_3 M^3 + \zeta_4 M^2 + \zeta_5 M + \zeta_6 \\ \zeta(m) = d(m) &= -\frac{61}{6149} \times 0.30103^5 - \frac{378}{106445} \times 0.30103^4 \\ &+ \frac{183}{4496} \times 0.30103^3 + \frac{49}{1636} \times 0.30103^2 - \frac{745}{10984} \times 0.30103 \\ &- \frac{557}{20987} = -0.04318709 \\ \zeta(m) = e(m) &= \zeta_1 m^5 + \zeta_2 m^4 + \zeta_3 m^3 + \zeta_4 m^2 + \zeta_5 m + \zeta_6 \end{aligned}$$

$$\zeta(m) = e(m) = -\frac{15}{96638} \times 2^5 + \frac{40}{15613} \times 2^4 - \frac{45}{2617} \times 2^3 + \frac{121}{1956} \times 2^2 - \frac{419}{2958} \times 2 + \frac{846}{1151} = 0.59761967$$

$$\zeta(m) = f(m) = \zeta_1 m^5 + \zeta_2 m^4 + \zeta_3 m^3 + \zeta_4 m^2 + \zeta_5 m + \zeta_6$$

$$\zeta(m) = f(m) = -\frac{6}{17893} \times 2^5 + \frac{104}{24163} \times 2^4 - \frac{163}{7461} \times 2^3 + \frac{1425}{24728} \times 2^2 - \frac{843}{7865} \times 2 - \frac{7069}{4355} = -1.72369159$$

5. Using Eq. (11), the parameter Λ is as:

$$\Lambda = a(m)Z^5 + b(m)Z^4 + c(m)Z^3 + d(m)Z^2 + e(m)Z + f(m)$$

$$\Lambda = -0.00047465 \times 1.22871176^5 + 0.00389708 \times 1.22871176^4 - 0.00543746 \times 1.22871176^3 - 0.04318709 \times 1.22871176^2 + 0.59761967 \times 1.22871176 - 1.72369159 = -1.05712358$$

6. According to Aq. (13), the aspect ratio $\bar{\eta}$ in the rough model is as:

$$\bar{\eta} = 3\pi \cdot 10^\Lambda = 3 \times \pi \times 10^{-1.05712358} = 0.82631863$$

7. According to Eq. (6), the watted perimeter \bar{P} is:

$$\bar{P} = b(1 + 2\bar{\eta}\sqrt{1+m^2}) = 2 \times (1 + 2 \times 0.82631863 \times \sqrt{1+2^2}) = 9.3908185 \text{ m}$$

The water area \bar{A} was easily calculated using Eq. (7) such that:

$$\bar{A} = b^2 \bar{\eta}(1 + m\bar{\eta}) = 2^2 \times 0.82631863 \times (1 + 2 \times 0.82631863) = 8.76769432 \text{ m}^2$$

The hydraulic diameter $\bar{D}_h = 4\bar{A}/\bar{P}$ is then:

$$\bar{D}_h = 4 \frac{\bar{A}}{\bar{P}} = 4 \times \frac{8.76769432}{9.3908185} = 3.73458153 \text{ m}$$

According to Eq. (20), Reynolds number \bar{R} is:

$$\bar{R} = \frac{4Q}{P\nu} = \frac{4 \times 3}{9.3908185 \times 10^{-6}} = 1277843.89$$

8. Using Eq. (19), the non-dimensional correction factor of linear dimension ψ was easily calculated as:

$$\begin{aligned}\psi &\cong 1.35 \left[-\log \left(\frac{\varepsilon/\overline{D}_h}{4.75} + \frac{8.5}{R} \right) \right]^{-2/5} \\ &= 1.35 \left[-\log \left(\frac{10^{-3}/3.73458153}{4.75} + \frac{8.5}{1277843.89} \right) \right]^{-2/5} = 0.7603501\end{aligned}$$

9. Assign to the rough model the following new value of linear dimension:

$$\bar{b} = b/\psi = 2/0.7603501 = 2.63036724 \text{ m}$$

According to Eq. (9), the corresponding value of the relative conductivity Q^* is:

$$Q^* = \frac{Q}{\sqrt{gib^{-5}}} = \frac{3}{\sqrt{9.81 \times 10^{-4} \times 2.63036724^5}} = 8.53583049$$

10. According to steps 2 and 5, one obtains the parameters Z and Λ such that:

Using Eq. (14), the parameter Z is as:

$$Z = \log Q^* = \log 8.53583049 = 0.93124578$$

Using Eq. (11) and the values of M and $\zeta(m)$ calculated respectively in steps 3 and 4, one obtains the parameter Λ such that:

$$\Lambda = a(m)Z^5 + b(m)Z^4 + c(m)Z^3 + d(m)Z^2 + e(m)Z + f(m)$$

$$\Lambda = -0.00047465 \times 0.93124578^5 + 0.00389708 \times 0.93124578^4$$

$$- 0.00543746 \times 0.93124578^3 - 0.04318709 \times 0.93124578^2$$

$$+ 0.59761967 \times 0.93124578 - 1.72369159 = -1.20640626$$

11. Thus, Eq. (13) gives the aspect ratio η as :

$$\bar{\eta} = \eta = 3\pi \cdot 10^\Lambda = 3 \times \pi \times 10^{-1.20640626} = 0.58595581$$

12. Finally, the required value of the normal depth is:

$$y_n = b\eta = 2 \times 0.58595581 = 1.17191162 \text{ m} \approx 1.172 \text{ m}$$

13. This step aims to verify the validity of the calculations by determining the longitudinal slope of the channel using Eq. (1). The energy slope so calculated should be equal to the slope given in the problem statement.

According to the rough model method, the friction factor f is related to the non-dimensional correction factor ψ by the following formula (Achour, 2013; 2014a; 2014b; 2014c; 2014d; 2014e; 2015a; 2015b; Lakehal and Achour, 2014; Achour and Bedjaoui, 2006; 2012; 2014;

Achour and Sehtal, 2014; Achour and Riabi, 2014; Riabi and Achour, 2014; Achour and Khattaoui, 2014):

$$f = \psi^5 / 16 = 0.7603501^5 / 16 = 0.01588357$$

The water area A was easily calculated using Eq. (7) such that:

$$\begin{aligned} A &= b^2 \eta (1 + m \eta) = 2^2 \times 0.58595581 \times (1 + 2 \times 0.58595581) \\ &= 5.090576921 m^2 \end{aligned}$$

According to Eq. (6), the watted perimeter P is:

$$P = b \left(1 + 2\eta \sqrt{1 + m^2} \right) = 2 \times \left(1 + 2 \times 0.58595581 \times \sqrt{1 + 2^2} \right) = 7.24094809 \text{ m}$$

The hydraulic diameter $D_h = 4A/P$ is then:

$$D_h = 4 \frac{A}{P} = 4 \times \frac{5.090576921}{7.24094809} = 2.81210519 m$$

Finally, according to Eq. (1), the longitudinal slope i is:

$$i = \frac{f}{D_h} \frac{Q^2}{2gA^2} = \frac{0.01588357}{2.81210519} \times \frac{3^2}{2 \times 9.81 \times 5.090576921^2} = 9.9983 \cdot 10^{-5}$$

As one can observe, the longitudinal slope so calculated is equal to the slope given in the problem statement.

CONCLUSIONS

Simple equations of hydraulics have helped to solve the problem of computing the normal depth y_n in a trapezoidal channel, by applying them to a referential rough model whose geometric and hydraulic characteristics are known. The rough model method or RMM is based on practical data, easily measurable, and does not take into account neither Chezy's coefficient nor Manning's roughness coefficient. The method uses simple hydraulic relations, such as Darcy-Weisbach relation, Colebrook-White equation and Reynolds number formula. The Darcy-Weisbach relationship was applied to a referential rough model, whose friction factor has been arbitrarily chosen. These equations were subsequently used to derive explicitly the searched normal depth by the use of a non-dimensional correction factor of linear dimension whose major role has been highlighted. The practical example we suggested showed the reliability of the RMM as well as its simplicity and efficiency.

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