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INCEPTION POINT AND AIR-WATER FLOW CHARACTERISTICS OVER STEPPED SPILLWAY: NUMERICAL STUDY

LE POINT D'INCEPTION ET LES CARACTERISTIQUES DE L'ECOULEMENT EAU-AIR SUR LES DEVERSOIRS EN MARCHE ESCALIER : ETUDE NUMERIQUE

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ABSTRACT

Stepped spillway is hydraulic structure designed to dissipate considerable kinetic energy because it's characterised by the large value of the surface roughness. On stepped chutes with skimming flow regime, the flow is highly aerated which can reduce the risk of cavitation. The air entrainment starts where the boundary layer attains the free surface of flow; this point is called "*point of inception*". In the present numerical study, the Reynolds-Averaged Navier–Stokes equations are coupled with $k-\varepsilon$ turbulence standard model to simulate the water flow over stepped spillway by using the software Ansys Fluent. Volume of fluid (VOF) model is used as a tool to track the free surface flow. This research aims to find new formulas for describe the variation of water depth and the positions of the inception point. In addition, to study the characteristics of flow over stepped spillways. The found numerical results agree well with experimental results.

Keywords: Ansys Fluent, VOF Model, Air entrainment, Stepped Spillway, Standard $k-\varepsilon$ Model,

RÉSUMÉ

Le déversoir en marche escalier est un ouvrage hydraulique conçu pour dissiper une quantité considérable de l'énergie cinétique en raison de la grande valeur de la rugosité de surface. L'écoulement extrêmement turbulent sur les canaux en marche escalier est hautement aéré qui peut réduire les risques liés à la cavitation. L'entraînement d'air commence lorsque la couche limite atteint la surface libre de l'écoulement; ce point est appelé «point d'inception». Dans cette étude numérique, les équations de Reynolds sont couplées avec le modèle de turbulence k −ε pour simuler l'écoulement de l'eau au-dessus d'un déversoir en marche escalier en utilisant le code de calcul Ansys Fluent. Le modèle multiphasique VOF (Volume Of Fluid) est utilisé comme outil pour modéliser l'interaction eau-air près de la surface libre. Cette recherche vise à trouver de nouvelles formules pour déterminer la variation de la surface libre et la position du point d'inception. Ainsi les caractéristiques de l'écoulement au-dessus les déversoirs en marche en escalier ont été étudiés dans cet article. Les résultats trouvés numériquement sont en bon accord avec les résultats expérimentaux.

Mots clés : Ansys Fluent, méthode VOF, entrainement d'air, déversoir en marche escalier, modèle (k-ε).

INTRODUCTION

Stepped spillway is an open channel characterised by the large value of the surface roughness which contribute to dissipate more kinetic energy and can reduce the size of stilling basin at the toe of the dam (Charles and Kadavy, 1996, Rajaratnam, 1990, Christodoulou, 1993 and Chanson, 2001). The losses of total head occur by interaction between cavity flow and main stream skimming flows. The advantage of stepped spillways over smooth spillways is the presence of air which can reduce the risk of cavitation.

It is known that for minimum water discharge and for high length of step, nappe flow occur over a stepped spillway and can be characterised by a succession of free falling jets impinging on the steps and followed by a fully or partially developed hydraulic jump. For medium flow rates, the transition flow arises and is characterised by significant aeration, splashing, and chaotic appearance (Chanson, 2001). By increasing of water discharge, the skimming flow appears and is characterised by highly turbulence and the water flows as a coherent stream.

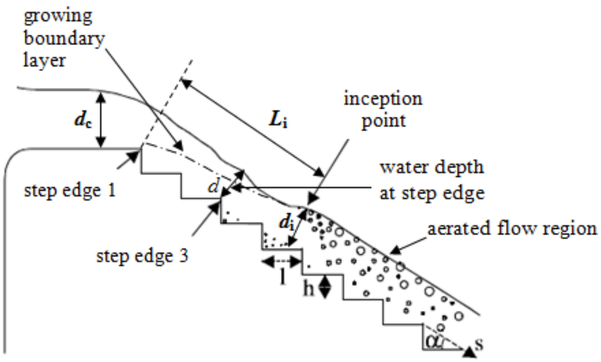

Figure 1 : Position of the inception point in stepped spillway

In the skimming flow regime, air entrainment occurs when the turbulent boundary layer thickness coincides with the water depth (Chanson, 1997). This location is called the inception point (e.g. Figure 1). At the inception point upstream, the flow is smooth and glassy whereas at the downstream of the inception point the flow becomes uniform as the depth of the air-water mixture grows. This position is characterised by two parameters: L_i and d_i (e.g. Fig. 1). The first is the distance from the start of growth of the boundary layer to the point of inception and other is the depth at the point of inception. Knowledge the position of the beginning of aeration in stepped channel is very important to determine the non-aerated zone, which is potentially prone to cavitation damage. The inception point of aeration of stepped spillways is placed further upstream than on smooth spillways. On smooth spillway, the position of the inception point is a function of the discharge and the roughness of the spillway. Wood et al. (1983) proposed an approach to estimate L_i and d_i. On stepped spillway, the position of the inception point is function of the discharge, spillway roughness, step geometry and spillway geometry. Chanson (2001) developed a method to determine the flow properties at the start of air entrainment with slopes greater or equal than 22°. Boes and Hager (2003) also derived a mathematical formula enabling the determination of the distance between the start of the turbulent boundary layer and the inception point. On steep stepped channel, the water depth is less than critical depth and regime flow is supercritical.

The boundary layer thickness (δ) is important element in determining the positions of inception point of free-surface aeration and it's defined as the perpendicular distance from the pseudo-bottom to where the velocity is 99% of the free-stream velocity. Zhang and Chanson (2016) defined relationships to determine the evolution of boundary layer thickness with slopes equal 45°:

$$\frac{\delta}{L} = 0.15 \left(\frac{L}{ks}\right)^{-0.37} \quad (1)$$

where δ is the boundary layer thickness, L is the streamwise distance from the start of the growth of the boundary layer, k_s is the roughness height.

Most experimental research has been developed to characterise the flow over stepped spillway. Today, with the use of high-performance computers and more efficient computational fluid dynamics (CFD) codes, the flow over spillways can be investigated numerically (Table 1) in reasonable time and with reasonable expense.

In this study, the water surface profile was compared by critical depth to determine the regime of gradually varied flow. This paper aims to develop new relationships for determining the variation of water level upstream of the inception point and the distance from the spillway crest to this point, at the same time the contour map of stream function, vorticity and shear strain rate are presented. The simulation results were compared with the experimental data of Zhang and Chanson (2015, 2016).

Table 1: Detailed numerical investigations of air–water flow in stepped chutes.

study	software	Turbulence model	Chute slope	Simulation Results
Chen et al (2002)	NA	*k–ε*	53.13°	Free surface Distribution of velocity and pressure
Bombardelli et al. (2011)	3D-Flow	RNG k-ε LES	53.13°	Water depth profile Boundary layer thickness, velocity and kinetic energy
Cheng et al. (2006)	Fluent	RNG k-ε	53.13°	Air entrainment Distribution of velocity and pressure
Chinnarasri et al (2012)	Fluent	realisable k-ε	26.56°	Distribution of velocity and turbulence intensity
Van alwon et al (2017)	ANSYS Fluent v16.2	realisable k-ε	45°	Air entrainment and pressure
Mohammed et al. (2012)	3D-Flow	RNG k-ε LES	50°	Air entrainment Velocity and pressure
Bentalha et al (2015,2017 and 2018)	Fluent V 6.3	*k–ε*	14° 22° 26°	Inception point location and air entrainment Velocity , static pressure and cavitation
Present study	ANSYS Fluent V 15.0	*k–ε*	45°	Inception point location and free surface Stream function, turbulent viscosity and shear strain

NUMERICAL MODEL

Ansys Fluent computational fluid dynamics (2014) is used to solve the Reynolds-Averaged Navier-Stokes equations are based on momentum and mass conservation of multi-phase flow over stepped spillway. The standard $k-\varepsilon$ turbulence model is adopted to enclose the equations.

Continuity equation:

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u_i}{\partial x_i}=0 \tag{2}$$

Momentum equation:

$$\frac{\partial \rho u_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\rho u_i u_j\right)=-\frac{\partial p}{\partial x_i}+\rho g_i+\frac{\partial}{\partial x_j}\left\{(\mu+\mu_t)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right\} \tag{3}$$

Turbulence kinetic energy equation (k):

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_i}\right]+G_k-\rho\varepsilon \tag{4}$$

Turbulence dissipation rate energy equation (ε):

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\frac{\partial}{\partial x_i}(\rho\varepsilon u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right]+C_{\varepsilon 1}\frac{\varepsilon}{k}G_k-C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k} \tag{5}$$

Where, Gk is production of turbulent kinetic energy which can be given as

$$G_k=\mu_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j}=2\,\mu_t S_{ij}\,\frac{\partial u_i}{\partial x_j} \tag{6}$$

μt is the turbulent viscosity that satisfies

$$\mu_t=\rho C_\mu\frac{k^2}{\varepsilon} \tag{7}$$

Cμ=0.09 is a constant determined experimentally;

σk and σε are turbulence Prandtl numbers for k and ε equation respectively, σk =1.0, σε=1.3,

$C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are ε equation constants, $C_{\varepsilon 1}$=1.44, $C_{\varepsilon 2}$=1.92.

The volume of fluid (VOF) method is applied to simulate the free surface between water and air (Ansys Fluent 2014). In this approach, the tracking interface between air and water is accomplished by the solution of a continuity equation for the volume fraction of water:

$$\frac{\partial \alpha_w}{\partial t} + \frac{\partial \alpha_w u_i}{\partial x_i} = 0\,;\, 0 \le \alpha_w \le 1 \tag{8}$$

Where, α_w is volume fraction of water.

In each cell, the sum of the volume fractions of air and water is unity. So, volume fractions of air denote α_a can be given as

$$\alpha_a = 1 - \alpha_w \tag{9}$$

The geometry of numerical model and boundary conditions are shown in figure 2. The stepped spillway contains 12 identical steps, with 0.1 m height and 0.1 m length by step. The channel slope is θ = 45°.

The two-dimensional numerical domain was divided into unstructured grids (triangular cell) that had a high adaptability to the complex geometry and boundary. Triangular meshes with 1 cm^2 are used.

The boundary conditions in this study are velocity inlet as water inlet and air inlet, outlet as a pressure outlet type. All of the walls as a stationary, no-slip wall. The viscosity layer near to the wall dealt with the standard wall function. The boundary conditions for the turbulent quantities such as k and ε can be calculated from (Fluent 2014):

$$k = \frac{3}{2}\left(U_{avg} I\right)^2 \tag{10}$$

$$\varepsilon = C_u^{3/4} \frac{k^{3/2}}{0.07 D_H} \tag{11}$$

Where, I is turbulence intensity can be estimated from the following formula derived from an empirical correlation for pipe flows:

$$I = 0.16 (Re_{DH})^{-1/8} \tag{12}$$

U_{avg} is the mean velocity of water flow inlet and D_H is the hydraulic diameter.

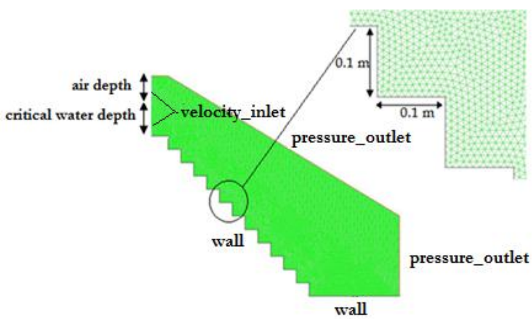

Figure 2: Boundary conditions and numerical model of a stepped spillway

RESULTS AND DISCUSSION

The results of the numerical simulation are compared with those obtained from the physical model. In this study, the position of the inception point are computed and compared with the experimental data by Zhang and Chanson (2015) for $0.7 \leq dc/h \leq 1.7$, where *dc* is critical flow depth and *h* is step height.

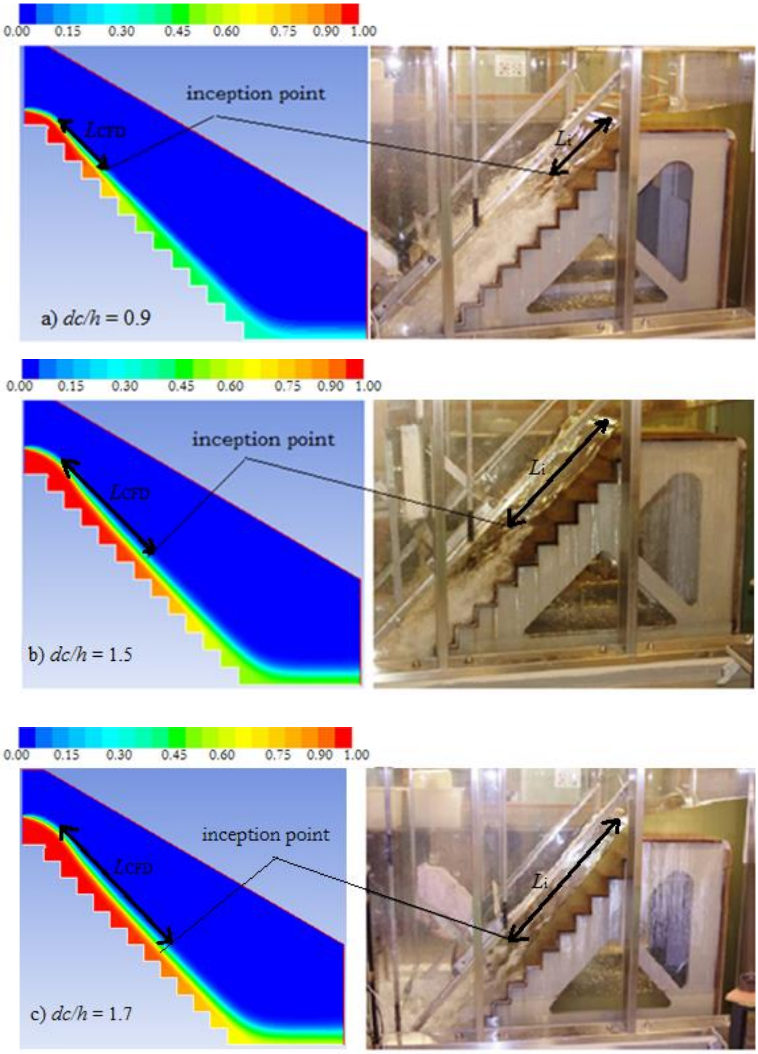

Figure 3: Free surface obtained by Ansys Fluent

Figure 3 compare the start of air entrainment obtained by numerical model and by experimental for different discharges. As can be seen from this figure, the calculated inception point is well agreed with that of measurement. At the inception point, the degree of turbulence was large enough to entrain air into the black water flow (Cheng et al, 2006), and then the volume fraction of water becomes less than unity (Bentalha and Habi, 2015). Boes and Hager (2003), suggest that, the mean air concentration at the inception point is approximately 0.22. This figure indicates also, that the inception point moves toward the basin floor when the discharge increases. Table 2 summarises the characteristics of the inception point of free-surface aeration found by Zhang and Chanson (2015), and by using Ansys Fluent.

Table 2: Measured and computed inception point

	Zhang and Chanson (2016)		Ansys Fluent		Error (%)	
dc/h	L_i *(m)*	d_i *(m)*	L_{CFD} *(m)*	d_{CFD} *(m)*	$\left\lvert\frac{Li - L_{CFD}}{Li}\right\rvert 10^2$	$\left\lvert\frac{di - d_{CFD}}{di}\right\rvert 10^2$
0.9	0.57	0.033	0.43	0.035	24.56	6.06
1	0.57	0.041	0.56	0.043	1.57	4.87
1.1	0.57	-	0.56	0.045	1.57	
1.3	0.85	0.049	0.78	0.052	8.23	6.12
1.5	0.85	0.061	0.91	0.060	7.06	1.64
1.7	1.13	0.062	1.16	0.061	2.65	1.61

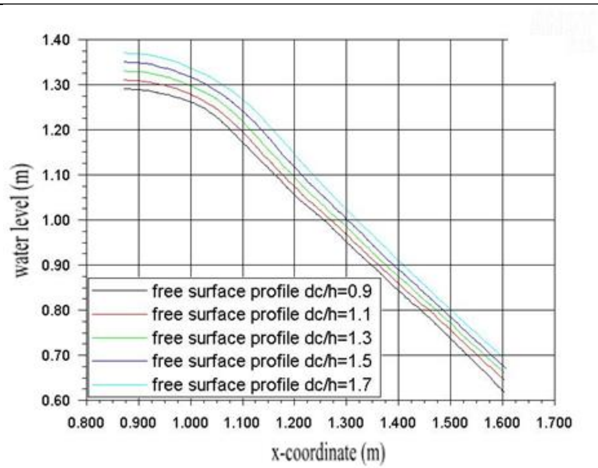

Figure 4: Free surface obtained by Ansys Fluent

Table 2 shows good agreement between the observed and numerical results. As can be seen from this table, the calculated water depth at the inception point is very close to the experimental value. Although the difference between the numerical and experimental inception point locations (L_i and L_{CFD}) are small (see also Fig. 3), except at low discharge the difference is slightly higher (24.56 %). This difference may be due to the VOF model which underestimates the value of air concentration **(**Afshin and Mitra 2012).

Figure 4 shows the simulated water surface profile for $0.7 \leq dc/h \leq 1.7$,where *dc* is critical flow depth and *h* is step height along the stepped spillways with slopes θ = 45°. It is clear that water flow depth increases by increasing of flow rates. The water depth in steep stepped spillways is always less than critical depth it means that the regime flow is supercritical (see figure 5).

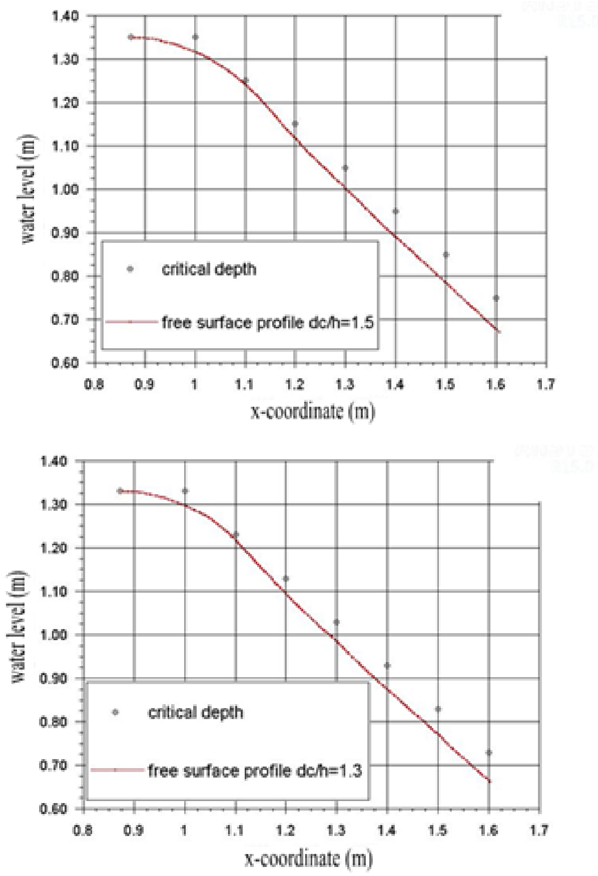

Figure 5: Comparison among critical and water depth

The dimensionless water depth at step edge upstream the inception point obtained by simulation and measured by Zhang and Chanson (2016) are depicted in figure 6. A very satisfactory agreement can be observed between both results. This result is qualitatively similar to those presented by Bombardelli et al. (2011). The profile of normalized water surface elevation was best fitted by the following equations (see figure 6):

$$\frac{d}{L} = \left(\frac{d_c}{h}\right)^{1.2} \left(\frac{L}{ks}\right)^{-1.28} \tag{13}$$

Where d is water depth; L is the streamwise distance from the spillway crest; h is step height; $ks = h\cos(\theta)$ and dc is critical flow depth.

The start of air entrainment is defined by the point where the boundary layer thickness reaches the water depth ($\delta \approx d$), so the distance from the start of growth of boundary layer to the inception point of air L_{incp} can be obtained by equality equation (13) and equation (1):

$$\frac{\delta}{L_{incp}} \approx \frac{d}{L_{incp}} \Rightarrow 0.15 \left(\frac{L_{incp}}{ks}\right)^{-0.37} \approx \left(\frac{d_c}{h}\right)^{1.2} \left(\frac{L_{incp}}{ks}\right)^{-1.28}$$

which can be rearranged to give

$$\frac{L_{incp}}{ks} \approx 8.0426 \left(\frac{d_c}{h}\right)^{1.32} \tag{14}$$

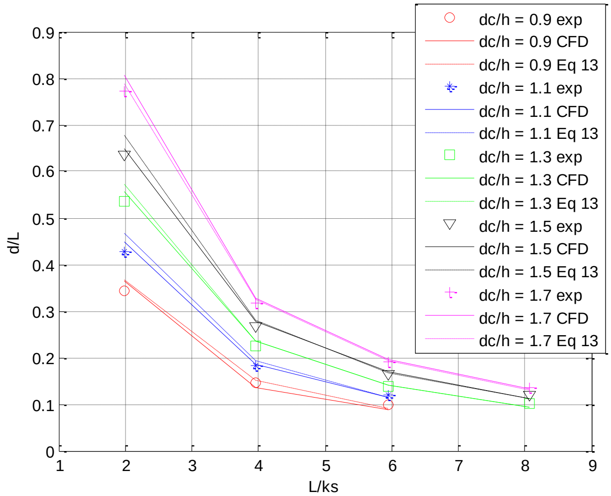

Figure 6: Comparison between equation (13) and normalised water depth obtained by experimental (exp) and by Ansys Fluent (CFD)

From figure 7, the agreement between the locations of inception point found by Zhang and Chanson (2016), by using Ansys Fluent and equation (14) is very good and it's applicable for chute slope equal to 45°.

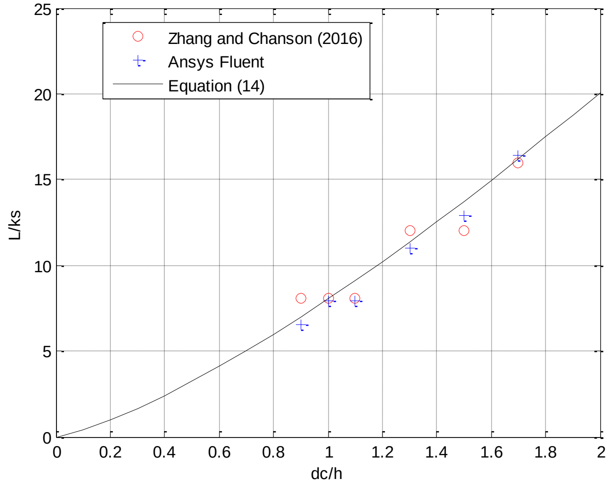

Figure 7: Comparison between dimensionless locations of inception point and the correlation

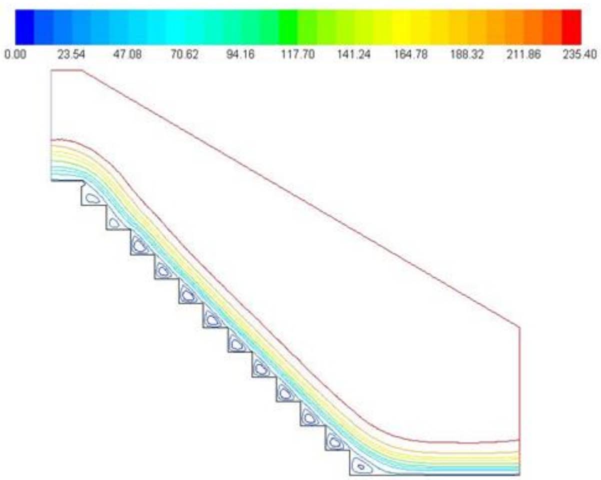

Figure 8: Stream function along stepped spillways for *q*=0.22m^2 s^{-1}

Figure 8 present the isolines of stream function along stepped spillway for *q*=0.22 m^{2}s^{-1}. This figure shows the development of recirculating vortices in step corner. Most of the energy is dissipated by momentum transfer between the skimming flow and the eddy in the interior of the step.

Figure 9 display contour map of turbulent viscosity in stepped spillway for q=0.22 m^2s^{-1}. It can be observed the increasing of turbulent viscosity along the stepped spillway which is the result of the development of recirculating vortices in step corner.

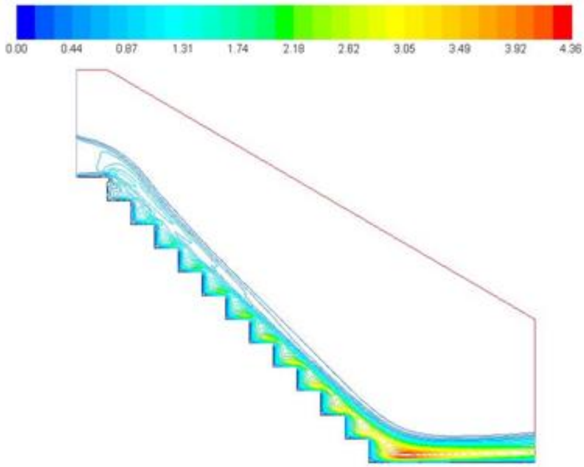

Figure 9: Contour map of turbulent viscosity for q=0.22 m^2s^{-1}.

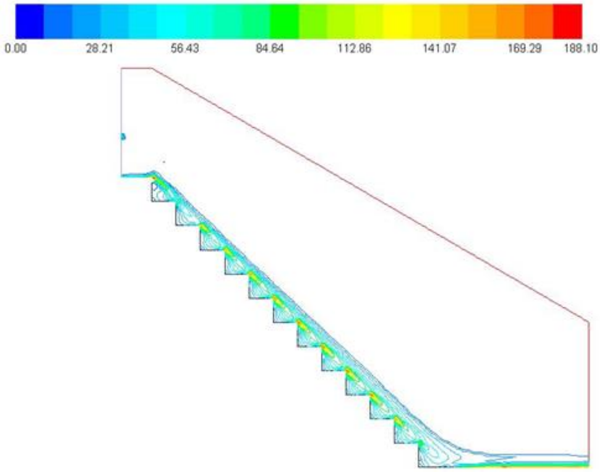

Figure 10 : Voritcity along the stepped spillway for q=0.22m^2 s^{-1}

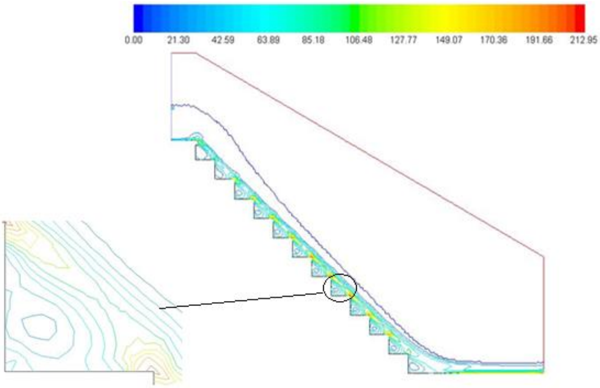

Figure 11: Shear strain rate along the stepped spillway for *q*=0.22m^2 s^{-1}

Figure 10 and 11 present the isolines of shear strain rate contour S_{ij} and magnitude of voritcity ω_z along stepped spillway for q=0.22 m^{2}s^{-1}. As can be seen from these figures, the maximum of ω_z and S_{ij} are located near the pseudo-bottom and is due to clockwise rotation . These results are qualitatively similar to those presented by Quian *et al.* (2009).

CONCLUSION

In this study, flow over stepped spillway was simulated by using Ansys Fluent. Free surface was treated by VOF model and turbulence flow was estimated by k-ε Standard Model. According to the simulation results, the type of flow over stepped channel is supercritical and water depth increase by increasing of depth. Good agreement is found between numerical and experimental results. The calculated dimensionless water depth is well agreed with that of measurement. Two relationships are established (equation 13 and 14) for determining the variation of dimensionless water depth upstream of inception point and the location of the inception point for channel slope θ = 45°. The comparison of computed values with experimental data and thus formulas are well. Due to the interaction between skimming flow and the eddy, the stepped spillway can effectively dissipate the energy. The peak value of shear strain rate and vorticity are located near the pseudo-bottom and is due to clockwise rotation.

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