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DARCY'S EXPERIMENTAL EMPIRICAL RELATION AND ITS EXTENSION

LA RELATION EMPIRIQUE EXPÉRIMENTALE DE DARCY ET SON EXTENSION

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ABSTRACT

Darcy's law is widely used to describe the steady-state laminar incompressible single-phase fluid flow in a fully saturated porous medium at the macroscopic-scale. However, in reality, we will be dealing with transient non-laminar compressible multi-phase fluid flow through a saturated porous medium. In this context, it is important to understand the original framework of Darcy's law; and subsequently, we need to understand clearly, under what circumstances the classical Darcy's law was extended in order to consider the (a) the differential form of Darcy's law; (b) the non-linear relation between pressure gradient and fluid velocity; (b) the transient nature of fluid flow; (c) the fluid flow through heterogeneous and anisotropic reservoirs or aquifers. It has been reemphasized from the present study that the presence of weak inertial effect along with the laminar fluid regime causes the 'non-linear' relation between the macroscopic pressure gradient and the macroscopic fluid velocity, while the strong inertial effect paves the way for the onset of transient nature of fluid flow.

Keywords: Darcy's law; porous medium; differential form; transient flow; inertial effect.

RESUME

La loi de Darcy est largement utilisée pour décrire le flux de fluide monophasé laminaire incompressible à l'état stationnaire dans un milieu poreux entièrement saturé à l'échelle macroscopique. Cependant, en réalité, nous aurons affaire à un écoulement de fluide multiphasique compressible non laminaire transitoire à travers un milieu poreux saturé. Dans ce contexte, il est important de comprendre le cadre original de la loi de Darcy; et par la suite, nous devons comprendre clairement, dans quelles circonstances la loi classique de Darcy a été étendue afin d'examiner a) la forme différentielle de la loi de Darcy; (b) la relation non linéaire entre le gradient de pression et la vitesse du fluide; (b) la nature transitoire de l'écoulement de fluide; c) l'écoulement du fluide à travers des réservoirs ou aquifères hétérogènes et anisotropes. Il a été souligné à nouveau dans la présente étude que la présence d'un faible effet inertiel avec le régime de fluide laminaire provoque la relation «non linéaire» entre le gradient de pression macroscopique et la vitesse du fluide macroscopique, tandis que le fort effet inertiel ouvre la voie à l'apparition de la nature transitoire de l'écoulement de fluide.

Mots-clés: Loi de Darcy; milieu poreux; forme différentielle; écoulement transitoire; effet inertiel.

INTRODUCTION

Darcy's law is the widely used equation that describes the fluid flow in a fully saturated porous medium at the macroscopic-scale. The original Darcy's law provide the expression for a steady-state laminar incompressible single-phase fluid flow. However, in reality, we are forced to deal with flow through porous media under transient non-laminar compressible multi-phase fluid flow. In this context, it becomes essential to understand the original version of Darcy's law and subsequently, we need to clearly understand how the classical Darcy's law was extended in order to consider the non-linearity and transient nature of fluid flow. Also, the concept of flow field and force field as introduced by King Hubbert in 1957 provides the foundation for extending the Darcy's law to be applicable in a non-homogeneous and anisotropic medium. Despite plenty of literature review available on Darcy's equations and its limitations, details on the extension of Darcy's law is very limited; and therefore, the authors have made an attempt to converge all the possible extensions associated with Darcy's law. In addition, Darcy's law is widely used even for a fractured reservoir for different applications (Suresh Kumar and Ghassemi, 2005, 2006; Suresh Kumar and Sekhar 2005; Suresh kumar et al., 2006; Sekhar and Suresh Kumar, 2006; Sekhar et al., 2006; Ghassemi and Suresh Kumar, 2007; Suresh Kumar, 2008; Suresh Kumar et al., 2008, 2009, 2014a; 2014b, 2015, 2016); and hence, a sound understanding on the origin and its extension to various forms remains very critical and fundamental in order to better characterize the aquifer or reservoir of interest. Thus, the focus of the present manuscript is (a) to deduce the relation between flow field and force field as explained by Hubbert; (b) to provide the details of the Darcy's experimental set up with a clarity on how to measure the manometer readings; (c) to dissect and project the rock and fluid properties that are associated with the proportionality constant used by Darcy in terms of their measurable and non-measurable quantities; (d) to deduce the differential form for of Darcy's law; (e) to deduce the non-linear form of Darcy's law; and (f) to deduce the transient or unsteady nature of Darcy's law.

HUBBERT'S FLOW FIELD AND FORCE FIELD

The physical expression for Darcy's law is given by eqn. (1) as suggested by King Hubbert (1957). Equation (1) remains valid for (Newtonian) liquids; and for gases at pressures greater than 20 atmospheres. In eqn. (1), 'N' represents a factor in order to account for the shape of fluid paths; and in turn, the shape of the solid grains, and, it is a dimensionless factor; 'd' represents the characteristic length of the connected pore structure, through which, the continuous fluid flow takes place, and it carries a dimension of length. Since, it is practically, not feasible to get the details on the pore-geometry, the data on the details of the solid grains is considered instead, assuming that the mean pore-size of the pore-space will be approximately equivalent to the mean pore size of the associated solid grains. Thus, both 'N' and 'd' in eqn. (1) pertain to the solid-rock properties. Density (ρ) and viscosity (μ) in eqn. (1) pertain to the fluid properties. The term 'σ' represents the 'volume conductivity' of the system as given in eqn. (2). The term $\vec{E}$ represents the impelling force per unit mass acting upon the fluid as given in eqn. (3). Albeit, we have $\vec{g}$ in the expression of $\vec{E}$ as given in eqn. (3), Darcy's law would remain valid, only, for those flow velocities resulting from dominant viscous forces with insignificant inertial forces. Thus, there are essentially two superposed physical fields in describing the fluid flow through a porous medium, one with the vector $\vec{E}$ representing the force field, while the other vector $\vec{q}$ representing the flow field as given in eqn. (4). The equations (1) – (4) remains valid for incompressible fluid flow with the property as given in eqn. (5). On the other hand, if the fluid density remains a function of pressure only, then, the force field becomes a function of ‘Φ’ called the ‘fluid potential’ as given in eqn. (6); and this ‘fluid potential’ represents the ‘energy per unit mass of the fluid’ with insignificant inertial effects, while with significant compressibility; or, it simply represents the Bernoulli’s equation for a steady, compressible flow with insignificant kinetic energy; and hence, we will not be having an exact differential term for ‘flow energy’ (pressure effect) as given in eqn. (7). Now, substituting eqn. (6) in eqn. (4) yields eqn. (8)

$$\vec{q} = \left(N\, d^2\right)\left(\frac{\rho}{\mu}\right)\left[\vec{g} - \left(\frac{1}{\rho}\right)\nabla p\right] = \sigma\,\vec{E} \tag{1}$$

$$\sigma = \left(N\, d^2\right)\left(\frac{\rho}{\mu}\right) \tag{2}$$

$$\vec{E} = \left[\vec{g} - \left(\frac{1}{\rho}\right)\nabla p\right] \tag{3}$$

$$\vec{q} = \sigma\,\vec{E} \tag{4}$$

$$\mathrm{Curl}\,\vec{E} = 0 \tag{5}$$

$$\vec{E} = -\nabla\Phi \tag{6}$$

$$\Phi = g\,z + \int \frac{dp}{\rho} \tag{7}$$

$$\vec{q} = \sigma\,\vec{E} = -\sigma\nabla\Phi \tag{8}$$

Equation (9) represents the conservation of steady-state, incompressible fluid flow in a porous medium with a constant porosity. Thus, a distinct continuity equation can be deduced for the ‘flow field’ as given in eqn. (9).

$$div\;\vec{q} = \nabla.\vec{q} = 0 \tag{9}$$

By comparing equations (8) and (9), it can be inferred that Darcy’s law connects the ‘flow field’ with the ‘force field’; and hence, it becomes feasible to characterize both the homogeneous as well as heterogeneous porous reservoirs.

It can be clearly seen from the above derivation that the ‘flow field’ as given in eqn. (9) has been deduced based on the assumption that the fluid remains incompressible; and thus, the 'fluid density' term got disappeared (from its spatial derivative). On the other hand, eqns. (6) and (7) associated with the 'force field' as suggested by King Hubbert (1957) has been deduced based on the assumption that the 'fluid density' does not remain incompressible; and hence, the integral form associated with the 'flow energy' term in eqn. (7). Thus, it can be clearly concluded that the 'flow field' and 'force field' as deduced by King Hubbert (1957) have varying nature of 'fluid densities', while both the 'flow field' and the 'force field' have been related with one another as given in eqn. (8).

DARCY'S EXPERIMENT

At the time of Darcy, there was a clear understanding in order to design and deduce a suitable size (diameter) of the pipe, through which the required amount of water (per day) can be transferred from one location to another location associated with the respective frictional losses of the pipe flow. However, there was no such design to deduce the required size (cross-sectional-area) of a porous medium (soil), through which, the required quantum of water (per day) can be transferred from one location to another location associated with the respective frictional losses of the flow through a porous medium or soil. In fact, Darcy tried to apply the already existing pipe flow theory to the porous medium flow but in vain; and hence, Darcy proceeded to obtain the required values associated with the flow through a porous medium by experimental means.

The Darcy's experiment essentially consisted of 3 different parts of a cylindrical vertical column. The top part pertains to the region, where the water gets into the left side of the vertical column and it acts as an inlet. In the same top portion, an input manometer has been attached on the right side of the vertical column in such a way that the centre of the inlet water pipe on the LHS and the centre of the input manometer on the RHS of the vertical column remains perfectly horizontal and coincides with each other. The central or intermediate portion of the cylindrical vertical column (with vertical length 'l' & full horizontal cross-sectional area 'A') pertains to the saturated soil column in which the fluid flow takes place vertically downwards from top to bottom. The third bottom portion is similar to the top portion, where a water outlet is connected and subsequently the water gets collected over a small water container. Thus, the fluid flow rate of water that comes out through the outlet may be measured in terms of volume over the respective time. An output manometer is connected in this bottom region in such a way that the centre of the water outlet pipe and the centre of the output manometer remains perfectly horizontal and coincides with each other. Now, it is very important to be noted that the reference level or the datum for both the input and output manometer measurements remains the same and it pertains to the bottom end of the saturated soil column. Thus, we have only water in the top and bottom regions, while the intermediate region consists of a fully saturated soil through which the fluid flow takes place. The vertical height corresponding to the water level in the input manometer from the datum corresponds to the height 'h_1', while the vertical height corresponding to the water level in the output manometer from the datum corresponds to the height 'h_2'.

The experiments were conducted in between 29th October 1855 and 2nd November 1855, while a few additional experiments were performed between 18th and 19th February 1856. The experiments comprised several series of observations with each series consisting of a different sand that is completely saturated with water. The experimental setup consisted of a cylindrical vertical column (iron pipe) with 0.35 m diameter and 3.5 m vertical height. The inlet and outlet valves were adjusted such that there will be a vertical fluid flow from top to bottom with varying fluid discharge rates (flow rate). For each flow rate, the respective manometer readings were taken; and it was recorded as the 'difference in pressure' expressed in terms of 'meters of water' (measured above the base of the vertical soil column). Thus, a plot was constructed with increasing 'discharge' from 0 to 30 litres/minute (along y-axis); and the respective 'drop in head across the sand' from 0 to 15 m (along x-axis). And, the results showed a beautiful linear relation between the 'discharge' and 'drop in head across the sand' for all the different series. The profiles remained as a straight line, originating from the origin (0,0) with varying slopes as a function of varying sands. Thus, Darcy observed that the water flows vertically downward through the saturated sand column; and he stated that the volume of water 'Q' flowing through the saturated sand column per unit time can be given as expressed in eqn. (10).

$$Q = -KA\frac{h_2 - h_1}{l} \qquad (10)$$

In eqn. (10), the LHS indicates the volume of water collected over the respective period of time; h_1 and h_2 represent the heights corresponding to the manometer readings; 'l' represents the thickness of the vertical soil column; and 'K' represents a proportionality factor. Darcy, then, rearranged eqn. (10) to get the volume of water passing through unit cross-sectional area per unit time as given in eqn. (11).

$$\frac{Q}{A} = q = -K\,\frac{h_2 - h_1}{l} \tag{11}$$

From eqn. (11), the following points can be observed:

The volumetric fluid flow rate (Q) is directly proportional to the changes in water levels between the input and output manometers (h_1-h_2).

The volumetric fluid flow rate (Q) is directly proportional to the full cross-sectional area (A) of the vertical cylindrical column.

The volumetric fluid flow rate (Q) is inversely proportional to the vertical length or thickness (l) of the saturated soil column.

It can be noted from eqn. (11) that Darcy's formulation was a simple algebraic equation in the absence of any differential form. Also, Darcy named 'K' as a simple proportionality constant in the absence of naming anything explicitly for 'K'.

PROPORTIONALITY CONSTANT FROM DARCY'S LAW

The proportionality constant associated with the Darcy's law was assumed to depend on both rock as well as fluid properties. With reference to fluid property, the proportionality constant was assumed to be directly proportional to the 'fluid specific weight', while the same is inversely proportional to the 'fluid viscosity'. It can be noted that both these fluid properties can directly be measured with ease. With reference to the rock property, the proportionality constant was assumed to be directly proportional to the square of the mean grain size (d_{mean}), while the same is inversely proportional to the 'tortuosity' (τ) & the 'specific surface area' (S_s) of the solid grains. It can be noted that these rock properties cannot directly be measured with ease; and hence, these rock properties were later collectively called as 'permeability'. Thus, eqn. (11) gets transformed into eqn. (12) as follows by incorporating the respective rock and fluid properties of the proportionality constant.

$$\frac{Q}{A} = q = -K\,\frac{h_2 - h_1}{l} = -\left\{(k)\left(\frac{\rho\, g}{\mu}\right)\right\}\left(\frac{h_2 - h_1}{l}\right) \tag{12}$$

From eqn. (12), the difference between the notations, 'K' [Hydraulic conductivity, which is a function of both rock as well as fluid properties] & 'k' [intrinsic 'permeability', which is a function of rock property only] can be clearly noted. It should be noted that the SI unit of 'K' is 'm/day', while that of 'k' is 'm^2.

DIFFERENTIAL FORM OF DARCY'S LAW

Equation (12) can be rewritten as expressed in eqn. (13) using the conventional hydrostatic principle $p = \rho g h$.

$$q = -\left\{\left(\frac{k}{\mu}\right)\right\}\left(\frac{p_2 - p_1}{l}\right) \tag{13}$$

Assuming 'l' to be equal to that of 'Δx'; and expressing the changes in pressures in 'differential form' yields eqn. (14).

$$q = -\left(\frac{k}{\mu}\right)\left(\frac{\Delta p}{\Delta x}\right) \tag{14}$$

Equation (14) can be expressed in 'differential form' when the changes in pressures over the respective length happens over a very small distance. In other words, the resultant changes in pressure will be smooth and continuous; and also, the magnitude of the changes in pressure will remain very small, when estimated between its neighbouring/successive/adjacent nodes. Of course, this is a very critical assumption behind extending the simple algebraic Darcy's law into the complex differential equation. Thus, as the Limit Δx→0, the changes in pressures between the successive points separated over the distance 'Δx' can be approximated using the differential equation as expressed in eqn. (15).

As Limit Δx→0;

$$\frac{p(x+\Delta x) - p(x)}{\Delta x} = -\frac{dp}{dx} \tag{15}$$

Thus, the 'differential form' of the equation as expressed in eqn. (15) can be substituted in eqn. (14) in order to yield eqn. (16).

$$q = -\left(\frac{k}{\mu}\right)\left(\frac{dp}{dx}\right) \tag{16}$$

In eqn. (16), the negative sign on the RHS indicates that the 'direction of fluid flow' is opposite to that of the 'direction of the pressure gradient'. Equation (16) is the modified Darcy's law that is widely used in petroleum industry in order to characterize the flow of hydrocarbons associated with a petroleum reservoir. However, eqn. (16) is valid for a one-dimensional single-phase fluid flow.

From eqn. (16), it is clear that the fluid flux per unit cross sectional area per unit time corresponds to 'q'; and this flux is known as 'Darcy flux'. Thus, it should be noted that 'Q' corresponds to the volumetric fluid flow rate in terms of volume per unit time (m^3/day), while 'q' corresponds to the Darcy flux (m^3/day/m^2 or m/day). However, it should be noted that eqn. (16) is valid only for a case with fluid flow through a porous medium with insignificant gravity effect. In reality, either the groundwater aquifer or a petroleum reservoir would not remain either purely horizontal or purely vertical. It will be in an inclined position with reference to the horizontal stratum. In other words, the aquifer or reservoir would be tilted at an angle with reference to its horizontal section. In such cases, fluid may flow either towards gravity (from top to bottom) or fluid may flow against the gravity (from bottom to the top). In the present work, let us assume that the fluid flows within the aquifer or reservoir against the gravity. Thus, the fluid flows from the region of higher potential to the region of lower potential. Now, the hydraulic heads at the inlet (h_1) and at the outlet (h_2) needs to be estimated. The 'hydraulic head' at any given point (entry point @ the centre of the reservoir/aquifer thickness) pertains to the difference between the 'datum head' (D → the depth or elevation at the surface level that is lying above the aquifer) and 'pressure head' (p/γ) measured from that datum up to the centre of the aquifer/reservoir inlet/outlet, assuming that the datum (or the reference level) lies above the aquifer or reservoir. The inclined length of the aquifer/reservoir between the inlet and the outlet points pertains to the 'length of the saturated aquifer or reservoir'.

Hydraulic head at inlet: $h_1 = D_1 - \left(\frac{p_1}{\gamma}\right)$ (17)

Hydraulic head at outlet: $h_2 = D_2 - \left(\frac{p_2}{\gamma}\right)$ (18)

Hydraulic gradient:

$$\left(\frac{1}{l}\right)(h_1 - h_2) = \left(\frac{1}{l}\right)\left[\left(D_1 - \frac{p_1}{\gamma}\right) - \left(D_2 - \frac{p_2}{\gamma}\right)\right] = \left(\frac{1}{l}\right)\left[(D_1 - D_2) - \frac{1}{\gamma}(p_1 - p_2)\right] \quad (19)$$

Now, eqn. (19) can be written in 'difference form' as given in eqn. (20).

$$\left(\frac{1}{l}\right)(h_1 - h_2) = \left(\frac{1}{l}\right)\left[(\Delta D) - \frac{1}{\gamma}(\Delta p)\right] \quad (20)$$

Substituting eqn. (20) in eqn. (12) yields eqn. (21).

$$q=-\left\{(k)\left(\frac{\rho g}{\mu}\right)\right\}\left(\frac{h_2-h_1}{l}\right)=-\left\{(k)\left(\frac{\rho g}{\mu}\right)\right\}\left(\frac{1}{l}\right)\left[\frac{1}{\gamma}(\Delta p)-(\Delta D)\right] \quad (21)$$

Equation (21) can be rewritten as given in eqn. (22).

$$q=-\left\{\left(\frac{k}{\mu}\right)\right\}\left(\frac{1}{l}\right)[(\Delta p)-\gamma(\Delta D)]=-\left(\frac{k}{\mu}\right)\left[\frac{\Delta p}{l}-\gamma\frac{\Delta D}{l}\right] \quad (22)$$

Now, by introducing differential calculus, and as the limit '*l*' or 'Δ*l*' or 'Δx' tends to 'zero'; then, the quotient of the 'pressure difference' (Δp) over its 'flow length' (*l or* Δ*l or* Δx) becomes equal to the negative pressure gradient. Thus, eqn. (22) can be written in 'differential form' as expressed in eqn. (23).

$$q=-\left(\frac{k}{\mu}\right)\left[\frac{\partial p}{\partial l}-\gamma\frac{\partial D}{\partial l}\right] \quad (23)$$

For multi-dimensional fluid flow, eqn. (23) can be represented in a more general form using gradient operator as expressed in eqn. (24).

$$q=-\left(\frac{k}{\mu}\right)[\nabla p-\gamma\,\nabla D] \quad (24)$$

Equation (24) can be extended to characterize the multi-phase fluid flow using the expression as given in eqn. (25).

$$q=-k\left(\frac{k_r}{\mu}\right)[\nabla p-\gamma\,\nabla D] \quad (25)$$

In eqn. (25), 'k' represents the 'absolute permeability tensor', while 'k_r' represents the 'relative permeability'. The product of the 'absolute permeability' and the 'relative permeability' yields the 'effective permeability'. Thus, the 'Darcy flux' for multi-phase, multi-dimensional fluid flow can be defined as expressed in eqn. (26).

$$\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = -\left(\frac{k_r}{\mu}\right)\begin{bmatrix} k_{xx}\, k_{xy}\, k_{xz} \\ k_{yx}\, k_{yy}\, k_{yz} \\ k_{zx}\, k_{zy}\, k_{zz} \end{bmatrix}\begin{bmatrix} \dfrac{\partial p}{\partial x} - \gamma\dfrac{\partial D}{\partial x} \\ \dfrac{\partial p}{\partial y} - \gamma\dfrac{\partial D}{\partial y} \\ \dfrac{\partial p}{\partial z} - \gamma\dfrac{\partial D}{\partial z} \end{bmatrix} \tag{26}$$

NON-LINEAR & TRANSIENT FORMS OF DARCY'S LAW

Darcy's law describes a steady-state fluid flow through a saturated porous medium. Thus, original Darcy's law is governed by the elliptic PDEs in the absence of any time frame or transient nature. At steady-state conditions, the Darcy flux [also called 'Darcy velocity' or 'Macroscopic flow velocity' or 'Macroscopic mean velocity' or 'Nominal velocity' or 'Superficial velocity'] has a direct linear relationship with the fluid pressure gradient. This pressure gradient needs to be looked at the macroscopic-scale (and not at the microscopic-scale as used in NSE); and hence, it should be addressed properly as 'macroscopic pressure gradient'. Thus, the original Darcy's law assumes that the time dependency of the 'Darcy flux', i.e., the 'macroscopic mean fluid velocity' and its associated 'macroscopic pressure gradient' remains insignificant. However, in reality, the 'macroscopic pressure gradient' does not remain to be independent of time, but, it strongly becomes a function of time as we very frequently encounter a rapid or abrupt change in the 'macroscopic pressure gradient' in various field applications (in the absence of considering any turbulence). Thus, Darcy's law had to be modified that takes into account the 'time dependency' of the 'macroscopic pressure-gradient' in order to use the same law in real field applications. Further, the fluid flow can be classified into linear and non-linear flows depending on whether the 'macroscopic pressure gradient' remains 'linear' or 'non-linear' with reference to the 'macroscopic mean fluid velocity'. For steady and linear fluid flow, the fluid flow should be driven by dominant viscous forces in the absence of capillary and gravity forces; and the macroscopic mean fluid velocity should remain low in such a way that the Pore Reynolds Number should not exceed unity for Darcy's law to be valid. Under such circumstances, the 'macroscopic pressure gradient' is directly proportional to the 'macroscopic mean fluid velocity' as given in eqn. (27).

$$\nabla p = aU \tag{27}$$

Equation (27) represents that the 'macroscopic pressure gradient' is linearly balanced by the 'macroscopic mean fluid velocity'. The coefficient 'a' in eqn. (27) represents the forces resulting from the associated rock and fluid properties. It can further be noted that the fluid flow can still remain 'laminar' in the absence of any 'eddies' or 'turbulence', but still with significant inertial effect. Thus, the pore Reynolds number may exceed unity that describes 'laminar' fluid flow through a porous medium with significant inertial effect. In such cases, the 'macroscopic pressure gradient' cannot be balanced by the 'macroscopic mean fluid velocity' by linear means; and the relation between the above two parameters becomes non-linear. Forchheimer in 1901 corrected eqn. (27) and introduced an additional momentum resulting from the inertial effect; and this additional momentum varied non-linearly (i.e., quadratically) with the 'macroscopic pressure gradient' rather than having a linear variation as given in eqn. (28).

$$\nabla p = aU + bU^2 \tag{28}$$

The coefficient 'b' associated with eqn. (28) is related with the forces exerted by rock and fluid properties resulting from 'weak' inertial effect. Thus, the presence of 'weak inertial effect' along with the laminar fluid flow introduces a non-linear effect in Darcy's equation.

It can be critically noted that the 'transitional fluid regime' can be characterized either by weak inertia or strong inertia. When the fluid flow is characterized by laminar flow with weak inertial effect, eqn. (28) can be used. However, when the laminar flow in a porous medium is characterized by the strong inertial effect, a complete 'phase shift' between 'macroscopic pressure gradient' and the 'macroscopic mean fluid velocity' happens resulting from the acceleration of the fluid particles initiated by the strong inertial effect. Thus, under strong inertial effect, the fluid particle at a particular location gets accelerated; and this acceleration of the fluid particle keeps varying as a function of time. This variation in the fluid acceleration is associated with the variation in the 'macroscopic mean fluid velocity' as expressed in eqn. (29).

$$\nabla p = aU + bU^2 + c\frac{\partial U}{\partial t} \tag{29}$$

In equation (29), the coefficient 'c' is related with the forces exerted by rock and fluid properties resulting from 'strong' inertial effect. Thus, the presence of 'strong inertial effect' along with the laminar fluid flow introduces the 'transient' effect in Darcy's equation.

For fluid flows through porous media with significant inertial effect, eqn. (29) can be simplified by getting rid-off the non-linear term present in eqn. (29); and the simplified form can be expressed as given in eqn. (30).

$$\nabla p = aU + c\frac{\partial U}{\partial t} \tag{30}$$

It can be noted that the estimation of the coefficient 'c' is not straight forward and it is relatively difficult to get it by laboratory-scale experimental means.

CONCLUSIONS

1. Since Darcy's law connects the 'flow field' with the 'force field'; it becomes feasible to characterize not only the homogeneous aquifers or reservoirs but also the heterogeneous porous aquifers or reservoirs.
2. The Darcy's law did not consider any gravity effect despite the fact that the experiment was conducted in a vertical cylindrical saturated sand pack; and not in a horizontal set up.
3. The proportionality constant associated with the Darcy's law is a complex function of measurable fluid properties; and non-measurable rock properties called the 'intrinsic permeability'.
4. The original algebraic form of Darcy's equation gets translated into a complex partial differential equation by assuming that the resultant changes in pressures between any two successive points in an aquifer or reservoir remains very small; and subsequently, the resulting spatial distribution of fluid pressure is assumed to follow a smooth and continuous profile in the absence of any steep gradient.
5. It has been reemphasized that the presence of 'weak inertial effect' along with the laminar fluid flow introduces a non-linear effect in Darcy's equation.
6. It has been reemphasized that the presence of 'strong inertial effect' along with the laminar fluid flow introduces the transient or unsteady nature in Darcy's equation.

REFERENCES

FORCHHEIMER P. (1901). Wasserbewegung durch boden, Zeitschrift Des Vereines Deutscher Ingenieure, Vol.45, p.1781.

KING HUBBERT M. (1957). Darcy's Law and the Field Equations of the Flow of Underground Fluids, Hydrological Sciences Journal, Vol.2, Issue 1, pp.23-59. DOI:10.1080/02626665709493062

GHASSEMI A., SURESH KUMAR G. (2007). Changes in fracture aperture and fluid pressure due to thermal stress and silica dissolution/precipitation induced by heat extraction from subsurface rocks, Geothermics (Elsevier Science Publications), Vol.36, Issue 2, pp.115-140.

DOI:10.1016/j.geothermics.2006.10.001 http://www.sciencedirect.com/science/article/pii/S0375650506000666 http://dx.doi.org/10.1016/j.geothermics.2006.10.001

SEKHAR M., SURESH KUMAR G., MISHRA D. (2006). Numerical Modeling and Analysis of Solute Velocity and Macrodispersion for Linearly and Nonlinearly Sorbing Solutes in a Single Fracture with Matrix Diffusion, Journal of Hydrologic Engineering (ASCE), Vol. 11, Issue 4, pp. 319-328.

DOI:http://dx.doi.org/10.1061/(ASCE)1084-0699(2006)11:4(319) http://ascelibrary.org/doi/full/10.1061/(ASCE)1084-0699(2006)11:4(319) http://dx.doi.org/10.1061/(ASCE)1084-0699(2006)11:4(319)

SEKHAR M., SURESH KUMAR G. (2006). Modeling Transport of Linearly Sorbing Solutes in a Single Fracture: Asymptotic Behavior of Solute Velocity and Dispersivity, Geotechnical and Geological Engineering (Springer Science Publications), Vol. 24, Issue 1, pp. 183-201.

DOI:10.1007/s10706-004-3053-7 http://link.springer.com/article/10.1007/s10706-004-3053-7

SURESH KUMAR G., SEKHAR M. (2005). Spatial Moment Analysis for Transport of Nonreactive Solutes in a Fracture-Matrix System, Journal of Hydrologic Engineering (ASCE), Vol. 10, Issue 3, pp. 192-199.

DOI:http://dx.doi.org/10.1061/(ASCE)1084-0699(2005)10:3(192) http://ascelibrary.org/doi/abs/10.1061/(ASCE)1084-0699(2005)10%3A3(192) http://dx.doi.org/10.1061/(ASCE)1084-0699(2005)10:3(192)

SURESH KUMAR. G., GHASSEMI A. (2005). Numerical Modeling of Non-Isothermal Quartz Dissolution/Precipitation in a Coupled Fracture-Matrix System, Geothermics (Elsevier Science Publications), Vol. 34, Issue 4, pp. 411-439.

DOI:10.1016/j.geothermics.2005.04.003
http://www.sciencedirect.com/science/article/pii/S0375650505000507
http://dx.doi.org/10.1016/j.geothermics.2005.04.003

SURESH KUMAR G., GHASSEMI A. (2006). Spatial Moment Analysis for One-Dimensional Nonisothermal Quartz Transport and Dissolution/Precipitation in a Fracture-Matrix System, Journal of Hydrologic Engineering (ASCE), Vol. 11, Issue 4, pp. 338-346.

DOI:http://dx.doi.org/10.1061/(ASCE)1084-0699(2006)11:4(338)
http://ascelibrary.org/doi/abs/10.1061/(ASCE)1084-0699(2006)11%3A4(338)
http://dx.doi.org/10.1061/(ASCE)1084-0699(2006)11:4(338)

SURESH KUMAR G., SEKHAR M., MISRA D. (2006). Time Dependent Dispersivity Behavior of Non-Reactive Solutes in a System of Parallel Fractures, Hydrology and Earth System Sciences Discussions (Copernicus Publications), Vol. 3, Issue 3, pp. 895-923.

http://www.hydrol-earth-syst-sci-discuss.net/hessd-2006-0048/

SURESH KUMAR G. (2008). Effect of Sorption Intensities on Dispersivity and Macro-dispersion Coefficient in a Single Fracture with Matrix Diffusion, Hydrogeology Journal (Springer), Vol.16, Issue 2, pp. 235-249.

DOI:10.1007/s10040-007-0234-5 http://link.springer.com/article/10.1007/s10040-007-0234-5

SURESH KUMAR G., SEKHAR D MISHRA D. (2008). Time dependent dispersivity of linearly sorbing solutes in a single fracture with matrix diffusion, Journal of Hydrologic Engineering (ASCE), Vol.13, Issue 4, pp. 250-257.

DOI:http://dx.doi.org/10.1061/(ASCE)1084-0699(2008)13:4(250)
http://ascelibrary.org/doi/full/10.1061/(ASCE)1084-0699(2008)13:4(250)
http://dx.doi.org/10.1061/(ASCE)1084-0699(2008)13:4(250)

SURESH KUMAR G. (2009). Influence of Sorption Intensity on Solute Mobility in a Fractured Formation, Journal of Environmental Engineering (ASCE), Vol.135, Issue 1, pp. 1-7.

DOI:http://dx.doi.org/10.1061/(ASCE)0733-9372(2009)135:1(1)
http://ascelibrary.org/doi/abs/10.1061/(ASCE)0733-9372(2009)135%3A1(1)
http://dx.doi.org/10.1061/(ASCE)0733-9372(2009)135:1(1)

SURESH KUMAR G. (2014a). Mathematical Modeling on Transport of Petroleum Hydrocarbons in Saturated Fractured Rocks, Sadhana – Academy proceedings in Engineering Sciences (Springer Publications), Vol. 39, Issue 5., pp. 1119-1139.

DOI:10.1007/s12046-014-0284-z http://rd.springer.com/article/10.1007/s12046-014-0284-z?no-access=true

SURESH KUMAR G. (2014b). Mathematical Modeling of Groundwater Flow and Solute Transport in a Saturated Fractured Rock using Dual-Porosity Approach, Journal of Hydrologic Engineering (ASCE), Vol. 19, Issue 12., pp. 04014033-1 – 04014033-8.

DOI:10.1061/(ASCE)HE.1943-5584.0000986 http://ascelibrary.org/doi/10.1061/%28ASCE%29HE.1943-5584.0000986

SURESH KUMAR G. (2015). Subsurface Transport of Nuclear Wastes in the Indian Subcontinent, ISH Journal of Hydraulic Engineering (Taylor and Francis Publications), Vol. 21, Issue 2, pp. 162-176.

DOI:10.1080/09715010.2014.984249 http://dx.doi.org/10.1080/09715010.2014.984249

SURESH KUMAR G. (2016). Modeling Fluid Flow through Fractured Reservoirs: Is it different from Conventional Classical Porous Medium?, Current Science (Current Science Association & Indian Academy of Sciences), Vol. 110, Issue 4., pp. 695-701.

DOI:10.18520/cs/v110/i4/695-701 http://www.currentscience.ac.in/Volumes/110/04/0695.pdf