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DISCHARGE COEFFICIENT OF SHAFT SPILLWAY UNDER SMALL HEADS

COEFFICIENT DE DEBIT DES EVACUATEURS EN PUITS SOUS FAIBLES CHARGES

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ABSTRACT

Among components of hydraulic dam constructions, the crucial part is the spillway, from which reliable operation depends on the trouble-free operation. In the event of a structural failure in the construction of the facility or an incorrect calculation of its carrying capacity, an emergency situation may arise.

To ensure trouble-free reliable operation of the spillways, it is very important to correctly design it and calculate the capacity of the facilities themselves, allowing water to be discharged to the downstream during periods of floods and rainfall events.

The present study deals with the analysis of the existing most usable formulas of calculation of weir flow capacity and theoretical development of the weir flow efficiency of a shaft spillway under small heads on the crest.

From the values obtained, it is possible to deduce the same conclusions as those of weirs with a sharp edge with or without a vacuum for small heads.

We can assume that the polynomial approximation of the experimental values made by the authors for the flow coefficient reflects the physics phenomenon of flow through the crest of the receiving funnel shaft spillway.

Keywords: Discharge coefficient, flow, shaft spillway, receiving funnel.

RESUME

Parmi les constructions hydrauliques les plus importantes dans un barrage, c’est l'évacuateur de crue. En cas de défaillance structurelle dans la construction de l'installation ou de calcul incorrect de sa capacité de charge, une situation d'urgence peut survenir.

Pour assurer un fonctionnement fiable et sans problème des évacuateurs, il est très important de bien calculer le débit et la capacité des installations elles-mêmes, ce qui permet de déverser l'eau en aval pendant les périodes d'inondation et les inondations pluviométriques.

La présente étude porte sur les conditions hydrauliques et l'utilisation des formules les plus connues pour déterminer les coefficients de débit des évacuateurs en puits avec un entonnoir de réception d’eau à entrée libre dans un premier cas et un entonnoir à paroi d’orientation du flux dans un deuxième cas.

A partir des valeurs obtenues, il est possible de déduire les mêmes conclusions que celles des déversoirs à arête vive avec ou sans vide pour de petites charges.

Nous pouvons supposer que l'approximation polynomiale des valeurs expérimentales faite par les auteurs sur le coefficient d'écoulement reflète le phénomène physique de l'écoulement à travers la crête du déversoir par le biais de l’entonnoir de réception.

Mots clés : coefficient de débit, écoulement, évacuateur en puits, entonnoir de réception.

INTRODUCTION

One of the main structures of reservoir hydro systems is a weir designed to discharge excess water. The costs for the construction of spillways should meet the requirements of reliability and safety of the spillway system operation, in connection with which they are calculated for the discharges of rare frequency, forcing to increase the size of the spillway structures and increase the cost of the entire complex. To reduce the cost of construction of spillway structures, the spillways used that require a minimum amount of expensive construction materials (Moise, 1970) and (Novak et al, 2007).

Shaft spillways are widely used in the practice of hydraulic engineering because of their advantages such as: compact design, high flow and automatic operation. They refer to surface spillways; the determining factors for the selection of this spillway are: the formation of sudden floods - increase in speed with maximum flow, seismicity of the construction area. (Moise ,1970).

Shaft spillway is also known as morning glory spillway. It is one of the spillway types used to pass additional waters and floods from the upstream to the downstream of dams. It is composed of a circular crest that directs the flow to an inclined or vertical axis, which is connected to a low gradient tunnel. It is one of the major water conveyance that is an emergency spillway. Photographs 1 and 2 show a shaft spillway in operation, and the funnel-shaped mouth.

Photo 1 : Shaft spillway- vertical shaft with funnel-Shaped mouth

Photo 2 : Shaft spillway in operation under small head over the crest

The morning glory spillway may operate with free flow (Photo 2), or, when designed properly, it can operate submerged. For free-flow, the discharge characteristics are similar to those of a straight overflow-dam section; that is, an increase in discharge is proportional to the three-halves power of the head (ASCE, 1956). When submerged, the flow characteristics change completely; an increase in discharge is proportional to the square root of the head. After submerged flow commences, a further increase in head on the crest results in a very limited increase in discharge. Thus, if a morning-glory spillway is designed to operate submerged, an additional factor of safety is necessary to guard against the spillway capacity ever being exceeded.

One of the parameters that characterizes spillways in general and in particular the shaft spillway is the discharge coefficient denoted C_d . It represents the ratio of the actual discharge to the theoretical discharge. It is generally find out from laboratory measurements. The variation of the discharge laboratory values, with regard to the relative total head *H/P* where *P* is the spillway height and *H* is the total head over the spillway crest, generally follows a polynomial equation ending up with a constant. This represents in fact the discharge coefficient value for small head, i.e. *H* theoretically tending towards zero.

The purpose of this paper is to develop a theoretical method of the discharge coefficient calculation for a polygonal section weir with a cylindrical head for two cases: first, the water intake inlet is free, second in the presence of a flow guide piers. The discharge coefficient of the shaft spillway, operating under small head, is worked out theoretically, through a rigorous approach. This takes into account the effect of the Reynolds number governing the laminar flow regime. The final result is a quadratic equation in C_d , where the influence, although relative, of the capillary number, is highlighted.

REVIEW OF LITERATURE

One of the primary objectives of the calculation fo hydraulic structures is to determine the capacity of the spillway. All types of spillways are calculated according to the following general dependence (Pavlovsky, 1939):

$$Q = C_d b\sqrt{2g}H^{3/2} \tag{1}$$

Where:

C_d = Discharge coefficient depending on the spillway design;

b = spillway length;

g = 9.81m/s^2 – acceleration due to gravity;

H = Total head on the spillway crest.

The discharge coefficient C_d is not amenable to theoretical determination, and therefore, is determined experimentally. The discharge coefficient of free flow shaft spillways with a practical head is recommended to be calculated by the generalized Moise relation (Slissky, 1986):

$$C_d = \left(0.597 - 0.136\frac{H_d}{R}\right)\sigma_1\sigma_2\sigma_3 k \tag{2}$$

Where

σ_1 = coefficient, consider the conditions of water supply;

σ_2 = coefficient, consider the depth of water in front of the intake funnel;

σ_3 = coefficient, consider the ratio H/H_d of the actual head H to the design head H_d;

k = Coefficient considering the effect of a counter-rotary design in the intake funnel.

R= radius of water receiving funnel.

The discharge coefficient of weirs with are calculated according to the table of Rozanov (1958), depending on the ellipticity noted e/r_f, taking into account the pressure coefficient of incompleteness, the values of which are also determined according to the corresponding table [*e* -axis length of an ellipse, r_f –fictitious intake weir radius] (Recommendations for hydraulic calculation of weirs,1974).

The coefficient *k* values for incomplete pressure are given for $H/H_d \geq 0.3$ by Moise (1970) and the discharge coefficient values are given for $H_0/r_f \geq 0.1$ and $H/H_d \geq 0.3$ by Rozanov (1958). There are no recommendations for determining flow coefficients at lower values of these parameters. At the same time, structures such as mine and trench spillways are designed in large numbers to automatically spillways without installing gates on their crest (Rozanov, 1958). In this case, their work begins immediately after raising the water level in the reservoir by an amount that ensures overcoming the surface tension forces on the crest of the spillway.

An increase in both head and flow rate entails the transformation of the flood hydrograph, which ultimately determines the required flow rate of the spillway.

When calculating spillways operating according to a sharp-edge spillway scheme, empirical equations are used in which the discharge coefficient C_d is a function of the head *H* on the spillway crest.

Automatic action, not equipped with gates for regulating the flow, begin to work with almost zero head, and the estimated flow is missed with a significant transformation of the flood hydrograph which affects the maximum flow rate and the size of the spillway structure.

All the equations for determining the discharge coefficients of the weirs, which are given in the manuals and reference works on hydraulics, were obtained following the mathematical processing of the experimental data. The equations approaching these results cover only the range of experiences.

For weirs without lateral compression, Bazin's formula (1898) for C_d is:

$$C_d = \left(0.405 + \frac{0.003}{H}\right) \cdot \left[1 + 0.55 \frac{H^2}{(H+P)^2}\right] \tag{3}$$

The SIA, The Swiss society of engineers and architects (1926) gives C_d for a sharp-edged weir as:

$$C_d = 0.41\left(1 + \frac{1}{1000H + 1.6}\right) \cdot \left[1 + 0.5 \frac{H^2}{(H+P)^2}\right] \tag{4}$$

Rehbock (1929) proposed the following C_d formula for weirs with circular section of the funnel:

$$C_d = \frac{2}{3} \cdot \left[0.312 + \sqrt{0.30 - 0.01 \cdot (5 - \frac{H}{r})^2} + 0.09 \cdot \frac{H}{p}\right] \tag{5}$$

while Chougaev's formula (1971) is:

$$C_d = 0.402 + 0.054 \frac{H}{P} \tag{6}$$

Hugly (1937) gave the C_d following relationship for a spillway with lateral compression:

$$C_d = \left(0.405 + \frac{0.0027}{H} - 0.03\frac{B-b}{B}\right)\cdot\left[1 + 0.55\frac{b^2}{B^2}\cdot\frac{H^2}{(H+P)^2}\right] \quad (7)$$

Moise (1970) proposed empirical equations to determine the discharge coefficient of a shaft spillway as a function of the H / Hc ratio (within the limits of the calculated relative heads Hc / R = 0.2,0.3,0.4 and 0.5) or R-radius of the water receiving funnel. He gave approximations of third-degree polynomials when H / Hd = 0, as follows:

$$C_{d\,H/R=0.5} = 0.009\left(\frac{H}{H_d}\right)^3 - 0.544\left(\frac{H}{H_d}\right)^2 + 0.751\left(\frac{H}{H_d}\right) + 0.225$$

(8)

$$C_{d\,H/R=0.5} = 0.061\left(\frac{H}{H_d}\right)^3 - 0.424\left(\frac{H}{H_d}\right)^2 + 0.735\left(\frac{H}{H_d}\right) + 0.207 \quad (9)$$

$$C_{d\,H/R=0.4} = -0.208\left(\frac{H}{H_d}\right)^3 - 0.097\left(\frac{H}{H_d}\right)^2 + 0.546\left(\frac{H}{H_d}\right) + 0.230 \quad (10)$$

$$C_{d\,H/R=0.2} = 0.503\left(\frac{H}{H_d}\right)^3 + 0.520\left(\frac{H}{H_d}\right)^2 + 0.178\left(\frac{H}{H_d}\right) + 0.290 \quad (11)$$

Rozanov (1959) also gave relations determining the discharge coefficients Cd and $K_{H/Hd}$: coefficient of load incompleteness as a function of the relative design load

H_d / r_f or r_f is the relative radius of weir of the spillway.

The equations of the flow coefficients by introducing the ellipsoidal value of the head of the weir with vacuum e/f (e/f = 3, 2 and 1) are as follows:

$$C_{d\,e/f=3} = 0.003\left(\frac{H_d}{r_f}\right)^3 - 0.03\left(\frac{H_d}{r_f}\right)^2 + 0.119\left(\frac{H_d}{r_f}\right) + 0.406 \quad (12)$$

$$C_{d\,e/f=3} = -0.002\left(\frac{H_d}{r_f}\right)^3 - 0.0005\left(\frac{H_d}{r_f}\right)^2 + 0.061\left(\frac{H_d}{r_f}\right) + 0.430 \quad (13)$$

$$\mathrm{C}_{d\ \mathrm{e/f}=3} = 0.006\left(\frac{H_d}{r_f}\right)^3 - 0.038\left(\frac{H_d}{r_f}\right)^2 + 0.111\left(\frac{H_d}{r_f}\right) + 0.412 \qquad (14)$$

This approximation allowed extrapolating the data up to $H / H_d = 0$ by the use of the head incompleteness coefficient k $_{H/Hd}$ which is given by the equation:

$$K_{\frac{H}{H_d}} = 0.0441\left(\frac{H}{H_d}\right)^3 - 0.192\left(\frac{H}{H_d}\right)^2 + 0.373\left(\frac{H}{H_d}\right) + 0.775 \qquad (15)$$

When comparing research results concerning the determination of discharge coefficients for weirs in shafts with a polygonal transverse profile, Rozanov (1958) gave a Rehbock equation as a function of the relative load H'= H / r = 0.3, r - radius of curvature of the weir in second degree polynomial form as follows:

$$C_d = 0.005H'^2 + 0.106H' + 0.265 \qquad (16)$$

The author has carried out model hydraulic studies of a weir in a polygonal section well with a cylindrical intake for two cases: first, the water reception inlet is free, second in the presence of a flow guide piers (Gouryev 2009). The equations for the discharge coefficients obtained are as follows:

- **Free entrance of intake funnel**

$$\mathrm{C}_d = -0.25\left(\frac{H}{H_c}\right)^2 + 0.508\frac{H}{H_c} + 0.246 \qquad (17)$$

- **With flow guide piers**

$$\mathrm{C}_d = -0.15\left(\frac{H}{H_c}\right)^2 + 0.49\frac{H}{H_c} + 0.232 \qquad (18)$$

THEORETICAL CONSIDERATIONS

Consider the problem of varying the discharge coefficient C_d while reducing to zero the head H on the crest of the weir threshold. In this case, the threshold ratio is as $r / H \to \infty$, and the movement of water in a small area on the crest of the threshold can be considered as movement in a large weir threshold.

Let us select on the threshold crest two vertical sections limiting a compartment of length and width b = 1m (Fig. 1). At the first section, the flow depth is h_1 and at the second section, the flow depth is h_2. The slope of the free surface is I (Fig.1).

Denote by *q* the flow rate per unit width of the weir crest. In this case, the water velocity in the first section is $V_1 = q / h_1$, and $V_2 = q / h_2$ in the second section.

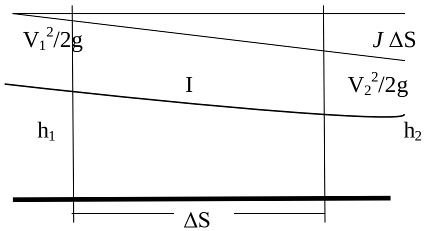


Figure 1: water movement at small depths

Applying the Bernoulli equation between the two sections shown in Fig.1 results in:

$$h_1 + \frac{\alpha_1 \cdot V_1^2}{2g} = h_2 + \frac{\alpha_2 \cdot V_2^2}{2g} + J \cdot \Delta S \tag{19}$$

where α_1 and α_2 are the Coriolis coefficients which can be considered as equal such that $\alpha_1 = \alpha_2 = \alpha$.

Let us assume, for any depth *h* along ΔS, what follows:

$$h_2 = kh_1 \tag{20}$$

k can be considered as a correction factor such as $0 \le k \le 1$. According to the Eq.(20), the depth h_2 is a fraction of *h*.

The slope of the energy grade line *J* is defined by the Darcy-Weisbach equation. Thus, substituting D_h by 4*R* where *R* is the hydraulic radius, the Darcy-Weisbach equation can be expressed as:

$$J = \lambda \frac{V^2}{8gR} \qquad (21)$$

where λ is the friction factor also known as the Darcy-Weisbach friction factor.

The flow described in fig. 1 can be considered as a two-dimensional flow so that the hydraulic radius *R* corresponds to the depth of the flow. Let us consider the average depth h_a between the depths h_1 and h_2 so as to write that:

$$R = h_a = \frac{h_1 + h_2}{2} \qquad (22)$$

Eqs. (20) and (22) lead to:

$$R = h_1 \frac{1+k}{2} \qquad (23)$$

The average velocity V_a of the flow can also be written as:

$$V = V_a = \frac{q}{h_a} \qquad (24)$$

where:

$$h_a = h_1 \frac{1+k}{2} \qquad (25)$$

Eq. (24) can then be rewritten as:

$$V = V_a = \frac{q}{h_1} \frac{2}{1+k} \qquad (26)$$

Let us assume in the following section $h_1 = h$. Taking into account Eqs. (22), (23), (24) and (25), Eq. (21) can be expressed as:

$$J = \lambda \frac{q^2}{gh^3 \left(1+k\right)^3} \qquad (27)$$

Thus, Eq. (19) can be written as:

$$h+\frac{\alpha q^2}{2gh^2}=kh+\frac{\alpha q^2}{2gk^2h^2}+\lambda\frac{q^2}{gh^3\left(1+k\right)^3}\Delta S \tag{28}$$

Eq. (28) is reduced to:

$$(1-k)\cdot h+\frac{\alpha q^2}{2gh^2}\frac{(1-k^2)}{k^2}=\lambda\frac{q^2}{g\cdot h^3(1+k)^3}\Delta S \tag{29}$$

With the decrease of the head on the crest of the weir, the flow regime becomes laminar. In this case, the conventional approximation of the friction factor λ is defined by Poiseuille's law (1840) as:

$$\lambda=\frac{64}{\mathrm{Re}} \tag{30}$$

Where Re is the Reynolds number which can be written as follows:

$$\mathrm{Re}=\frac{VD_h}{\nu}=\frac{4qh}{h\nu}=\frac{4q}{\nu} \tag{31}$$

where D_h is the hydraulic diameter. Thus Eq. (30) becomes:

$$\lambda=\frac{64\nu}{4q}=\frac{16\nu}{q} \tag{32}$$

The flow rate per unit width of the weir crest is expressed as:

$$q=C_d\sqrt{2gH^3} \tag{33}$$

Where C_d is the discharge coefficient, and H is the total head above the weir crest. Inserting Eqs.(32) and (33) into Eq.(29) with the consideration $\alpha=2$ for laminar flow results in:

$$(1-k)-2\frac{H^3}{h^3}\frac{(1-k^2)}{k^2}=\frac{16\nu}{gh^4}\frac{\sqrt{2gH^3}}{(1+k)^3}C_d\Delta S \tag{34}$$

After some manipulations, Eq.(34) is rewritten as :

$$\frac{(1-k)h^3k^2}{2H^3(1-k^2)}-C_d^2=\frac{16\nu}{gh^4}\frac{\sqrt{2gH^3}}{(1+k)^3}\frac{h^3k^2}{2H^3(1-k^2)}C_d\Delta S \tag{35}$$

On the other hand, the total head above the weir crest is in a critical state allowing to write that:

$$H = 1.5h \tag{36}$$

Inserting Eq. (36) into Eq. (35) yields:

$$\frac{(1-k)k^2}{2(1.5)^3(1-k^2)} - C_d^2 = \frac{16\nu}{gh^4}\frac{\sqrt{2g}(1.5)^{3/2}h^{3/2}h^3}{(1+k)^3 2(1.5)^3 h^3(1-k^2)}C_d\Delta S \tag{37}$$

After simplifications, Eq. (37) is reduced to:

$$\frac{(1-k)k^2}{2(1.5)^3(1-k^2)} - C_d^2 = \frac{16\nu}{gh^{5/2}}\frac{\sqrt{2g}(1.5)^{3/2}k^2}{(1+k)^3 2(1.5)^3(1-k^2)}C_d\Delta S \tag{38}$$

Dividing the both sides of Eq.(38) by $(1+k)$, one may derive the following result:

$$\frac{k^2}{2(1.5)^3(1+k)} - C_d^2 = \frac{16\nu}{gh^{5/2}}\frac{\sqrt{2g}(1.5)^{3/2}k^2}{(1+k)^4 2(1.5)^3(1-k)}C_d\Delta S \tag{39}$$

Multiplying and dividing the both sides of Eq.(39) by $\sqrt{h}$ and ρ, one can be obtained Eqs (40) and (41):

$$\frac{k^2}{2(1.5)^3(1+k)} - C_d^2 = \frac{16\nu}{gh^3}\frac{\sqrt{2}\sqrt{gh}(1.5)^{3/2}k^2}{(1+k)^4 2(1.5)^3(1-k)}C_d\Delta S \tag{40}$$

$$\frac{k^2}{2(1.5)^3(1+k)} - C_d^2 = \frac{16\nu\rho}{\rho gh^3}\frac{\sqrt{2}\sqrt{gh}(1.5)^{3/2}k^2}{(1+k)^4 2(1.5)^3(1-k)}C_d\Delta S \tag{41}$$

Rearranging Eq. (41) results in :

$$\frac{k^2}{2(1.5)^3(1+k)} - C_d^2 = \frac{16\nu\rho}{\left(\frac{\rho gh^3}{\Delta S}\right)}\frac{\sqrt{2}\sqrt{gh}(1.5)^{3/2}k^2}{(1+k)^4 2(1.5)^3(1-k)}C_d \tag{42}$$

Let's denote $V = \sqrt{gh}$ which has velocity units and represents in fact the shallow water wave velocity. Thus, Eq.(42) becomes with $\nu\rho = \mu$:

$$\frac{k^2}{2(1.5)^3(1+k)} - C_d^2 = \frac{16\mu V}{\left(\frac{\rho g h^3}{\Delta S}\right)} \frac{\sqrt{2}(1.5)^{3/2} k^2}{(1+k)^4 2(1.5)^3 (1-k)} C_d \tag{43}$$

where μ is the dynamic viscosity. In Eq.(43), one can easily identify the following dimensionless number σ which represents in fact the surface tension:

$$\sigma = \frac{\rho g h^3}{\Delta S} \tag{44}$$

On the other hand, one can define from Eq.(43), the following dimensionless number :

$$C_a = \frac{\mu V}{\sigma} \tag{45}$$

In fact, C_a is the capillary number which represents the relative effect of viscous drag forces versus surface tension forces. Eq.(43) is then written as :

$$\frac{k^2}{2(1,5)^3(1+k)} - C_d^2 = \frac{16 C_a \sqrt{2}(1,5)^{3/2} k^2}{(1+k)^4 2(1,5)^3 (1-k)} C_d \tag{46}$$

Capillary numbers are usually large for high-velocity flows and low for low-velocity flows. Regarding the C_a value, a rule of thumb says that it should not exceed 10^{-5}. When the capillary number is lower than 10^{-5}, the flow behavior is determined based on capillary forces.

According to Eq.(20), when h_1 tends towards h_2, the factor k tends to unity. But, this will generate an infinite limit for the right-hand side of Eq.(46). To avoid this inconvenience, one may tend the factor k to 0.999 instead of 1. With $k = 0.999$, Eq.(46) becomes :

$$C_d^2 + 384.9 C_a C_d - 0.07396 = 0 \tag{47}$$

This is the quadratic equation that governs the discharge coefficient C_d as a function of the capillary number C_a, for a shaft spillway operating under a small head. Table 1 gives C_d for some values of C_a according to Eq.(47).

Table 1: Discharge coefficient C_d for some values of C_a according to Eq.(47)

C_a	10^{-8}	10^{-7}	10^{-6}	10^{-5}
C_d	0.27195	0.27194	0.27177	0.27004

As it can be seen, the discharge coefficient C_d varies in the confined range [0.270 ; 0.272] when the capillary number C_a increases from 10^{-8} to 10^{-5}. The discharge coefficient C_d decreases with the increase of C_a which is in accordance with the physical phenomenon. The increase in C_a is due to the increase in viscous forces, which causes the slowing down of the flow and therefore a decrease in C_d.

DISCUSSION OF RESULTS

According to the Eqs.(5) and (6), when tending theoretically the head H to zero, the discharge coefficient C_d is as 0.357 and 0.402 respectively, implying a deviation of about 13%. Thus, there is an uncertainty when computing the discharge coefficient for the small heads. Moreover, Eqs.(3) and (7) give $C_d \to \infty$ for $H \to 0$, while Eq.(4) leads to $C_d = 0.666$ that can not be reliable..

Moise (1970) has plotted a graph derived from laboratory tests. This graph is shown in figure 2. On the graph m represents the discharge coefficient C_d used in the present study. The total head H_p corresponds in fact to the design head H_d. Four values of H_d / R have been set and figure 2 shows the variation of the discharge coefficient C_d with the relative head H / H_c where H_c is the calculated head which is used to design the inner profile of the shaft spillway. The obtained experimental values of the discharge coefficient were adjusted to polynomials of degree three. According to the chosen values of H_d / R, the discharge coefficient C_d for the head H tending towards zero varies from 0.207 to 0.290. The results reported in Table 1 locate the discharge coefficient C_d in this range. Regarding the coefficient of incompleteness introduced by Rozanov in Eq.(15), its value is $K_{H/H_d} = 0.775$ for $H \to 0$. The discharge coefficient values, corresponding to $H \to 0$, derived from Eqs.(12), (13) and (14), must be corrected for the effect of K_{H/H_d} as follows :

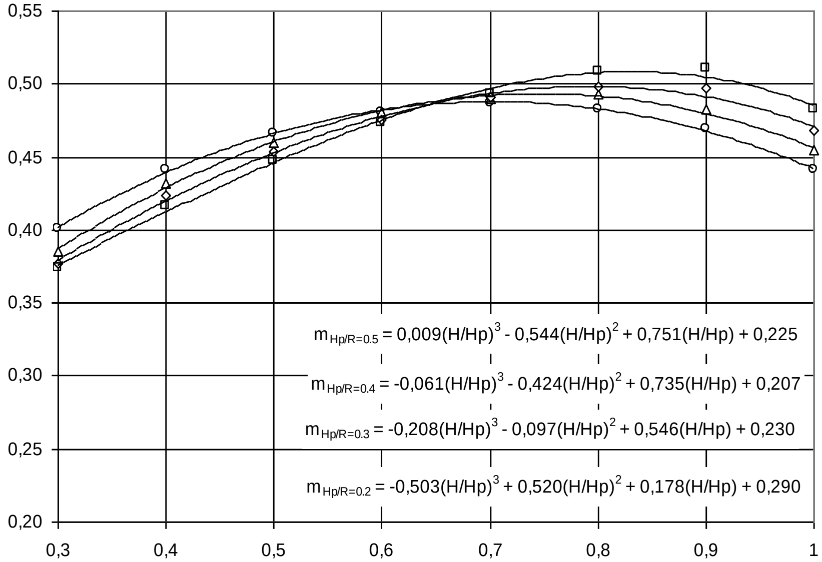
$$C_{d,H\to 0} = C_{d,e/f} K_{H/H_d \to 0} \tag{48}$$

The final result is as:

$C_{d1} = 0.775 \times 0.406 = 0.314$; $C_{d2} = 0.775 \times 0.430 = 0.333$; and
$C_{d3} = 0.775 \times 0.412 = 0.319$

Note that these values are slightly higher than those obtained by Moise and the present study (Table 1)..

According to Eq.(16), the discharge coefficient C_d for $H \to 0$ is equal to 0.265 which is of the same order of magnitude as the value obtained in the present study and reported in table 1.



H/H_c

Figure 2: $C_d = f$ (H/H$_c$) according to Moise (1970)

CONCLUSION

The main objective of this study was to examine the possibility of calculating analytically the value of the discharge coefficient C_d of a shaft spillway operating under small head, theoretically corresponding to H tending towards zero. Based on a simple diagram, the Bernoulli equation was applied between two sections taken on the spillway crest. In view of the laminar nature of the flow, the effect of the Reynolds number has been taken into account. The theoretical development, judiciously carried out, led to a quadratic equation in C_d including the effect of the capillary number C_a [Eq.(47)]. Varying the capillary number from 10^{-8} to 10^{-5}, it has been observed that the obtained C_d values range is [0.270 ; 0.272], suggesting an average value of 0.271. This theoretical value is very close to that given by experimental relationships for H tending towards zero, such as the reliable formulas of Moise and Rozanov.

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