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THEORETICAL CONSIDERATIONS ON FLOW REGIME DEPENDENCY OF THE HAZEN-WILLIAMS COEFFICIENT

CONSIDERATIONS THEORIQUES SUR LA DEPENDANCE DU COEFFICIENT DE HAZEN-WILLIAMS AU REGIME D'ECOULEMENT

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ABSTRACT

The Hazen-Williams equation is still widely used nowadays, despite its applicability limits that many research workers have highlighted. The equation contains a constant coefficient C which depends only on the material of the pipe. However, many studies have asserted that C should depend on both the relative roughness of the pipe and the Reynolds number. This dependence of C on the relative roughness and the Reynolds number was highlighted by Liou, in particular, through a relationship that interests the present study. A readjustment of Liou's relationship led to an implicit dimensionless equation that was translated into a C dimensionless diagram. Furthermore, the derived dimensionless relationship from the transformation of the Hazen-Williams equation, along with the dimensionless diagram, speeds up the slope of the energy grade line calculation. The dimensionless C relationship reaches a maximum for a given value of the relative roughness. A deep analysis of this relationship led to the successful establishment of the explicit dependence of C maximum on the relative roughness.

Keywords: Hazen-Williams coefficient, Darcy-Weisbach, Galileo number, Dimensionless diagram.

RESUME

L'équation de Hazen-Williams est encore largement utilisée de nos jours, malgré ses limites d'applicabilité que de nombreux chercheurs ont mises en évidence. L'équation contient un coefficient C constant qui ne dépend que du matériau de la conduite. Cependant, de nombreuses études ont affirmé que C devrait dépendre à la fois de la rugosité relative de la conduite et du nombre de Reynolds caractérisant l'écoulement. Cette dépendance de C vis-à-vis de la rugosité relative et du nombre de Reynolds a été mise en évidence par Liou, notamment, à travers une relation qui intéresse la présente étude. Un réajustement de la relation de Liou a conduit à une équation implicite adimensionnelle qui a été traduite par un diagramme également adimensionnel de C. De plus, la relation sans dimension dérivée de la transformation de l'équation de Hazen-Williams, avec le diagramme sans dimension, accélère le calcul de la pente de la ligne d'énergie. La relation de C sans dimension atteint un maximum pour une valeur donnée de la rugosité relative. Une analyse approfondie de cette relation a conduit à l'établissement réussi de la dépendance explicite de C maximal vis-à-vis de la rugosité relative.

Mots-clés : Coefficient de Hazen-Williams, Darcy-Weisbach, Nombre de Galilée, Diagramme adimensionnel.

INTRODUCTION

The Hazen-Williams formula, usually intended for head losses pressurized flow calculation, is an old empirical relationship used since 1902 and remains nowadays one of the most popular exponential formulas in America and in the Middle East. The formula was an important tool in the practice of the hydraulic engineer practicing especially in the fields of sanitary engineering and water supply systems. The Hazen-Williams formula, although much appreciated, has been used regardless of the limits of its validity and the conditions of its design. In this regard, some research workers have widely discussed the limits of the applicability of the Hazen-Williams relationship, such as Vennard (1961), Streeter and Wylie (1985), Street et al. (1996), Potter and Wiggert (1997) and also Liou (1998) who advises against using the Hazen-Williams equation.

The Darcy-Weisbach equation is much more rational and without applicability restrictions than the Hazen Williams equation is. Thus, from the large database of the Hazen-Williams C values, some authors have attempted to estimate the relative roughness of the pipe which will then be used in the Darcy-Weisbach

relation, Diskin(1960), Liou (1998), Christensen (1998), Travis and Mays (2007).

Due to the fact that the Hazen-Williams equation is still widely used nowadays, the purpose of this technical note is to provide to its strong proponents a general formula and a dimensionless diagram that facilitates the determination of the *C* coefficient required for the proper use of the equation. The *C* coefficient is contained in a non-dimensional parameter C^* which depends on the relative roughness and the Reynolds number. This was made possible thanks to the readjustment of the Liou's *C* relationship (1998). Relations for the rapid calculation of energy slope are proposed. They come from the transformation of the Hazen-Williams and Darcy-Weisbach equations. These relations are presented in dimensionless terms and bring out in particular the effect of the dimensionless Galileo number on the Darcy-Weisbach energy slope.

THEORETICAL CONSIDERATIONS

The general form of the Hazen-Williams equation relates the mean velocity of water in a pipe with the geometric properties of the pipe and slope of the energy grade line. In SI units, it can be expressed as (Bhave, 1991):

$$V = 0.849 C R_h^{0.63} S^{0.54} \tag{1}$$

Where *V* = velocity (m/s); *C* = Hazen-Williams coefficient ($m^{0.37}/s$); R_h= hydraulic radius (m) and *S* = slope of the energy grade line (m/m).

Regarding Hazen-Williams coefficient *C*, Swamee (2000) rightly recalls that *C* must have the dimensions of a length to the 0.37 power over time, meaning $m^{0.37}/s$. Adopting this dimension for *C* will then results in the dimensional homogeneity of Eq. (1).

For pipe-flow of diameter *D*, $R_h=D/4$ and Eq.(1) becomes:

$$V = 0.3545 C D^{0.63} S^{0.54} \tag{2}$$

Liou (1998) rearranged Eq.(2) in the form of $S=KV^2/(2gD)$ and identified *K* as the Darcy-Weisbach friction factor *f*. This approach allowed expressing *C* as follows:

$$C = 14.07 f^{-0.54} R^{-0.08} D^{-0.01} \nu^{-0.08} \tag{3}$$

Eq (3) translates what must be *C* coefficient so that the equations of Hazen-Williams and Darcy-Weisbach give the same result. Some research workers

have used the same approach in the past, such as Vennard (1958), Diskin (1960), Barlow and Markland (1975), Swamee and Jain (1946) and Jain et al. (1978).

In fact, in Eq. (3), the constant 14.07 contains the gravitational acceleration g (m/s^2).In order to preserve the dimensional homogeneity of Eq.(3), let's extract from it the quantity $g^{0,54}$. Eq. (3) can then be rewritten in the following form:

$$C = 4.101 g^{0.54} f^{-0.54} R^{-0.08} D^{-0.01} \nu^{-0.08} \tag{4}$$

One can thus define the dimensionless parameter C^* such that:

$$C^* = \frac{C}{g^{0.54} D^{-0.01} \nu^{-0.08}} \tag{5}$$

Then, Eq. (3) allows writing that:

$$C^* = \frac{4.101}{f^{0.54} R^{0.08}} \tag{6}$$

In equation (6), f is governed by the Colebrook-White relation (1939), so that the right-hand side of the equation is a function of the relative roughness ε/D and the Reynolds number R. Applying Eq. (6), it was, therefore, possible to design the dimensionless diagram $C^*(\varepsilon/D,R)$ shown in Fig.1 which can be used to evaluate C value required to compute the slope of the energy grade line S_{HW} according to the Hazen-Williams equation. This is one of the three problems that arise in pipe-flows analysis. For this, the following data are given in practice Q,D,ε,ν, where Q = discharge (m^3/s) and ν = kinematic viscosity (m^2/s). The Reynolds number R is such that $R=4Q/(\pi D\nu)$.

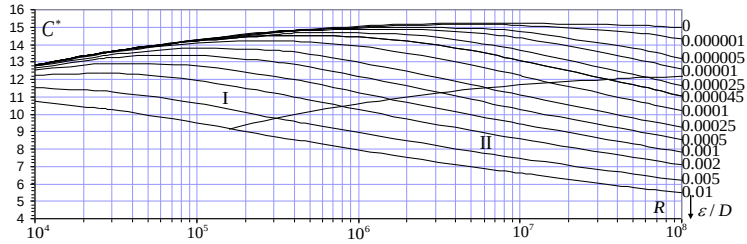

Figure 1: Dimensionless diagram $C^*(\varepsilon/D,R)$ for pipe-flow according to Eq. (6) I: Transition zone, II: Turbulence-rough zone

The friction factor f in Eq. (6) was calculated according to the implicit Colebrook-White relation (1939). However, several formulas exist in the

literature that approximate the Colebrook-White relation. These formulas were recently recalled by Zeghadnia et al. (2019) providing indications on their limitations as well as on their accuracy. One of them can be used for the calculation of C^* according to Eq. (6) if one prefers avoiding the use of the dimensionless diagram shown in Fig.1.

Multiplying and dividing the numerator and the denominator of the right-hand side of Eq. (5) by $D^{1.63}/\nu$, results in:

$$C^* = \frac{R^*}{Ga^{0.54}} \tag{7}$$

where $R^*=CD^{1.63}/\nu$ is a Reynolds number and $Ga=gD^3/\nu^2$ is the dimensionless Galileo number which represents the ratio between the gravity forces and the viscous forces. Therefore, C^* corresponds to the ratio of two dimensionless numbers. Eqs. (6) and (7) allow writing that:

$$\frac{R^*}{Ga^{0.54}} = \frac{4.101}{f^{0.54}R^{0.08}} = \phi\left(\varepsilon / D, R\right) \tag{8}$$

what amounts to writing, from a functional point of view, that:

$$C = \nu D^{-1.63}\psi\left(f, R, Ga\right) \tag{9}$$

Since R and R^* are known parameters, so one should then use the following simple relation for S_{HW}, derived from the transformation of the Hazen-Williams equation:

$$S_{HW} = \left(\frac{2.821R}{R^*}\right)^{1.852} \tag{10}$$

Thus, through the Eq. (10), S_{HW} can be calculated without knowing the Hazen-Williams coefficient C value.

One might as well use the following transformed Darcy-Weisbach equation:

$$S_{DW} = \frac{f}{2}\frac{R^2}{Ga} \tag{11}$$

NUMERICAL EXAMPLE

Let's consider a high-density polyethylene pipe (HDPE) with the following characteristics, given by the literature:

$\varepsilon = 0.0000015m$, $D = 0.30m$, $C = 145m^{0.37}/s$ (mean value),

$T = 20°C$ ($\nu = 0.0000010023m^2/s$ for water).

Let it flow in the pipe the following discharges:

$Q_1 = 0.05m^3/s$, $Q_2 = 0.1m^3/s$, $Q_3 = 0.15m^3/s$, $Q_4 = 0.20m^3/s$,
$Q_5 = 0.25m^3/s$, $Q_6 = 0.30m^3/s$, $Q_7 = 0.35m^3/s$, $Q_8 = 0.40m^3/s$

The objective is to determine the relative error committed in the computation of the energy grade line slope when one considers the Hazen-Williams and Darcy-Weisbach equations, i.e. Eq. (10) and Eq. (11). The relative error can be expressed as $(S_{HW} - S_{DW})/ S_{DW}$ and varies with the Reynolds number R. S_{HW} is computed using Eq.(10) in which $R=4Q/(\pi Dv)$ and $R^*=CD^{1.63}/v$ corresponding to $C=145m^{0.37}/s$. S_{HW} is thus computed for all the given discharges.

As for S_{DW}, it is calculated according to Eq.(11). The friction factor f is given by the Colebrook-White relation for the known values of ε/D and R. The dimensionless Galileo number is $Ga=gD^3/v^2$ which takes then a constant value for all the considered discharges. The result derived from these calculation steps is shown in Fig.2.

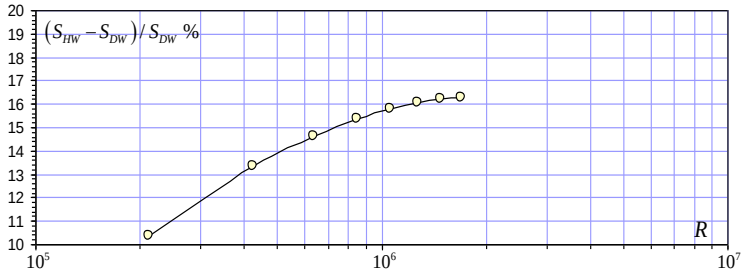

Figure 2: Relative error (%) between S_{HW} and S_{DW}

Fig. 2 clearly shows a significant error on the energy slope S, varying between 10% and 16%, when using the Hazen-Williams relationship instead of the Darcy-Weisbach equation which is considered as a rational reference equation. According to the Hazen-Williams equation, when the relative error on the

calculation of S_{HW} varies in this range, the relative error on the calculation of the discharge Q varies between 5.4% and 8.6%. On the other hand, the relative error on the calculation of the diameter D will vary between 2% and 3.3%. In order that Hazen-Williams and Darcy-Weisbach equations give almost the same value, the following C values must be adopted for each considered Reynolds number. These values were calculated according to Eq. (6).

$$C_1 \approx 153,\ C_2 \approx 155,\ C_3 = 156,\ C_4 = C_5 = C_6 = C_7 = C_8 \approx 157.$$

The relative deviation between the above C values and C=145 then varies in the range of 8.5% and 11%. Thus, applying the Hazen-Williams equation will result in a relative error varying between 3.2% and 4.2% in the diameter D calculation. The slope of the energy grade line S_{HW} will be subjected to an error varying between 15.7% and 20% while the error will vary between 8.5% and 11% in the discharge Q calculation.

MAXIMUM OF THE HAZEN-WILLIAMS COEFFICIENT

It is interesting to notice that curves of the dimensionless diagram $C^*(\varepsilon/D, R)$ in Fig.1 reach a maximum for each relative roughness which corresponds to a certain Reynolds number value. It is then practical to express C^* as a function of ε / D, writing that $C^*_{\max} = F(\varepsilon / D)$.

A formal study of inflexion points of the C^* function allows the determination of these maximums. Let consider Eq.(6) and replace f by its expression according to Colebrook-White. That is:

$$C^* = 4.101\left[-2\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{f^{0.5}R}\right)\right]^{1.08} R^{-0.08} \tag{12}$$

Differentiating Eq. (12) with respect to R at the first order approximation, considering here f as constant to avoid continued fractions, one can isolate extrema of this function. The solution retained for corresponding R as the root of the resulting equation in the turbulent flow regime is:

$$R\left(C_{\max}^{*}\right)=\frac{-2.287\,D\,\mathrm{W}\left(2.661384\cdot10^{6}\,\dfrac{\varepsilon}{D}\right)}{f^{0.5}\varepsilon\left[\mathrm{W}\left(2.661384\cdot10^{6}\,\dfrac{\varepsilon}{D}\right)-13.5\right]} \tag{13}$$

In which *W* is the *Lambert* Function defined as the inverse function of $z = we^{w}$ i.e. $w=W_k(z)$ for some integer *k*. Since the argument of the function is a real number, W-function has two branches namely W_0 and W_{-1}. Since it is a transcendental function, formal solution of the Lambert W-function can be expressed only in endless form. However, it can be approximated using series expansion (Brkić, 2011) or other numerical techniques (Chapeau-Blondeau and Monir, 2002). The solution of Eq. (13) is obtained iteratively regarding to its implicit form. Nevertheless, approximate evaluation formulas of *f* can be used with confidence in this case.

In order to express maximum values of C^{*} as a function of ε/D, a series of solutions of Eq. (13) were generated and fitted for explicit direct use. As a result, with a correlation coefficient R^2=0.9983, this function can be expressed as a second-order exponential decay model as:

$$C_{\max}^{*}=F\left(\varepsilon/D\right)=10.592+1.536\,e^{-\frac{\varepsilon/D}{0.0001176}}+2.993\,e^{-\frac{\varepsilon/D}{0.00411}} \tag{14}$$

Accordingly, Eq.(7) allows writing that:

$$\frac{R_{\max}^{*}}{Ga^{0.5}}=F\left(\varepsilon/D\right) \tag{15}$$

Thusly, with regard to the definition of R^{*}, one may write:

$$C_{\max}=\nu D^{-1.63}Ga^{0.54}F\left(\varepsilon/D\right) \tag{16}$$

Finally, from a functional point of view, Eq. (16) can be expressed as:

$$C_{\max}=\nu D^{-1.63}\xi\left(\varepsilon/D,Ga\right) \tag{17}$$

CONCLUSIONS

From the readjustment of the Liou's relationship which expresses the Hazen-Williams coefficient $C(\varepsilon/D,R)$, a C^* parameter has been identified as a ratio of two dimensionless numbers [Eq.(7)], one of them contains the Hazen-Williams dimensionless parameter, denoted R^*, defined as a Reynolds number linked to C. The second parameter, denoted Ga, is none other than the dimensionless Galileo number which represents the ratio of the gravity forces to the viscous forces. The $C^*(\varepsilon/D,R)$ implicit relationship [Eq.(6)] has been translated into a dimensionless diagram that can be used to evaluate C, provided ε/D and R are given. The C values deduced from the diagram are such that the Hazen-Williams and the Darcy-Weisbach equations give the same result, especially with regard to the slope of the energy grade line. The second equation should be considered as a rational reference equation. The transformation of the Hazen-Williams and Darcy-Weisbach equations led to writing the slope of the energy grade line in a dimensionless form, involving the following dimensionless numbers R^*, Ga and R. The obtained equations are simple to use and speed up the calculation. The diagram $C^*(\varepsilon/D,R)$ has shown that C^* reaches a maximum for a given value of the relative roughness ε/D. A deep study of the relationship has shown, with an excellent correlation coefficient, that $C^*_{\max}$ is governed by a second-order exponential decay model whose mathematical expression has been presented [Eq.(14)]. As a result, the C_{max} relationship has been deduced showing the different parameters on which C_{max} is dependent [Eq.(17)].

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