



PROPER RELATIONSHIP OF MANNING'S COEFFICIENT IN A PARTIALLY FILLED CIRCULAR PIPE

RELATION APPROPRIÉE DU COEFFICIENT DE MANNING DANS UNE CONDUITE CIRCULAIRE PARTIELLEMENT REMPLIE

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ABSTRACT

The Manning's n coefficient represents the friction applied to the flow by the inner wall of a channel or a pipe. For a correct design of hydraulic systems, the designer should have an appropriate value of this coefficient. The present study aims to establish the proper Manning's n relationship including all the parameters that affect the flow in a partially filled circular pipe, such as the relative roughness, the slope of the energy grade line, and the kinematic viscosity. A new dimensionless parameter, acting as a Reynolds number, is introduced reflecting the relative effect of friction forces versus viscous forces. The study highlights the significant role of this parameter in the variation of the Manning's n coefficient with the relative flow depth.

Keywords: Manning's coefficient, Darcy-Weisbach, Partially filled pipe, Dimensionless diagram.

RESUME

Le coefficient n de Manning représente le frottement appliqué à l'écoulement par la paroi intérieure d'un canal ou d'une conduite. Pour une conception correcte des systèmes hydrauliques, le concepteur doit avoir une valeur

appropriée de ce coefficient. La présente étude vise à établir la relation n de Manning appropriée, incluant tous les paramètres qui affectent l'écoulement dans une conduite circulaire partiellement remplie, tels que la rugosité relative, la pente de la ligne d'énergie et la viscosité cinématique. Un nouveau paramètre sans dimension, agissant comme un nombre de Reynolds, est introduit reflétant l'effet relatif des forces de frottement par rapport aux forces visqueuses. L'étude met en évidence le rôle important de ce paramètre dans la variation du coefficient n de Manning en fonction de la profondeur relative de l'écoulement.

Mots-clés : Coefficient de Manning, Darcy-Weisbach, Conduite partiellement remplie, Diagramme adimensionnel.

INTRODUCTION

The roughness coefficient represents the friction applied to the flow by the inner wall of a channel or a pipe (Chow, 1959). The determination of the appropriate value of the roughness coefficient in the channels and conduits is essential for carrying out the correct calculation of the flow characteristics such as velocity, dimensions, and slope. A value higher than that required leads to oversizing the structure, while a value below that required can lead to a hydraulically deficient structure. For the designer, having a correct value for the roughness coefficient is then essential. What is also essential is how the roughness coefficients are determined. The recent literature often gives roughness coefficients which are sometimes significantly different from those provided by the older literature. This is also the case with research that often comes up with values of roughness coefficients contrasting those usually used. The determination of the roughness coefficient by laboratory tests does not reflect reality because the tests are carried out under ideal conditions. Water used for testing is clean without debris and the experimental channels are straight without bends or other obstructions. Manning's n laboratory values are systematically corrected before using them in actual installed conditions. The correction factor, called design factor, could vary between 20% and 30% (ACPA, 2000; 2012). It is therefore a significant correction that is made on the Manning's n laboratory values. As an indication, for smooth pipes engineers use Manning's n value ranging between 0.012 and 0.013, while Manning's n value has been found varying between 0.009 and 0.01 for this state of pipe. For some pipe material, recommended n design values can be selected from tables given by many institutions and authors such as the University of Minnesota (1950), Barfuss and Tullis (1989), the American concrete pipe association (2000), and the US department of transportation (2012).

The Manning's n roughness coefficient has always been considered to be a constant value depending on the type of material constituting the channel or the pipe. Even today, engineers continue to view n as a constant whose value comes from the tables. Yet, seventy years ago Camp (1946) drew up a chart showing, in particular, the variation of the mean values of Manning's n as a function of the relative flow depth in partially filled circular pipes with different diameters. This diagram, reproduced elsewhere in manuals and reports by the American Society of Civil Engineers (1992), is recognized today and is used by many professionals. Many studies later confirmed the fact that Manning's coefficient varies as function of flow depth (Pomeroy, 1967; Yen, 1992; Meky et al., 2015).

Manning's versatile formula, developed in the 1890s, came twenty years later to replace that of Ganguillet-Kutter which was developed in the 1869s. The later and the associated aid chart design were considered to be quite cumbersome. This is the coefficient n of the Manning's formula that interests the present study. The main objective of this is to examine the variation of this coefficient in partially filled circular pipe not only as a function of the relative flow depth but also as a function of other flow parameters such as the relative roughness, the slope of the energy grade line and the kinematic viscosity of the flowing liquid. For doing so, the Manning's formula (1891) is faced with a dimensionally consistent uniform flow relationship given by Achour and Bedjaoui (2006). In this relation, a dimensionless number is introduced which gives a measure of the ratio of friction forces to viscous forces and consequently the relative importance of these types of forces on the variation of the Manning's n coefficient.

GEOMETRICAL PROPERTIES

Fig.1 shows a partially filled circular pipe with the flow depth h and the diameter D .

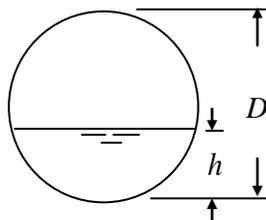


Figure 1: Circular pipe partially filled

Let us define $\eta = h / D$ as the aspect ratio or the ratio of depth to diameter. The wetted area, the wetted perimeter P and subsequently the hydraulic radius $R_h = A / P$ can be written respectively as:

$$A = \frac{D^2}{4} \sigma(\eta) \varphi(\eta) \tag{1}$$

$$P = D\sigma(\eta) \tag{2}$$

$$R_h = \frac{D}{4} \varphi(\eta) \tag{3}$$

where:

$$\sigma(\eta) = \cos^{-1}(1 - 2\eta) \tag{4}$$

$$\varphi(\eta) = 1 - \frac{2(1 - 2\eta) \sqrt{\eta(1 - \eta)}}{\cos^{-1}(1 - 2\eta)} \tag{5}$$

It can be derived from the equation (5) that for a half-full pipe $\varphi(0.5) = 1$ and for a full pipe $\varphi(1) = 1$. Accordingly, Eq. (3) allow writing that:

$$\frac{R_h}{R_{h,f}} = \varphi(\eta) \tag{6}$$

which also amounts to writing that for both a half-full pipe and a full pipe, corresponding respectively to $\varphi(0.5) = 1$ and $\varphi(1) = 1$ as stated above:

$$\frac{R_h}{R_{h,f}} = 1 \tag{7}$$

The subscript “*f*” denotes the full flow condition. Eqs. (5), (6) and (7), in particular, will be used for the rest of the study.

THEORETICAL CONSIDERATIONS

The dimensionally consistent uniform flow relationship $Q(S, \varepsilon, g, A, R_h, \nu)$ can be established using the Rough Model Method (Achour and Bedjaoui, 2006) or by combining the rational equations of Darcy-Weisbach (1854) and Colebrook (1939). Q is the discharge, S is the slope of the energy grade line, ε

is the absolute roughness, g is the acceleration due to gravity and ν is the kinematic viscosity. These three relationships can be written respectively as:

$$Q = -4\sqrt{2g} A\sqrt{R_h S} \log\left(\frac{\varepsilon}{14.8R_h} + \frac{10.04}{R^*}\right) \quad (8)$$

where :

$$R^* = 32\sqrt{2} \frac{\sqrt{gR_h^3 S}}{\nu} \quad (9)$$

$$S = f \frac{V^2}{8gR_h} \quad (10)$$

$$\frac{1}{\sqrt{f}} = -2 \log\left(\frac{\varepsilon}{14.8R_h} + \frac{2.51}{R\sqrt{f}}\right) \quad (11)$$

In Eqs (10), f is the friction factor also called the Darcy-Weisbach friction factor which is governed by Eq.(11), while V is the mean flow velocity. Special attention should be paid to the dimensionless number R^* which Eq.(9) refers to. It seems that the literature makes no explicitly mention of this number. There is also no indication of its order of magnitude. It does not correspond to the Reynolds number $R=VD_h/\nu$ usually used, where D_h is the hydraulic diameter. What the literature indicates, however, is that the quantity $\sqrt{gR_h S}$ corresponds to the shear velocity u^* also called friction velocity (Schlichting, 1979) having units of velocity. Therefore, Eq.(9) can be written in the following form:

$$R^* = 32\sqrt{2} \frac{u^* R_h}{\nu} \quad (12)$$

With regard to the form of the Eq.(12), the dimensionless number R^* would give a measure of the ratio of friction forces to viscous forces and consequently the relative importance of these types of forces. It is a form of conversion by which the shear stress is re-written in units of velocity. The quantity $u^* R_h/\nu$ can, therefore, be considered as the shear Reynolds number which also corresponds, to within a constant, to the dimensionless number R^* . This could, therefore, be appropriate for quantifying the variation as a function of the flow depth of the roughness coefficients opposite to the moving of flows such as Manning's n .

The Manning's formula expresses the mean flow velocity, written in SI units, as (Chow, 1959):

$$V = \frac{1}{n} R_h^{2/3} S^{1/2} \tag{13}$$

where n is the manning’s roughness coefficient. Multiplying and dividing the right-hand side of Eq.(13) by $g^{0.5}$ and rearranging results in:

$$\frac{u^*}{V} = \frac{n\sqrt{g}}{R_h^{1/6}} \tag{14}$$

The literature indicates that the u^*/V ratio is such that (Daugherty and Franzini, 1977):

$$\frac{u^*}{V} = \sqrt{\frac{f}{8}} \tag{15}$$

Thus:

$$u^* = V \sqrt{\frac{f}{8}} \tag{16}$$

Multiplying the both sides of Eq.(16) by $32\sqrt{2}R_h / \nu$ and knowing that $R_h = D_h/4$, one may derive the following result:

$$32\sqrt{2} \frac{u^* R_h}{\nu} = 32\sqrt{2} \frac{VD_h / \nu}{4\sqrt{8}} \sqrt{f} \tag{17}$$

Eq. (17) can be reduced to:

$$R^* = 4R\sqrt{f} = \zeta(\varepsilon / R_h, R) \tag{18}$$

On one hand, Eq.(18) indicates that the dimensionless number R^* is, to within a constant, the product of the two well known dimensionless numbers R and \sqrt{f} , and Eq.(18) reveals that R^* is a function of both ε/R_h and R , on the other hand. Extracting $R\sqrt{f}$ and $1/\sqrt{f}$ from Eq.(18) and inserting them in Eq.(11) gives:

$$4 \frac{R}{R^*} = -2 \log \left(\frac{\varepsilon}{14.8R_h} + \frac{2.51}{R^* / 4} \right) \tag{19}$$

Rearranging Eq. (19) results in:

$$R = -\frac{1}{2} R^* \log \left(\frac{\varepsilon}{14.8 R_h} + \frac{10.04}{R^*} \right) \quad (20)$$

The same equation was established by Achour and Bedjaoui (2006) using the Rough Model Method. This is the fundamental relationship relating R^* to the Reynolds number R and the relative roughness ε/R_h . It is implicit in R^* and a dimensionless chart $R^*(\varepsilon/R_h, R)$ can be designed as was done for Moody's diagram (Moody, 1944). This diagram will allow reading R^* for the given value of both Reynolds number R and the relative roughness ε/R_h . Moreover, the value thus determined of R^* will be used to calculate the shear velocity u^* according to the Eq.(12). If one wants to avoid the implicit calculation of R^* imposed by the form of Eq. (20), the explicit Eq. (9) would then be the most appropriate provided R_h , S and ν are given, which is generally the case in practice. If both Reynolds number R and the relative roughness ε/D are given, Eq.(18) is strongly recommended for the computation of R^* . The downside is that the friction factor f is governed by the implicit Eq.(11). However, one can use one of the explicit approximate relationships available in the literature (Zeghadnia et al., 2019).

PROPER RELATIONSHIP OF MANNING'S n

With $V = Q/A$, the Eq.(13) can be rewritten as :

$$Q = \frac{1}{n} A R_h^{2/3} S^{1/2} \quad (21)$$

By equating Eqs.(8) and (21), one can write :

$$Q = \frac{1}{n} A R_h^{2/3} S^{1/2} = -4\sqrt{2g} A \sqrt{R_h S} \log \left(\frac{\varepsilon}{14.8 R_h} + \frac{10.04}{R^*} \right) \quad (22)$$

After simplifications, Eq.(22) is reduced to:

$$\frac{1}{n} = -4\sqrt{2g} R_h^{-1/6} \log \left(\frac{\varepsilon}{14.8 R_h} + \frac{10.04}{R^*} \right) \quad (23)$$

Introducing the dimensionless number representing the right-hand side of Eq.(14), Eq.(23) can be rewritten as :

$$\frac{n\sqrt{g}}{R_h^{1/6}} = \frac{1}{4\sqrt{2}} \left[-\log \left(\frac{\varepsilon}{14.8R_h} + \frac{10.04}{R^*} \right) \right]^{-1} \tag{24}$$

On the other hand, taking into account Eq.(3), Eqs.(9) becomes:

$$R^* = 32\sqrt{2} \frac{\sqrt{gR_h^3S}}{\nu} = 4\sqrt{2} \frac{\sqrt{gD^3S}}{\nu} [\varphi(\eta)]^{3/2} \tag{25}$$

For the full-pipe, corresponding to $\varphi(1) = 1$, Eq.(25) can be rewritten as:

$$R^* = R_f^* [\varphi(\eta)]^{3/2} \tag{26}$$

Where:

$$R_f^* = 4\sqrt{2} \frac{\sqrt{gD^3S}}{\nu} \tag{27}$$

According to Eq.(26), it is worth noting that the ratio R^* / R_f^* depends exclusively on the relative flow depth $\eta = h / D$.

Inserting Eqs.(3) and (26) into Eq.(24), and rearranging, gives the following results:

$$\frac{n\sqrt{g}}{D^{1/6}} = \frac{\sqrt{2}}{8} [\varphi(\eta)]^{1/6} \left[-\log \left(\frac{\varepsilon / D}{3.7\varphi(\eta)} + \frac{0.222}{R_f^* [\varphi(\eta)]^{3/2}} \right) \right]^{-1} \tag{28}$$

Eq.(28), presented in dimensionless terms, is the fundamental relationship which governs the Manning's n coefficient in a partially filled circular pipe. It can be plotted on a Cartesian coordinate graph where the x-axis represents the dimensionless number $n\sqrt{g} / D^{1/6}$ and the y-axis represents the filling rate $\eta = h / D$. For this, the value of the relative roughness ε / D is fixed and R_f^* is varied. One may thus obtain a series of curves that have a resemblance to those of Figs. 2 and 3. By following this procedure, it can be observed the influence of the modified Reynolds number R_f^* on the Manning's n coefficient for a given value of ε / D .

For the full-pipe, Eq.(28) gives :

$$\frac{n_f \sqrt{g}}{D^{1/6}} = \frac{\sqrt{2}}{8} \left[-\log \left(\frac{\varepsilon / D}{3.7} + \frac{10.04}{R_f^*} \right) \right]^{-1} \quad (29)$$

The ratio of Eqs.(28) and (29) leads to :

$$\frac{n}{n_f} = [\varphi(\eta)]^{1/6} \left[-\log \left(\frac{\varepsilon / D}{3.7\varphi(\eta)} + \frac{10.04}{R_f^* [\varphi(\eta)]^{3/2}} \right) \right]^{-1} \left[-\log \left(\frac{\varepsilon / D}{3.7} + \frac{10.04}{R_f^*} \right) \right] \quad (30)$$

Camp (1946) further stated that:

$$\frac{n}{n_f} = \left(\frac{R_h}{R_{h,f}} \right)^{1/6} \frac{f}{f_f} \quad (31)$$

which is incorrect. It must be corrected by writing the proper equation as:

$$\frac{n}{n_f} = \left(\frac{R_h}{R_{h,f}} \right)^{1/6} \left(\frac{f}{f_f} \right)^{0.5} \quad (32)$$

Eq.(32) can be easily worked out from the combination of Eqs.(10) and (13).

The following important result can be deduced from Eq.(30). For a half-full pipe, it was demonstrated previously that $R_h = R_{h,f}$ [Eq.(7)] and $\varphi(0.5) = 1$. Therefore, the quantities in parenthesis of the right-hand side of Eq.(30) are equal and their ratio gives 1. In other words, Eq.(30) necessarily leads to writing that $n = n_f$ when the pipe is half full. This result is not observed on the Camp's chart (1946) regarding the variation of Manning's n with the relative depth h/D . As an indication, Manning's n value according to Camp's chart is about $1.25 \text{ s} / \text{m}^{1/3}$ for $h/D = 0.5$. Fig.2 shows the variation of n/n_f with h/D in a smooth circular pipe partially filled, in accordance with Eq.(30), while Fig.3 indicates the variation of n/n_f in a rougher pipe.

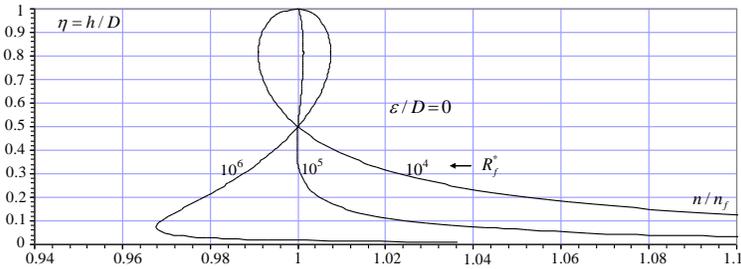


Figure 2: Variation of n/n_f with h/D in a smooth circular pipe, according to Eq.(30)

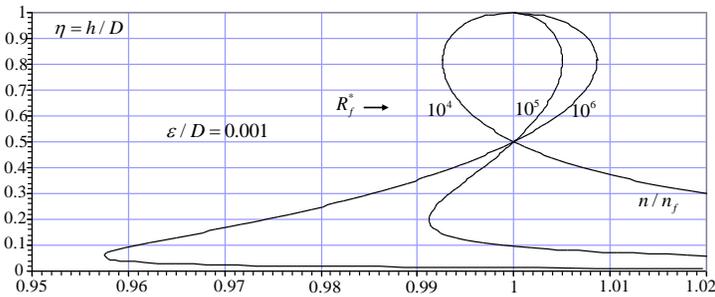


Figure 3: Variation of n/n_f with h/D in a circular pipe for $\epsilon/D=0.001$, according to Eq.(30)

DISCUSSION OF RESULTS

As can be observed in Figs.2 and 3, the variation of n/n_f in smooth or rougher pipes strongly depends on the value of R_f^* . It can be also observed that the more the relative roughness ϵ/D increases and the more the n/n_f ratio increases, meaning that n takes large values. This is confirmed in the literature.

As expected, all the obtained curves intersect the particular point $n/n_f=1$ for the half-full state of the pipe. It should also be noted that the Camp’s data are such that $n/n_f \geq 1$ throughout the range $0.026 \leq h/D \leq 1$. Figs. 2 and 3 show that n/n_f can be less than unity, depending on the value of R_f^* . This fact has been already reported in the literature (Pomeroy, 1967; Meko et al., 2015). The ratio n/n_f can also remain almost constant beyond the relative depth of 30%, in the case of smooth pipes as shown in Fig.2 for $R_f^*=10^5$. For shallow depths, n/n_f

undergoes an abrupt variation with h/D and the curves pass from negative to positive through an inflection point, until reaching the relative depth of 80%. This is particularly the case for high values of R_f^* . In the shallow depth flow region, extending beyond the inflection point, the size of the roughness has an important effect and the n value should be large. The inflection point gets closer and closer to the shallow depths as R_f^* increases as it can be observed in Fig.3. Due to the presence of the inflection point, the curves in Figs. 2 and 3 do not pass through the origin. In fact, in the current state of knowledge, the variation of n/n_f around the small flow depths is not well known. Even on the Camp's chart, designed with practical data, the curve showing the variation of n/n_f versus h/D is interrupted when approaching shallow depths, due to the fact that there is no available data for this area of flow. If the curves of n/n_f were to pass through the origin, this would mean that $1/n$ tends to infinity. In the same time, the hydraulic radius R_h tends to zero, that will involve a $\infty \times 0$ indeterminate form in Eq.(13). This indeterminacy can be lifted if it is accepted that around small depths n tends to take great values, what the curves in Figs. 2 and 3 show.

CONCLUSIONS

The aim of the study was to examine the influence of the flow parameters on the variation of the Manning's n coefficient in a partially filled circular pipe, for doing so, the Manning's equation expressing the discharge [Eq. (21)] has been faced with a dimensionally consistent uniform flow relationship derived from the combination of the rational Darcy-Weisbach and Colebrook equations [Eqs. (8) and (11)]. The derived final relation [Eq (24)], valid for all shapes of channels, contains in its left-hand side a dimensionless parameter related to the Manning's n coefficient and in its right-hand side a dimensionless parameter R^* , acting as a Reynolds number, representing, in fact, the ratio of the friction forces to viscous forces. It has been shown that R^* can be related to both the Reynolds number R and the relative roughness ε/R_h in accordance with an implicit relation [Eq.(20)]. The general relationship [Eq (24)] has been applied to the partially filled circular pipe. This resulted in an equation linking both the relative flow depth h/D , the relative roughness ε/D , and the dimensionless number R_f^* which is a characteristic of the flow when the pipe is full [Eq.(28)]. The same relationship was applied to the totally filled circular pipe [Eq.(29)] and the ratio between the latest equations led to the final Eq.(30). This has been plotted in Figs. 2 and 3 for a smooth pipe and for a rougher pipe respectively.

The filling rate h/D is representing on the Y-axis, while n/n_f is defined by the X-axis. Depending on the R_f^* values, the slope of the curves changes from negative to positive or vice versa. A change in slope from negative to infinity can even be observed in the case of the smooth pipe, indicating a constant value for n/n_f beyond a certain filling rate [Fig.(2)]. Beyond the filling rate of 80%, n/n_f decreases for the low values of R_f^* , while it increased for the larger values of R_f^* . Whatever the value of both the relative roughness ε/D and the number R_f^* , the largest values of n/n_f are observed around shallow depths [Figs.(2) and (3)].

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