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DISCUSSION

DISCHARGE COEFFICIENT OF SHAFT SPILLWAY UNDER SMALL HEADS

COEFFICIENT DE DEBIT DES EVACUATEURS EN PUITS SOUS FAIBLES CHARGES

By

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Discussers

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The discussers would like to thank the authors for their remarkable demonstration aimed at the analytical determination of the discharge coefficient value of a shaft spillway operating under shallow heads. In hydraulic engineering, the determination of the discharge coefficient is one of the underlying issues. As a general rule, it is found out from laboratory tests carried out above a lower limit threshold of the relative heads. The discussers would like to share their point of view relating to this subject by adopting a different theoretical approach which leads to a result of the same magnitude. For the analytical determination of the discharge coefficient, the theoretical approach consists to compute the coefficient of the vertical contraction of the flow considering curvature effects as well as the coefficient of velocity taking accounts the velocity distribution

THEORETICAL DEVELOPMENT

Let consider a flow through a shaft spillway under a small head as shown on Fig. (1). The flow discharge relationship for a unit length of the shaft perimeter is given by:

$$q = C_d \sqrt{2g} H^{3/2} \qquad (1)$$

where $H =$ total head which is the static head far from the shaft intake, $C_d =$ discharge coefficient defined as (Henderson, 1966):

$$C_d = \frac{2}{3} C_c C_v \qquad (2)$$

in which $C_c =$ coefficient of contraction and $C_v =$ velocity coefficient.

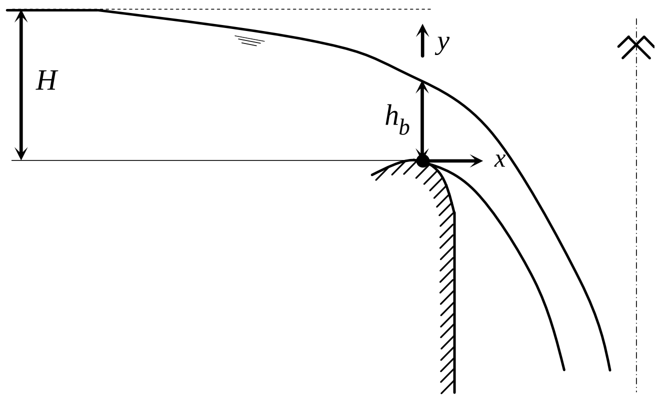

Figure 1: Definition sketch of the flow problem

Let first determine the contraction coefficient considering the streamline curvature effect at the vicinity of the intake. In this case, the generalized Bernoulli equation using Boussinesq approximation can be expressed as (Subramanya, 2009):

$$H = h + \alpha \frac{V^2}{2g} + \frac{V^2 h}{3g}\left(\frac{d^2 h}{dx^2}\right) \qquad (3)$$

Assuming that Coriolis factor $\alpha = 1$ for simplicity and H being constant in the neighbourhood of the shaft intake, Eq. (3) is rewritten in dimensionless form with respect to the total head at the funnel intake brink as follows:

$$1=\frac{h_b}{H}+\frac{q^2}{2gh_b^2H}+\frac{q^2}{3gh_bH}\left(\frac{d^2h}{dx^2}\right)_{h=h_b} \tag{4}$$

where velocity *V* is replaced by *q/h* from continuity equation. Noting the critical depth $h_c^3=q^2/g$, Eq.(4) reads then:

$$1=\frac{h_b}{H}+\frac{h_c^3}{2h_b^2H}+\frac{h_c^3}{3h_bH}\left(\frac{d^2h}{dx^2}\right)_{h=h_b} \tag{5}$$

The curvature expression of the water nappe at the brink is obtained by matching flow velocity with that of the free-jet. The free-jet trajectory of a falling nappe can be considered for the mean streamline for $h_b \to 0$ as:

$$y=-\frac{1}{2}gt^2=-\frac{1}{2}g\frac{x^2}{V^2} \quad \text{or} \quad y=-\frac{1}{2}g\frac{h_b^2}{q^2}x^2 \tag{6}$$

Equating curvatures at the brink for the free-jet and the approach water nappe, we get:

$$\left.\frac{d^2y}{dx^2}\right|_{y=h_b}=\left.\frac{d^2h}{dx^2}\right|_{h_b}=\frac{-h_b^2}{h_c^3} \tag{7}$$

Replacing Eq.(7) into Eq.(5) and expressing critical depth as a function of the total head as $h_c=2H/3$ results in:

$$\frac{h_b}{H}+\frac{4}{27}\frac{H^2}{h_b^2}-\frac{h_b}{3H}=1 \tag{8}$$

Knowing that the contraction coefficient $C_c=h_b/H$, Eq.(8) reads finally:

$$\frac{2}{3}C_c^3-C_c^2+\frac{4}{27}=0 \tag{9}$$

For which the physically consistent solution is $C_c=0.4629$.

Now, let determine the velocity coefficient at the shaft spillway intake. To do that, we must consider the boundary layer theory just on the intake. The approaching flow being potential, a sudden shear stress appears when flow touching the concrete intake funnel before falling down into the well. Thus, the velocity profile will mark a strong gradient near the wall (Figure 2). At this stage the boundary layer is not fully developed and boundary thickness is in the growing phase. For simplicity reason, let assume the power law velocity distribution for a fully developed flow of the form (Cheng, 2007):

$$\frac{u(z)}{U_{\max}} = \left(\frac{z}{h}\right)^{1/n} \tag{10}$$

in which $U_{\max}$ is the free stream velocity, n is an exponent depending of the Reynolds number of the incident flow. A frequent value $n = 7$ is generally used for which Eq.(7) is called Von Kármán-Prandtl 1/7 power law (Schlichting, 1968). However for a growing boundary layer this exponent is not known. On the other hand, it is known that in this growing stage the stiff velocity gradient suggests a choice of a greater value of n. Taking $n = 8$ and integrating over the flow depth at the brink, we have the mean velocity $\bar{u}$ as:

$$\bar{u} = \frac{U_{\max}}{h_b}\int_0^{h_b}\left(\frac{z}{h}\right)^{1/8} dz = \frac{8}{9}U_{\max} \tag{11}$$

The velocity coefficient being defined as the ratio of the real velocity $\bar{u}$ to the frictionless velocity $U_{\max}$, one gets $C_v = 8/9$.

Inserting values of C_c and C_v obtained herein into Eq.(2) the discharge coefficient C_d for the shaft spillway under small heads is then:

$$C_d = 0.2743 \tag{12}$$

Considering different values of n in Eq. (10) to describe strongest velocity gradients, the values of discharge coefficient are reported in table (1).

Table 1: Discharge coefficient C_d for different values of the exponent n

n	7	8	9	10
C_d	0.2700	0.2743	0.2777	0.2805

As it can be seen from table 1, the discharge coefficient value is not very sensitive for different values of n. Hence, we can see that the value of C_d for n = 8 is practically the same as the mean of these values.

The value of the discharge coefficient obtained by the Authors using classical energy considerations was $C_d = 0.271$ as an average value. This result is very close to that obtained herein by considering the streamlines curvature and boundary layer effects. Furthermore, if boundary layer effects are neglected i.e. $C_v = 1$, we obtain a discharge coefficient $C_d = 0.3086$ which is fairly in good agreement with Moise's and Rozanov's experimental data for small heads.

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REPLY BY THE AUTHORS

The Authors are indebted to Dr. Amara and Pr. Achour for their valuable contribution to the understanding of the mechanics of the complex problem of shaft spillway flows.

Indeed, the issue of determining the flow coefficient at small heads is not resolved unambiguously and there may be many solutions. The Authors proposed one of the options, discussers suggested another, and surely there are several other options for solving this problem. It is to be emphasized once again the importance of this issue specifically for unregulated spillways, which begin to work at 4-5 millimeters, and not from 0.

Moreover, in hydraulics, inertial forces play a large influence on the kinematic parameters of the flow and its dynamic characteristics. But it is very difficult to study these inertial forces, especially in gravity flows, since it is practically impossible to accurately determine the parameters of the free surface, taking into account the effect of inertial forces, curvature effects and even more so the parameters of the boundary layers.