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NEW FORMULATION OF THE DARCY-WEISBACH FRICTION FACTOR

NOUVELLE FORMULATION DU COEFFICIENT DE FROTTEMENT DE DARCY-WEISBACH

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ABSTRACT

The proper assessment of the friction factor f is of a great importance in the sound resolve of turbulent flow problems. The current rational formulation of f is that developed by Colebrook stating that f depends on the relative roughness ε / D_h and the Reynolds number R, through an implicit equation. The new formulation developed herein presents f as a function not of the usual Reynolds number R but of a dimensionless parameter, denoted R^*, representing the ratio of the friction forces to the viscous forces. Acting as a Reynolds number, it is shown that R^* is governed by an implicit equation of ε / R_h and R. The calculation of the friction factor value using the new formulation gives a maximum deviation of 0.25% in comparison with the exact value of f derived from Colebrook equation. At the end of an additional calculation step, the deviation drops down to a maximum of 0.04% only. This calculation step is recommended for solving problems requiring high accuracy. All the formulas developed herein can be classified in the category of short equations, easily memorized, handy, and of good accuracy.

Keywords: Friction factor, Darcy-Weisbach, Reynolds number, Pipe-flow.

RESUME

L'évaluation appropriée du coefficient de frottement *f* est d'une grande importance dans la bonne résolution des problèmes d'écoulement turbulent. La formulation rationnelle actuelle de *f* est celle développée par Colebrook en montrant que *f* dépend de la rugosité relative ε / D_h et du nombre de Reynolds *R*, à travers une équation implicite. La nouvelle formulation développée ici présente *f* en fonction non pas du nombre de Reynolds habituel *R* mais d'un paramètre sans dimension, noté R^*, représentant le rapport des forces de frottement aux forces visqueuses. Agissant comme un nombre de Reynolds, il est démontré que R^* est régi par une équation implicite de ε / R_h et de *R*. Le calcul de la valeur du coefficient de frottement à l'aide de la nouvelle formulation donne un écart maximal de 0,25% par rapport à la valeur de *f* dérivée de l'équation de Colebrook. À la fin d'une étape de calcul supplémentaire, l'écart tombe à un maximum de 0,04% seulement. Cette étape de calcul est recommandée pour résoudre les problèmes nécessitant une grande précision. Toutes les formules développées ici peuvent être classées dans la catégorie des équations courtes, facilement mémorisables, maniables et de bonne précision.

Mots clés : Coefficient de frottement, Darcy-Weisbach, nombre de Reynolds, Conduite.

INTRODUCTION

In a turbulent flow regime, the friction factor, denoted *f*, plays a very important role. It is a dimensionless parameter that relates the head loss in a pipe to its length/diameter ratio and dynamic pressure (Jaeger, 1956). It is governed by the well-known implicit Colebrook formula (1939) which states that *f* depends on the relative roughness ε / D_h and the Reynolds number *R*. Colebrook formula is one of the few relationships that has aroused so much interest probably due to its importance in solving a number of main problems such as pressure drop calculation in pipe-flow. Various approaches are available to solve the Colebrook equation and find the appropriate value of *f*. The best known of the time was the graphical solution using the Rouse and Moody diagrams, established respectively in the years 1943 and 1944. Due to the low accuracy, the value of *f* provided by the reading of theses charts should be as mere guidance value or approximate. With the advent of modern laptops, the Colebrook equation can be solved iteratively using an Excel spreadsheet or a programming solver, but this approach requires more computational time. Recently (Brkić, 2011), Lambert W function was used for the calculation of *f* and this approach seems to avoid iterative calculation and reduces the relative errors. This probably has a purely theoretical and mathematical interest, but has no practical impact in the daily work of an engineer who needs a fast and reliable calculation of the friction factor. It is also the major concern of students faced with their exam problems. There is also the trial-and-error method which is today completely obsolete. One of the research workers' most noticed and preferred approaches is to solve the Colebrook equation by an explicit approximate equation. In this regard, one may counts up in the literature no less than forty approximate formulas solving for *f*. These differ in both form and accuracy (Zeghadnia et al., 2019). The craze to find an easier-to-compute formula result still remains today. There are short and simple formulas but less accurate than the long and more developed formulas. The most effective will be the one which will admit the best trade-off between time of computation and precision of results. It is better to use short formulas that are less accurate than long formulas that are more accurate for at least three reasons. Long formula requires a longer computation time, they are not easy to memorize and present the risk of omitting terms when key stroking. The short formulas available in the literature generate a deviation in *f* of the order of 2% to 3% in the whole range of turbulent flow corresponding to Reynolds number $R \geq 2300$. Considering, for instance, the three main problems encountered in turbulent pipe-flow, 2% deviation in *f* causes the same deviation in the slope of the energy grade line computation, 1% in the discharge calculation, and only 0.4% in the determination of the pipe diameter. It is therefore not a "disaster".

In this technical note, a new formulation of the friction factor *f* is presented not dependent on the usual Reynolds number R but on a dimensionless number denoted R^*. This acts as a Reynolds number taking into account the effect of the friction forces. It can be, therefore, considered within a constant as the shear Reynolds number. It is shown that R^* depends on both the Reynolds number R and the relative roughness ε / D_h, through an implicit equation. A good approximate relationship has been found which allows calculating *f* with an acceptable accuracy when compared to the *f* value given by the Colebrook equation. If the problem under considerations requires a more accurate value of *f*, an additional step calculation is then necessary. This causes a significant drop down in the deviation in *f*. Thanks to the introduction of the dimensionless parameter R^*, it was possible to derive an explicit approximate relationship of the shear velocity u^*.

NEW FORMULATION OF THE FRICTION FACTOR

Using the Rough Model Method (Achour and Bedjaoui, 2006) or eliminating the friction factor *f* between the Darcy-Weisbach (1854) and Colebrook (1939) equations, the following dimensionally consistent uniform flow relationship can be worked out as:

$$Q = -4\sqrt{2g}\,A\sqrt{R_h S}\,\log\left(\frac{\varepsilon}{14.8R_h} + \frac{10.04}{R^*}\right) \tag{1}$$

where Q is the discharge, S is the slope of the energy grade line, ε is the absolute roughness, g is the acceleration due to gravity, A is the wetted area, R_h is the hydraulic radius and ν is the kinematic viscosity. The parameter R^* is a dimensionless number, acting as a Reynolds number, expressed as:

$$R^* = 32\sqrt{2}\,\frac{\sqrt{gR_h^3 S}}{\nu} \tag{2}$$

Eq.(1) is valid for any channel and pipe shape. It is also applicable in the whole domain of turbulent flow, including smooth, transitional, and rough flow regimes. The Darcy-Weisbach and Colebrook equations can be written as follows:

$$S = f\,\frac{V^2}{8gR_h} \tag{3}$$

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{14.8R_h} + \frac{2.51}{R\sqrt{f}}\right) \tag{4}$$

where V is the mean velocity of the flow.

The quantity $\sqrt{gR_h S}$ in Eq.(2) corresponds to the shear velocity u^* also called friction velocity (Schlishting, 1979) having dimension of velocity. Thus, Eq.(2) can be re-written in the following form:

$$R^* = 32\sqrt{2}\,\frac{u^* R_h}{\nu} \tag{5}$$

With regard to the form of the Eq.(5), the dimensionless number R^* would give a measure of the ratio of friction forces to viscous forces and consequently the relative importance of these kind of forces. On the other hand, Eq.(3) allows writing that:

$$u^{*2} = f\,\frac{V^2}{8} \tag{6}$$

or:

$$u^* = V\sqrt{\frac{f}{8}} \tag{7}$$

Multiplying both sides of Eq.(7) by $32\sqrt{2}R_h/\nu$ and knowing that $R_h = D_h/4$, one may derived the following result:

$$32\sqrt{2}\,\frac{u^{*}R_h}{\nu}=32\sqrt{2}\,\frac{VD_h/\nu}{4\sqrt{8}}\sqrt{f} \tag{8}$$

which is reduced to :

$$R^{*}=4R\sqrt{f} \tag{9}$$

When the proper values of R and f are known, Eq.(9) gives the exact value of R^{*}. It is worth noting that the equality $R^{*}=R$ is obtained for $f=1/16=0.0625$. This corresponds to the relative roughness $\varepsilon/D_h=0.037$ according to Eq.(4) for $R\to\infty$. The flow is in the fully rough domain.

With $V=Q/A$, Eq.(3) expresses the discharge Q as:

$$Q=\frac{\sqrt{8}}{\sqrt{f}}A\sqrt{gR_hS} \tag{10}$$

Comparing Eqs.(1) and (10) results in:

$$\frac{1}{\sqrt{f}}=-2\log\left(\frac{\varepsilon}{14.8R_h}+\frac{10.04}{R^{*}}\right) \tag{11}$$

The friction factor f is therefore presented as a function of both the relative roughness ε/R_h and the dimensionless number R^{*}.

Extracting $1/\sqrt{f}$ from Eq.(9) and inserting it in Eq.(11) gives:

$$R=-\frac{1}{2}R^{*}\log\left(\frac{\varepsilon}{14.8R_h}+\frac{10.04}{R^{*}}\right) \tag{12}$$

Eq.(12) shows that R^{*} is governed by an implicit function of both the relative roughness ε/R_h and the Reynolds number R. To avoid the implicit calculation induced by the Eq.(12), one may use the following explicit approximate relationship :

$$R^{*}=2R\left[-\log\left(\frac{\varepsilon/D_h}{3.7}+\frac{5.45}{R^{0.9}}\right)\right]^{-1} \tag{13}$$

As shown in Fig.1, the maximum deviation between Eqs.(12) and (13) is about 1% only. The deviation depends strongly on the relative roughness and the Reynolds number values.

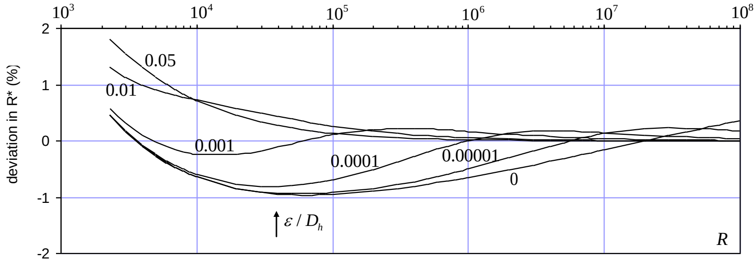

Figure 1: Deviation in R^* between Eqs.(12) and (13)

When the relative roughness ε / D_h and the Reynolds number R are given, Eq.(11) along with Eq.(13) allows computing the friction factor f with a maximum deviation of less than 0.25% as shown in Fig.2.

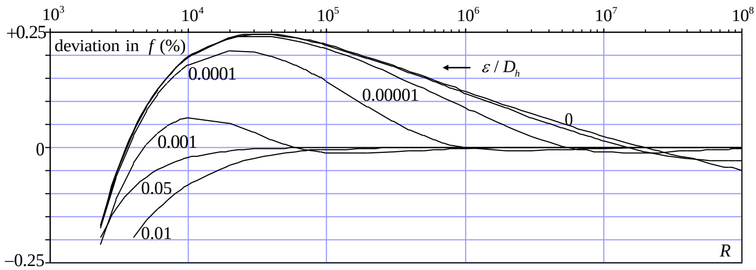

Figure 2: Deviation between Eqs.(4) and (11)

SHEAR VELOCITY

The shear velocity u^* can be expressed when combining Eqs.(5) and (13). Hence:

$$32\sqrt{2}\,\frac{u^* R_h}{\nu} = 2R\left[-\log\left(\frac{\varepsilon / D_h}{3.7} + \frac{5.45}{R^{0.9}}\right)\right]^{-1} \tag{14}$$

Rearranging Eq.(14) results finally in :

$$u^{*} = \frac{\nu R_h^{-1}}{16\sqrt{2}} R \left[-\log\left(\frac{\varepsilon / R_h}{14.8} + \frac{5.45}{R^{0.9}} \right) \right]^{-1} \tag{15}$$

Knowing that $R = 4VR_h / \nu$ where V is the mean velocity, Eq.(15) can be rewritten as :

$$\frac{u^{*}}{V} = \frac{1}{4\sqrt{2}} \left[-\log\left(\frac{\varepsilon / R_h}{14.8} + \frac{5.45}{R^{0.9}} \right) \right]^{-1} \tag{16}$$

IMPROVEMENT OF THE FRICTION FACTOR CALCULATION ACCURACY

To improve the friction factor f calculation accuracy, one may try to find a substitute relation to the equation (13), but it is not self-obvious. The simplest way is to consider an additional step in the calculation of f using Eqs.(9), (11), and (13). The calculation procedure can be described by the following steps:

1. Let's denote R_0^{*} the value of R^{*} given by Eq.(13) for the known values of both the Reynolds number R and the relative roughness ε / D_h. The corresponding value of f is $f_1(R_0^{*})$, worked out from Eq.(11). We have seen earlier that the maximum deviation caused by this first step calculation on the f value, when compared to Colebrook Eq.(4), is less than 0.25%.

2. Compute in this additional step $R_1^{*}(f_1)$ using Eq.(9), whence:

$$R_1^{*} = 4R\sqrt{f_1} \tag{17}$$

3. Introduction $R_1^{*}(f_1)$ so calculated into Eq.(11) gives f_2 as the second value of f.

It was observed that the maximum deviation between f_2 and f given by the Colebrook Eq.(4) is less than 0.04% as it is reported in Fig.3. This result was obtained for $R \geq 2300$ and $0 \leq \varepsilon / D_h \leq 0.05$, thus encompassing the whole domain of turbulent flow. Thus, at the end of the second calculation step, the deviation on f experienced a significant drop down from 0.25% to 0.04%. In fact, R_0^{*} served as an appropriate initial guess value rapidly converging the only two-steps iterative process. The computation can stop at this step because the relative error in f is largely sufficient to solve accurately practical problems.

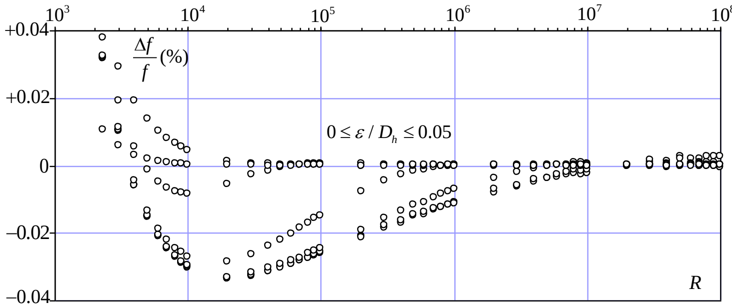

Figure 3: Deviation between Eqs.(4) and (11) at the end of an additional calculation step

NUMERICAL EXAMPLE

For the following data, compute the Darcy-Weisbach friction factor f using Eq.(11) along with Eq.(13), after the first calculation step and then after the second calculation step. What should be the deviation in comparison with the value of f given by Colebrook equation?

$$\varepsilon / D_h = 0.00001,\ R = 2{,}000{,}000$$

Let's assume the following definitions:

f_e = the "exact" value of f computed using Colebrook Eq.(4).

$f_{1,a}$ = the approximate value of f after the first step of calculation.

$f_{2,a}$ = the approximate value of f after the second step of calculation.

$R^*_{0,a}$ = initial guess value of R^* computed using the approximate Eq.(13) and giving $f_{1,a}$ by the use of Eq.(11).

The subscripts "1", "2", "0", "a", and "e" denote respectively "first step calculation" "second step calculation", "initial value", "approximate value", and "e" "exact value".

1. The iterative process applied to the implicit Eq.(4) of Colebrook gives f_e value as:

$f_e = 0.0107206$

2. The exact value of R^* is given by the implicit Eq.(12). The calculation shows that:

$$R_e^* = 828322.642$$

Note that the exact value of R^* can also be computed using Eq.(9) in which $f = f_e$.

3. The intial approximate value of R^* is easily worked out from Eq.(13). The final result is:

$$R_{0,a}^* = 825804.52$$

4. The deviation between the exact value of R^* given in step 2, i.e. R_e^*, and the approximate value computed in step 3, i.e. $R_{0,a}^*$, is then :

$$100 \times \left| \frac{R_{0,a}^* - R_e^*}{R_e^*} \right| = 100 \times \frac{825804.52 - 828322.642}{828322.642} = 0.304\%$$

5. The approximate value of *f*, i.e. $f_{1,a}$, is given by Eq.(11) in which the value of R^* is that of $R_{0,a}^*$ computed in step 3. Hence:

$$f_{1,a} = 0.01072536$$

6. The deviation between f_e and $f_{1,a}$ is then:

$$100 \times \left| \frac{f_{1,a} - f_e}{f_e} \right| = 100 \times \frac{0.01072536 - 0.0107206}{0.0107206} = 0.0443\%$$

7. This step aims to compute the deviation between f_e and $f_{2,a}$ after an additional calculation step following the procedure described above. Eq.(9) gives:

$$R_1^* = 4R\sqrt{f_{1,a}}$$

The final result is:

$$R_1^* = 828506.369$$

8. Introducing R_1^* in Eq.(11) gives a second approximate value of f, i.e. $f_{2,a}$, as:

$$f_{2,a} = 0.0107202$$

9. The deviation between f_e and $f_{2,a}$ is:

$$100\times\left|\frac{f_{2,a}-f_e}{f_e}\right| = 100\times\frac{0.0107202-0.0107206}{0.0107206} = 0.0037\%$$

CONCLUSIONS

A new formulation of the friction factor $f(\varepsilon/R_h, R^*)$ was presented [Eq(11)], where R^* is a dimensionless parameter, acting as a Reynolds number taking into account the effect of the friction forces. It was shown that R^* depends on both the Reynolds number R and the relative roughness ε/R_h through an implicit equation [Eq.(12)]. An approximate relationship has been derived giving R^*-value with a maximum deviation of 1% [Fig.(1)]. The friction factor $f(\varepsilon/R_h, R^*)$ was then calculated with a maximum deviation of 0.25% [Fig.(2)] when compared to the friction factor given by the Colebrook equation [Eq.(4)]. If the problem under considerations requires a more accurate friction factor value, an additional step calculation has been suggested to improve the accuracy causing the deviation on f to drop down significantly from 0.25% to 0.04% [Fig.(3)].

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