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NEW THEORETICAL CONSIDERATIONS ON THE CRITICAL FLOW IN A CIRCULAR CONDUIT (PART 1)

NOUVELLES CONSIDERATIONS THEORIQUES SUR L'ECOULEMENT CRITIQUE DANS UNE CONDUITE CIRCULAIRE (PARTIE 1)

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ABSTRACT

In a circular conduit of diameter D and of a given slope S_0, two critical states of the flow may occur for two different discharges. The first one is observed at shallow depths while the second one settles down at greater depths. This statement is the result of the in-depth study carried out on the smooth circular conduit of diameter D = 1m, taken as an example. For this conduit, all slopes S_0 greater than $S_0 = 0.00183813$ generate the two critical states of the flow. Slopes that are less than this value do not generate any critical state of the flow. The study reveals that the slope $S_0 = 0.00183813$ corresponds to the smallest slope that causes a single critical state of the flow. Other interesting conclusions, fundamental relationships as well as meaningful graphs are drawn from this study, after a detailed examination of the rational equations which govern the critical and normal flows.

Keywords: Circular conduit, critical depth, normal depth, slope, discharge.

RESUME

Dans une conduite circulaire de diamètre D et de pente donnée, deux états critiques de l'écoulement peuvent se produire pour deux débits différents. Le premier état critique est observé aux faibles profondeurs, tandis que le second s'installe à des profondeurs plus grandes. Cette affirmation est le résultat approfondie menée sur la conduite circulaire lisse de diamètre D = 1m, prise comme exemple. Pour cette conduite, toutes les pentes S_0 supérieures à $S_0 = 0.00183813$ génèrent les deux états critiques de l'écoulement. Les pentes qui sont inférieures à cette valeur ne génèrent aucun état critique de l'écoulement. L'étude révèle que la pente $S_0 = 0.00183813$ correspond à la plus petite pente qui génère un seul état critique de l'écoulement. D'autres conclusions intéressantes, des relations fondamentales et aussi des graphes significatifs sont tirés de l'étude, après un examen détaillé des équations rationnelles qui gouvernent les écoulements critique et normal.

Mots clés : Conduite circulaire, profondeur critique, profondeur normal, pente, débit.

INTRODUCTION

The critical depth denoted y_c, corresponds to a minimum specific energy for a given discharge or the depth at which the discharge is maximum for a given specific energy (Chow, 1959). Critical depth is an important parameter in understanding the characteristics of flow. If the actual depth is greater than critical depth, then the flow is considered as subcritical. The flow is said to be supercritical when the actual depth is less than the critical depth. The critical depth is also used in the classification of water surface profiles, along with the slope of the channel S_0 and the normal depth y_n. At critical flow conditions, the Froude number is equal to unity. It is this property that is used to determine the critical depth in open channels, also called the critical criterion (Swamee, 1993; Vatankhah and Bijankhan, 2010; Vatankha and Easa, 2011; Shang et al., 2019).

The critical depth, even if it is a particular depth, is a uniform depth that should depend on the characteristics of the flow such as the slope S_0 of the channel, the absolute roughness ε characterizing the state of the inner wall of a channel or a conduit, and the kinematic viscosity ν of the flowing liquid. According to our investigations, there is no study available on this subject.

The depth of a flow is said to be normal when the free surface of the water is parallel to the bottom of the channel, meaning that the slope of the channel S_0 and that S_w of the surface water are equal. The flow depth remains constant along the stream. When the gravitational force balances the friction drag along the channel, normal depth then occurs with no acceleration of the flow (Chow, 1959; Subramanya, 2007; Chaudhry, 2008; Moglen, 2015).

The calculation of the normal depth in the channels is essentially based on the Manning formula which has remained the most popular in recent years (Manning, 1891). Applying this relation to channels of different shapes often leads to an implicit relationship, and many research workers have proposed approximate relationships according to the shape of the channel (Vatankha and Easa, 2011; Shang et al., 2019). These formulas are virtually unusable in their current form since they contain Manning's *n* coefficient. It has been recognized, for a long time (Camp, 1946; Achour and Bedjaoui, 2006; Achour, 2014), that the flow resistance coefficients such that Manning's *n* varies according to the parameters of the flow, in particular the depth which is the sought unknown parameter of the problem. Select an appropriate value of the flow resistance coefficient without knowing the parameters thereof would be a real achievement. These formulas can only be used if the flow is in the rough zone for which the flow resistance coefficient is practically constant. In this case, one may then, as a first approximation, consider the value of the resistance coefficient given by the tables according to the nature of the material of the channel. To properly use these approximate relations, a preliminary calculation is necessary which consists in first calculating the Manning's *n* coefficient. To do this without taking into account the normal depth which is the sought parameter, the Rough Model Method (RMM) is the most appropriate and remains the only method available to solve this type of problem (Achour, 2014; Achour and Sahtal, 2014; Achour 2015).

The literature indicates that, regardless of the shape of the channel profile such as rectangular, trapezoidal, or circular, there is only one discharge that generates a unique critical state of the flow. However, the literature does not answer the question of how many discharges exist that generate so many critical flow states in a channel whose shape and slope are given. The present study gives a clear, reasoned, and persuasive answer to this question for the case of a circular conduit. The theoretical approach is based on two rational formulas, namely the dimensionally consistent uniform flow relationship and the criticality criterion. The first one has been derived from the Rough Model Method (Achour and Bedjaoui, 2006) that expresses the discharge Q with respect to all the parameters influencing the flow: $Q(S_0, \varepsilon, g, A, R_h, \nu)$. The effect of the kinematic viscosity ν is evidenced through a Reynolds number which is closely related to the shear Reynolds number giving then a measure of the ratio of friction forces to viscous forces (Achour and Amara, 2020). The second one is worked out from the property of the critical flow for which the Froude number is equal to unity. Eliminating the discharge Q between these two relations results in a firmly implicit but indubitable relationship: $\psi(\eta_c, \eta_n, S_0, \varepsilon / D, R_f^*) = 0$, where R_f^* is the shear Reynolds number at the full state of the conduit, η_c is the relative critical depth, and η_n is the relative normal depth. The relation was applied to a smooth circular conduit of diameter D = 1m, taken as an example. The graph of the variation of the critical depth η_c versus the normal depth η_n for various values of the slope S_0 revealed surprising properties of the flow. The study ends with the examination of the behaviour of the critical depth under the effect of various slopes. A theoretical relationship is deduced from the previous one by simply writing $\eta_n = \eta_c$. The deduced relationship allowed the plotting of a graph which gave a clear overview of the behaviour of the flow, and interesting conclusions were drawn.

RELATIONSHIP BETWEEN CRITICAL AND NORMAL DEPTHS

The criticality criterion for all the known shapes of channels is translated by the following relation (Subramanya, 2009):

$$\frac{\alpha Q^2 T_c}{g A_c^3 \cos\theta} = 1 \tag{1}$$

Where α is the energy correction factor which can be reasonably considered to be equal to unity, Q is the discharge, TT is the top width at the water surface, g is the acceleration due to gravity, A is the water area and θ is the inclination angle of the channel relative to the horizontal. The subscript "*c*" is related to the critical flow conditions.

Knowing that $S_0 = \sin\theta$, where S_0 is the channel slope, one may write:

$$\cos\theta = \cos(\arcsin S_0) = \sin(\arccos S_0) = \sqrt{1 - S_0^2} \tag{2}$$

Eq.(1) can be rewritten as :

$$Q = \frac{\sqrt{g} A_c^{3/2} \left(1 - S_0^2\right)^{1/4}}{\sqrt{T_c}} \tag{3}$$

T_c and A_c for a circular conduit can be written respectively as follows:

$$T_c = 2D\zeta(\eta_c) \tag{4}$$

$$A_c = \frac{D^2}{4}\sigma(\eta_c)\left[1 - \varphi(\eta_c)\right] \tag{5}$$

Where D is the internal diameter of the conduit and ζ, σ and φ are functions exclusively dependent on the relative critical depth η_c. These functions are expressed respectively as:

$$\zeta_c = \sqrt{\eta_c(1-\eta_c)} \tag{6}$$

$$\sigma(\eta_c) = \cos^{-1}(1-2\eta_c) \tag{7}$$

$$\varphi(\eta_c) = \frac{2(1-2\eta_c)\sqrt{\eta_c(1-\eta_c)}}{\cos^{-1}(1-2\eta_c)} \tag{8}$$

where :

$$\eta_c = \frac{y_c}{D} \tag{9}$$

y_c being the critical depth.

On the other hand, the dimensionally consistent uniform flow relationship $Q(S_0, \varepsilon, g, A, R_h, \nu)$ can be established using the Rough Model Method (Achour and Bedjaoui, 2006) or by combining the rational equations of Darcy-Weisbach (1854) and Colebrook (1939). ε is the absolute roughness, ν is the kinematic viscosity and R_h is the hydraulic radius. For normal flow conditions, the above functional relationship is expressed as:

$$Q = -4\sqrt{2g}A_n\sqrt{R_{h,n}S_0}\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R^*}\right) \tag{10}$$

with:

$$R^* = 32\sqrt{2}\frac{\sqrt{gR_{h,n}^3 S_0}}{\nu} \tag{11}$$

The subscript “*n*” is related to the normal flow conditions.

It was shown in an earlier study (Achour and Amara, 2020) that the dimensionless number R^* is closely related to the shear Reynolds number. Consequently, R^* would give a measure of the ratio of the friction forces to the viscous forces.

Since $R_h = A / P$, Eq.(10) becomes:

$$Q = -4\sqrt{2g}\frac{A_n^{3/2}}{P_n^{1/2}}\sqrt{S_0}\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R^*}\right) \tag{12}$$

By equalizing Eqs.(3) and (12) and rearranging, results in:

$$\frac{A_c^{3/2}}{\sqrt{T_c}} = -4\sqrt{2}\,\frac{A_n^{3/2}}{P_n^{1/2}}\,\frac{\sqrt{S_0}}{(1-S_0^2)^{1/4}}\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R^*}\right) \tag{13}$$

The greatest slope S_0 considered in this study is $S_0 = 5/100$. For this slope the quantity $(1-S_0^2)^{1/4}$ is equal to 0.99937 and can therefore be considered as equal to 1. Thus, one may write $(1-S_0^2)^{1/4} \approx 1$ for $S_0 \leq 5/100$ .Therefore, Eq.(13) becomes :

$$\frac{A_c^{3/2}}{\sqrt{T_c}} = -4\sqrt{2}\,\frac{A_n^{3/2}}{P_n^{1/2}}\sqrt{S_0}\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R^*}\right) \tag{14}$$

where P_n is the wetted perimeter.

This is the general relationship that combines both the characteristics of critical and normal flows, valid for all known channels and pipes shapes. For a partially filled circular conduit, the hydraulic radius can be expresses as:

$$R_{h,n} = \frac{D}{4}[1-\varphi(\eta_n)] \tag{15}$$

$\varphi(\eta_n)$ is given by Eq.(8) after substituting η_c by η_n . The wetted perimeter P_n is as:

$$P_n = D\,\sigma(\eta_n) \tag{16}$$

$\sigma(\eta_n)$ is governed by Eq.(7) for $\eta_c = \eta_n$.

Taking into account all these considerations, Eq.(14) is reduced to:

$$\frac{\left[\sigma(\eta_c)\right]^{3/2}\left[1-\varphi(\eta_c)\right]^{3/2}}{\left[\zeta(\eta_c)\right]^{1/2}} = -8\sigma(\eta_n)\left[1-\varphi(\eta_n)\right]^{3/2}\sqrt{S_0}\log\left(\frac{\varepsilon/D}{3.7[1-\varphi(\eta_n)]} + \frac{10.04}{R^*}\right) \tag{17}$$

with:

$$R^* = 4\sqrt{2}\,\frac{\sqrt{gD^3S_0}}{\nu}[1-\varphi(\eta_n)]^{3/2} \tag{18}$$

The filling rate η_n is defined as $\eta_n = y_n / D$ where y_n is the normal depth.

According to Eq.(8) for $\eta_c = \eta_n$, one may write $\varphi(\eta_n = 1) = 0$ which corresponds to the full state of the conduit. Consequently, Eq.(18) can be rewritten as:

$$R^* = R_f^*[1-\varphi(\eta_n)]^{3/2} \tag{19}$$

where:

$$R_f^* = 4\sqrt{2}\,\frac{\sqrt{gD^3 S_0}}{\nu} \tag{20}$$

The subscript "*f*" denotes the full state of the conduit. Finally, Eq.(17) is as:

$$\frac{[\sigma(\eta_c)]^{3/2}[1-\varphi(\eta_c)]^{3/2}}{[\zeta(\eta_c)]^{1/2}} = -8\sigma(\eta_n)[1-\varphi(\eta_n)]^{3/2}\sqrt{S_0}\log\left(\frac{\varepsilon/D}{3.7[1-\varphi(\eta_n)]} + \frac{10.04}{R_f^*[1-\varphi(\eta_n)]^{3/2}}\right) \tag{21}$$

Eq. (21) expresses the functional relationship $\psi(\eta_c, \eta_n, S_0, \varepsilon/D, R_f^*) = 0$. It is thus made up of five dimensionless parameters and it is doubly implicit with respect to η_c and η_n in particular. It encompasses all the parameters that influence the flow. The graphical representation of Eq. (21) relationship is not easy. Therefore, by choosing a conduit characterized by a diameter *D*, a slope S_0 and an absolute roughness ε of given values, it is then possible to graphically observe the variation of the critical depth η_c as a function of the normal depth η_n. This is what the next section proposes.

VARIATION OF THE CRITICAL DEPTH WITH RESPECT TO THE NORMAL DEPTH FOR A GIVEN CIRCULAR CONDUIT

In this section, the smooth circular pipe ($\varepsilon \to 0$) of diameter $D = 1m$ is considered as an example. The objective is to observe the variation of the critical depth as a function of the normal depth while varying the slope S_0. The calculations were carried out according to the implicit Eq.(21). The obtained results have allowed plotting the diagram of Fig. 1.

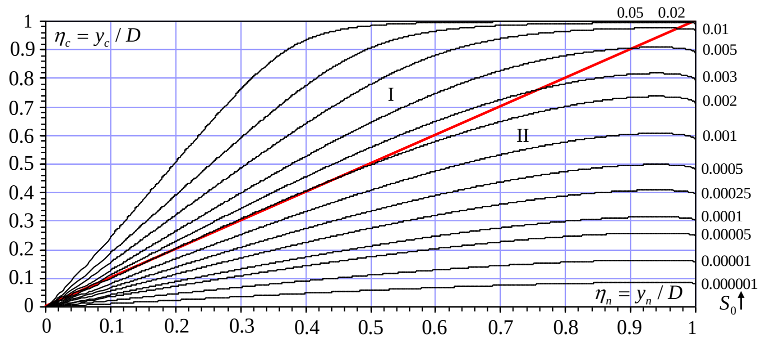

Figure 1: Variation of the critical depth versus the normal depth in a smooth circular conduit for various slopes. $D = 1m$, $\varepsilon \to 0$, $\nu = 10^{-6} m^2 / s$, **I:Supercitical flow zone, II : Subcritical flow zone, Red line (—) : First bisector corresponding to** $\eta_n = \eta_c$

Fig.1 indicates the supercritical zone of the flow located above the first bisector as well as the subcritical zone occupying the area below the first bisector. This latter represents all the points for which the flow is critical. Fig.1 shows that the curves pass through a maximum for η_c corresponding to η_n = 0.941 according to the calculations done. The in-depth examination of Eq.(21) and of Fig. 1 showed that the slope $S_0 = 0.00183813$ corresponds to the smallest slope which generates a single critical state of the flow as shown in Fig.2.

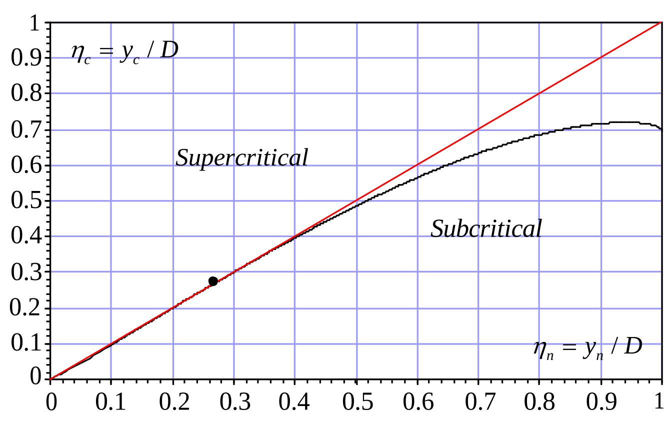

Figure 2: Variation of the critical depth versus the normal depth for the slope $S_0 = 0.00183813$, **(•)** $\eta_n = \eta_c = 0.26944 \approx 0.27$, $\eta_n(\eta_{c,\max}) \approx 0.941$, $\eta_{c,\max} = 0.72$

Slopes less than $S_0 = 0.00183813$ do not generate any critical state, while slopes greater than this value generate two critical states of the flow, corresponding to two different discharges. The first critical state is observed for shallower depths, while the second critical state is established at greater depths. The second critical state can be seen in Figure 1, while the first critical state is not clearly visible. This is represented on a larger scale in Figs. 3 for the slope $S_0 = 0.002$. For the same slope, Fig.4 shows the second critical state observed at a higher depth.

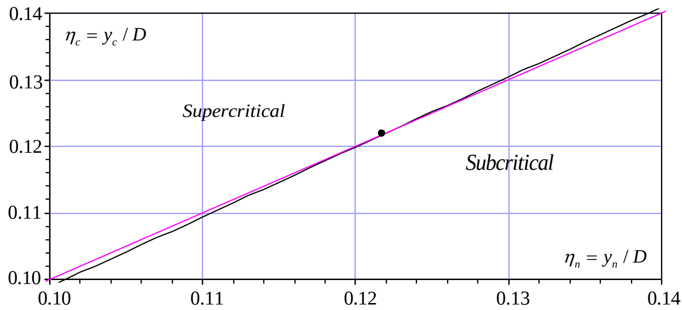

Figure 3: Variation of the critical depth versus the normal depth for the slope $S_0 = 0.002$ **, (•)** $\eta_n = \eta_c = 0.12176 \approx 0.122$

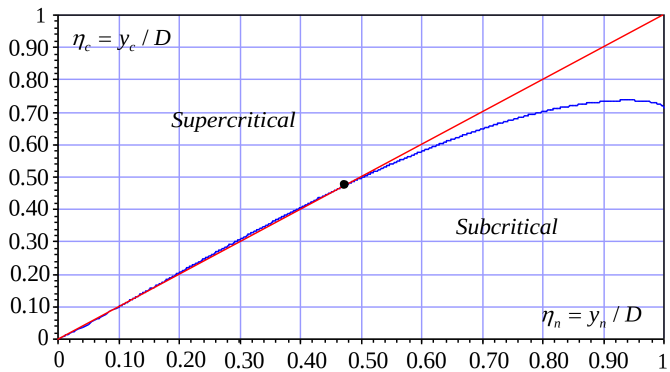

Figure 4: Variation of the critical depth versus the normal depth for the slope $S_0 = 0.002$ **, (•)**$\eta_n = \eta_c = 0.47484 \approx 0.475$**,** $\eta_n(\eta_{c,\max}) \approx 0.941$**,** $\eta_{c,\max} \approx 0.737$

The criticality of the two states of the flow previously described was verified by the specific energy plot as shown on Figs.(5) and (6). The specific energy is written for any shape of channel as:

$$E_s = y + \frac{V^2}{2g} \tag{22}$$

V is the mean velocity of the flow of depth *y*. With *V=Q/A*, Eq.(22) is rewritten as:

$$E_s = y + \frac{Q^2}{2gA^2} \tag{23}$$

Taking into account all the above mentioned relations for the circular conduit, Eq.(23) results in:

$$E_s^* = \eta + \frac{8Q^{*2}}{\left[\sigma(\eta)\right]^2\left[1-\varphi(\eta)\right]^2} \tag{24}$$

where $E_s^* = E_s / D$ is the relative specific energy, $\eta = y / D$ and Q^* is the relative discharge expressed as $Q^* = Q / \sqrt{gD^5}$. One can easily show that Q^* is, to within a constant, the ratio of two known dimensionless numbers namely, the Reynolds number *R* characterizing the full state of the flow in the conduit and the Galileo number *Ga*. The final result being:

$$Q^* = \frac{\pi}{4}\frac{R}{\sqrt{Ga}} \tag{25}$$

with $R = 4Q / (\pi D \nu)$ and $Ga = gD^3 / \nu^2$ representing the ratio of the gravity forces to the viscosity forces.

The relative discharge Q^* can be computed according to the Eq.(12) applied for the circular conduit, Hence:

$$Q^* = -\frac{\sqrt{2}}{2}\sigma(\eta)\left[1-\varphi(\eta)\right]^{3/2}\sqrt{S_0}\log\left(\frac{\varepsilon / D}{3.7\left[1-\varphi(\eta)\right]} + \frac{10.04}{R_f^*[1-\varphi(\eta_n)]^{3/2}}\right) \tag{26}$$

For the two states of the flow shown in Figs. (3) and (4), corresponding to $\eta = 0.12176 \approx 0.122$ and $\eta = 0.47484 \approx 0.475$, Eq.(26) gives :

$Q^* = 0.22288965$ and $Q^* = 0.01574231$ respectively.

Figs. (5) and (6) clearly show that the relative specific energy is minimal for the two above relative discharges and that the corresponding relative depths are indeed those of the Fig.(3) and (4).

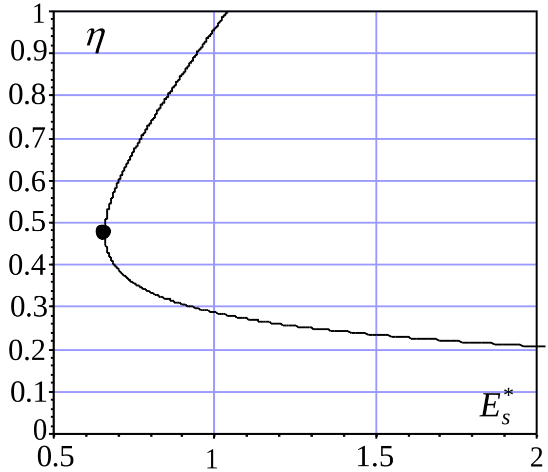

Figure 5: Relative specific energy curve E_s^* **according to Eq.(24) For** $Q^* = 0.22288965$ **. (•)** E_s^* **minimum =0.6587 ,** $\eta \approx 0.475$ **,**

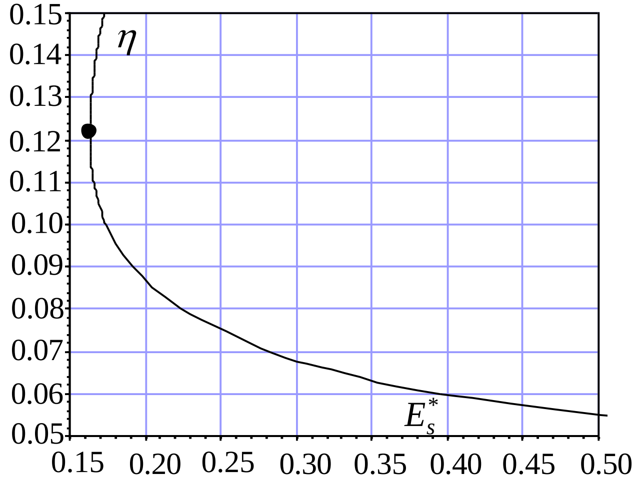

Figure 6: Relative specific energy curve E_s^* **according to Eq.(24). For** $Q^* = 0.01574231$ **. (•)** E_s^* **minimum = 0.1634,** $\eta \approx 0.122$ **,**

VARIATION OF THE CRITICAL DEPTH WITH RESPECT TO THE SLOPE

To express the general relationship which governs the critical flow in a channel or conduit of a given shape, one just have to replace the subscript "*n*" by the subscript "*c*" in the right-hand side of the Eq.(14), resulting in what follows after some simplifications:

$$\frac{\sqrt{P_c}}{\sqrt{T_c}} = -4\sqrt{2}\sqrt{S_0}\log\left(\frac{\varepsilon}{14.8R_{h,c}} + \frac{10.04}{R^*}\right) \tag{27}$$

According to Eq.(11), R^* is such that:

$$R^* = 32\sqrt{2}\,\frac{\sqrt{gR_{h,c}^3 S_0}}{\nu} \tag{28}$$

For the circular conduit, Eq.(27) leads to :

$$\frac{\left[\sigma(\eta_c)\right]^{1/2}}{\left[\zeta(\eta_c)\right]^{1/2}} = -8\sqrt{S_0}\log\left(\frac{\varepsilon / D}{3.7[1-\varphi(\eta_c)]} + \frac{10.04}{R_f^*[1-\varphi(\eta_c)]^{3/2}}\right) \tag{29}$$

This is the relationship which governs the critical flow in the circular conduit. Let's remember that R_f^* is given by Eq.(20). Eq.(29) implicitly describes the functional relationship $\xi(\eta_c, \varepsilon / D, S_0, R_f^*) = 0$. The objective sought in this section is to observe the variation of the relative critical depth η_c as a function of the slope S_0. To do so, let's use once again the example of the smooth circular conduit of diameter *D* = 1m flowing a liquid of $\nu = 10^{-6} m^2 / s$ as the kinematic viscosity. In this case, Eq. (29) has allowed establishing the graph of Fig. (7). The relative critical depth η_c is shown on the y-axis and the slope S_0 is on the x-axis.

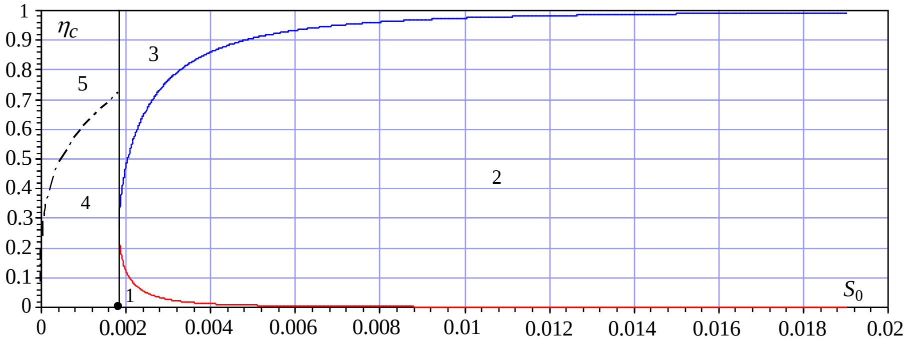

Figure 7: Variation of the relative critical depth η_c versus the slope S_0 for the smooth circular conduit of Diameter *D* =1m. (•) Smallest slope that generates a single critical state of the flow, i.e $S_0 = 0.00183813$

Fig. (7) gives a clear overview of the behaviour of the flow in the considered conduit. One can distinguish the red curve which corresponds to the first critical state occurring at shallow depths, and the blue curve which reflects the second critical state obtained at greater depths. Five zones of the flow are thus highlighted in figure 7 which can be interpreted as follows

Zone 1: Subcritical flow zone. The slopes are medium or strong causing two critical states of the flow. Theses slopes are greater than the smallest slope $S_0 = 0.00183813$ generating only one critical state of the flow. The first critical state occurs at the intersection of the vertical with the red curve. The second occurs at the intersection of the vertical with the blue curve. The flow is subcritical in zone 1, critical on the red curve, supercritical in zone 2, critical on the blue curve and becomes subcritical again in zone 3;

Zone 2 : Supercritical flow zone;

Zone 3: Subcritical flow zone;

Zone 4: Subcritical flow zone of weak slopes, less than $S_0 = 0.00183813$. These slopes do not generate any critical flow. The curve separating zones 4 and 5 corresponds to the maximum of the fictitious critical depth.

Zone 5: Supercritical flow area associated with weak slopes, where the uniform flow is improbable. Due to the low slopes and the supercritical nature of the flow regime, zone 5 can be the site of a hydraulic jump.

CONCLUSIONS

The study involved an in-depth examination of the critical flow in a circular conduit. The main purpose was to know how the critical depth varies as a function of all the parameters influencing the flow, such as the slope of the conduit, the roughness of the internal walls of the conduit, and the kinematic viscosity of the flowing liquid. In order to achieve this goal, the study used a combination of two rational relationships, namely, the dimensionally coherent uniform flow relationship and the equation derived from the criticality criterion.

This approach has led to the establishment of a conclusive and decisive relationship, albeit implicit, which can be translated by the following functional relationship: $\psi(\eta_c, \eta_n, S_0, \varepsilon / D, R_f^*) = 0$ [Eq.(21)]. The main particularity of this relation is that it allows deducing the relationship between the critical depth and all the other parameters which govern the flow, by simply writing $\eta_n = \eta_c$, resulting in the following functional relationship $\psi(\eta_c, S_0, \varepsilon / D, R_f^*) = 0$ [Eq.(29)]. Thanks to these relations, graphs revealing the behaviour of the flow in the smooth conduit of diameter D = 1m, taken as an example, have been elicited [Figs.(1) and (7)]. The graphs clearly show how the flow changes regimes, moving from subcritical to critical, from critical to supercritical and finally back to subcritical. The graphs also indicate for which slope of the conduit this change of regime takes place. It can also be observed that certain slopes of the conduit do not generate any critical state of the flow which in addition remains in the subcritical zone whatever the discharge. It can also worth noting that other slopes are the location of two critical states of the flow corresponding to two different discharges. One occurs at shallow depths, while the other appears at greater depths. Although unwieldy, the calculations nevertheless led to the conclusion that $S_0 = 0.00183813$ is the smallest slope of the conduit which generates a single critical state of the flow. All slopes less than this value do not cause any critical state of the flow, while slopes higher than this value generate two critical states of the flow that can be observed for each slope.

It is preferable and recommended that this study be completed by further investigations on the behaviour of the critical flow in a circular conduit by examining the influence of the diameter and of the absolute roughness. This is what the authors will try to clarify during the second part of the study.

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