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NEW THEORETICAL CONSIDERATIONS ON THE CRITICAL FLOW IN A CIRCULAR CONDUIT (PART 2)

NOUVELLES CONSIDERATIONS THEORIQUES SUR L'ECOULEMENT CRITIQUE DANS UNE CONDUITE CIRCULAIRE (PARTIE 2)

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ABSTRACT

This study constitutes the second part of a first study devoted to the critical flow in a smooth circular conduit. The first study published on this subject showed that two critical states occur in a circular conduit for a given diameter D and slope S_0. The first one is observed at shallow depths while the second one settles down at greater depths. The study considered the example of the smooth circular conduit of diameter D = 1m and concluded that when the slope S_0 is such that $S_0 > 0.00183813$, two critical states occur for two different discharges. Slopes that are less than this value do not generate any critical state of the flow. It was found that the slope $S_0 = 0.00183813$ corresponds to the smallest slope that causes a single critical state of the flow. The present study is interested in the influence of the diameter D on the variation of the critical depth as well as on the fate of the two critical states of the flow. The partially filled smooth circular conduit is still considered herein. This shows that there is a diameter D_1, the smallest, which generates only one critical state of the flow for a given slope of the conduit. All the conduit diameters D greater than D_1 generate two critical flow states, while diameters smaller than D_1 are the location of no critical flow. In addition, it has been demonstrated that the more the diameter of the conduit increases, the more the first critical state occurs at shallower depths. Conversely, the second critical state of the flow is observed at greater depths.

Using the appropriate theoretical relationship, it was possible to calculate the pair of values (D_1 ; S_0) which generates a single critical state of the flow. These values have been plotted graphically, which allowing to know whether the conduit is the seat of one or two critical states, or even of no critical state, from the known pair of values (D ; S_0).

Keywords: Circular conduit, critical depth, normal depth, slope, discharge, diameter.

RESUME

Cette étude constitue la deuxième partie d'une première étude consacrée à l'écoulement critique dans une conduite circulaire. La première étude publiée sur ce sujet a montré que deux états critiques se produisent dans une conduite circulaire de diamètre D et de pente S_0 donnés. Le premier s'observe à faible profondeur tandis que le second s'installe à plus grande profondeur. L'étude a pris l'exemple de la conduite circulaire lisse de diamètre D = 1m et a conclu que lorsque la pente est telle que $S_0 > 0.00183813$, deux états critiques se produisent pour deux débits différents. Les pentes inférieures à cette valeur ne génèrent aucun état critique de l'écoulement. Il a été constaté que la pente $S_0 = 0.00183813$ correspond à la plus petite pente qui provoque un seul état critique de l'écoulement. La présente étude s'intéresse à l'influence du diamètre D sur la variation de la profondeur critique ainsi que sur le devenir des deux états critiques de l'écoulement. La conduite circulaire lisse partiellement remplie est toujours considérée dans cette étude. Celle-ci montre qu'il existe un diamètre D_1, le plus petit, qui ne génère qu'un seul état critique de l'écoulement pour une pente donnée de la conduite. Tous les diamètres D de la conduite supérieurs à D_1 génèrent deux états d'écoulement critiques, tandis que les diamètres inférieurs à D_1 sont le lieu géométrique d'aucun écoulement critique. De plus, il a été démontré que plus le diamètre de la conduite augmente, plus le premier état critique se produit à des profondeurs moindres. A l'inverse, le deuxième état critique de l'écoulement est observé à des profondeurs plus importantes. En utilisant la relation théorique appropriée, il a été possible de calculer le couple de valeurs (D_1 ; S_0) qui génère un seul état critique de l'écoulement. Ces valeurs ont été reportées graphiquement, ce qui permet de savoir si la conduite est le siège de un ou de deux états critiques, voire même d'aucun état critique, à partir du couple de valeurs connu (D ; S_0).

Mots clés : Conduite circulaire, profondeur critique, profondeur normale, pente, débit, diamètre.

INTRODUCTION

The critical depth denoted y_c is the depth at which the specific energy is minimum for a given discharge or the depth at which the discharge is maximum for a given specific energy (Chow, 1959). The corresponding flow is called critical flow that be occurred in a channel by raising the channel bottom, and/or by decreasing the channel width.

Another parameter of great importance in free surface flow is the celerity c of a small wave generated by a disturbance or an obstruction, especially to define critical flow (French, 1985; Subramanya, 2009). The celerity c is defined as the velocity of the wave relative to the velocity V of the mass medium in which the wave is travelling. When the celerity c is equal to the flow velocity V, the flow is thus critical. The flow is said subcritical when $V < c$ and it is said supercritical when $V > c$. The critical flow is characterized by a Froude Number F equal to unity. The Froude number F is less than unity when the flow is subcritical or tranquil, meaning that the wave celerity exceeds the flow velocity. Waves can flow upstream and water can pond behind an obstruction. The Froude number is greater than unity when the flow is supercritical, meaning that the wave velocity is lower than the flow velocity. Waves cannot be generated upstream. Any disturbance occurring downstream of the flow has no effect on the upstream flow. The nature of the flow can also be known after comparing the critical and normal depths. If the actual depth is greater than critical depth, then the flow is considered as subcritical. The flow is said to be supercritical when the actual depth is less than the critical depth. The critical depth is also used in the classification of water surface profiles, along with the slope of the channel S_0 and the normal depth y_n (Chow, 1959).

When the flow is critical, the discharge and the flow depth have a unique relationship (Chaudhry, 2008), according to the criticality criterion. Using this property, several flow-measuring devices have been developed. These are called critical-flow meters (Henderson, 1966: Chaudhry, 2008). As a general rule, the equation which governs the critical flow in channels and conduits partially filled is implicit with respect to the critical depth which is the unknown parameter of the problem. The equation can be solved by a trial-and-error procedure or by using numerical methods. Several numerical methods are available for solving this equation (McCracken and Dorn, 1964), such as bisection method, Newton method also called the Newton-Raphson method, secant method, and the method of successive approximations. The critical depth may also be determined by the use of the design curves (Chow, 1959).

Regardless of the shape of the prismatic channel profile such as rectangular, trapezoidal, or circular, there is only one discharge that generates a unique critical state of the flow (Chow, 1959). On the other hand, there may be more than one critical depth for a specified discharge in a compound channel (Chaudhry and Bhallamudi, 1988). In their recent study, Achour and Amara (2020) showed the existence of two critical states in the partially filled circular conduit, corresponding to two different flow rates. One of the two critical states is observed at shallow depths, while the second occurs at greater depths.

The critical depth, even if it is a particular depth, is a uniform depth that should depend on the characteristics of the flow such as the slope S_0 of the channel, the absolute roughness ε characterizing the state of the inner wall of the channel or the conduit, and the kinematic viscosity ν of the flowing liquid. For the partially filled circular conduit, Achour and Amara (2020) were able to establish the implicit relationship $\psi(\eta_c, S_0, \varepsilon / D, R_f^*) = 0$, where $\eta_C = y_C / D$ is the relative critical depth, ε / D is the relative roughness, and R_f^* is the shear Reynolds number at the full states of the conduit denoted "*f*". It has been shown that R_f^* depends on the diameter D of the conduit, the slope S_0 of the conduit, the kinematic viscosity ν of the flowing liquid, and the acceleration due to gravity g. Thanks to this relation, a graph was drawn showing the variation of the critical depth as a function of the slope of the conduit. This graph corresponds to a partially filled smooth circular conduit of diameter D = 1m. This graph shows five zones in which the flow occurs and how it evolves. The first zone is that of the subcritical flow. This evolves towards the second zone which corresponds to the zone of the supercritical flow. This passage takes place via a first critical state of the flow. The flow then passes into the third zone corresponding to a subcritical flow. This passage takes place through the appearance of a second critical state of the flow. A fourth zone can be observed on the graph and corresponds to a subcritical flow zone generated by weak slopes S_0 less than 0.00183813. In this zone, the real critical state of the flow does not exist and the critical depths are only fictitious. All slopes S_0 greater than 0.00183813 generate two critical states of the flow, while the slope S_0 = 0.00183813 generates only one critical state. The fifth and last zone of the graph corresponds to a zone where the slopes are weak, i.e. less than 0.00183813, and where the flow is supercritical. Therefore, uniform flow is unlikely in this area which can be the site of a hydraulic jump or a backwater curve.

In the present study, the influence of the conduit diameter on the variation of the critical depth as well as on the fate of the two critical states of the flow is highlighted. By choosing a given slope S_0, the variation of the critical relative depth as a function of the normal relative depth is graphically represented for various values of the diameter D. Interesting conclusions are drawn concerning the evolution of the two critical states of the flow as a function of the variation of the diameter. In particular, it was possible to determine the pair of parameters D_1 and S_0 which generates a single critical state of the flow in the smooth circular conduit. All the conclusions drawn as well as all the plotted graphs are valid for a kinematic viscosity $\nu = 10^{-6}$ m²/s of the flowing liquid.

AVAILABLE FUNDAMENTAL RELATIONSHIPS

In their recent study, Achour and Amara (2020) established two fundamental relationships governing the critical flow in a partially filled circular pipe. These relationships are deduced from the general equations valid for any shape of channel or conduit. The first general relationship combines the characteristics of both critical and normal flows. It is expressed as follows:

$$\frac{A_c^{3/2}}{\sqrt{T_c}} = -4\sqrt{2}\,\frac{A_n^{3/2}}{P_n^{1/2}}\sqrt{S_0}\,\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R^*}\right) \tag{1}$$

Where A_c is the critical water area, T_c is the critical top width at the water surface, A_n is the normal water area, P_n is the normal wetted perimeter, S_0 is the slope of the channel, $R_{h,n}$ is the normal hydraulic radius, R^* is a dimensionless number which gives the measure of the ratio of the friction forces to the viscous forces. It is then closely related to the shear Reynolds number, and ε is the absolute roughness. R^* is expressed as:

$$R^* = 32\sqrt{2}\,\frac{\sqrt{gR_{h,n}^3 S_0}}{\nu} \tag{2}$$

Where g is the acceleration due to gravity, and ν is the kinematic viscosity of the flowing liquid.

The second relation expresses the critical flow in a channel or conduit of any shape. It is deduced from relation (1) in which the subscript "*n*" in the hand-right side of the equation is replaced by the subscript "*c*" which denotes the critical condition of the flow. After some simplifications, the final result is:

$$\frac{\sqrt{P_c}}{\sqrt{T_c}} = -4\sqrt{2}\sqrt{S_0}\,\log\left(\frac{\varepsilon}{14.8R_{h,c}} + \frac{10.04}{R^*}\right) \tag{3}$$

In this equation, R^* is such that:

$$R^* = 32\sqrt{2}\,\frac{\sqrt{gR_{h,c}^3 S_0}}{\nu} \tag{4}$$

Relation (3) represents the general relationship which governs the critical flow in a channel of any shape. All the parameters which influence the flow are taken into account. The critical parameters P_c, T_c, and $R_{h,c}$ depend on the geometry of the channel as well as on the relative critical depth.

Applied to the case of the partially filled circular conduit of diameter D, relations (1) and (3) are written respectively as (Achour and Amara, 2020):

$$\frac{[\sigma(\eta_c)]^{3/2}[1-\varphi(\eta_c)]^{3/2}}{[\zeta(\eta_c)]^{1/2}} = -8\sigma(\eta_n)[1-\varphi(\eta_n)]^{3/2}\sqrt{S_0}\log\left(\frac{\varepsilon/D}{3.7[1-\varphi(\eta_n)]}+\frac{10.04}{R_f^*[1-\varphi(\eta_n)]^{3/2}}\right) \tag{5}$$

$$\frac{[\sigma(\eta_c)]^{1/2}}{[\zeta(\eta_c)]^{1/2}} = -8\sqrt{S_0}\log\left(\frac{\varepsilon/D}{3.7[1-\varphi(\eta_c)]}+\frac{10.04}{R_f^*[1-\varphi(\eta_c)]^{3/2}}\right) \tag{6}$$

Where:

$$\sigma(\eta_c)=\cos^{-1}(1-2\eta_c) \tag{7}$$

$$\varphi(\eta_c)=\frac{2(1-2\eta_c)\sqrt{\eta_c(1-\eta_c)}}{\cos^{-1}(1-2\eta_c)} \tag{8}$$

$$\zeta_c=\sqrt{\eta_c(1-\eta_c)} \tag{9}$$

η_C is the relative critical depth which represents the ratio of the critical depth y_C to the diameter D of the conduit, such that:

$$\eta_c=\frac{y_c}{D} \tag{10}$$

The dimensionless number R_f^* is the shear Reynolds number at the full state of the conduit. It is expressed as:

$$R_f^*=4\sqrt{2}\,\frac{\sqrt{gD^3S_0}}{\nu} \tag{11}$$

INFLUENCE OF THE CONDUIT DIAMETER

To observe the influence of the diameter D of the conduit on the variation of the relative critical depth as well as on the evolution of the two critical states of the flow, it is necessary to use Eq.(5). A value of the slope S_0 is chosen as well as the kinematic viscosity ν of the flowing liquid. The diameter D of the conduit is varied thereafter. Therefore, the dimensionless number R_f^* is known, according to Eq.(11). Figure 1, resulting from this approach, illustrates the variation of the relative critical depth as a function of the relative normal depth for the chosen slope $S_0 = 0.002$ and the kinematic viscosity $\nu = 10^{-6} m^2 / s$, for various diameters D of the conduit and ε→0.

Figure 1 shows that some curves do not intersect the first bisector, while others have two points of intersection. One is at shallow depths, while the other manifests itself at greater depths. This allows concluding that for the chosen slope S_0 = 0.002, there are conduit diameters which are the seat of two critical states of the flow, while other diameters do not generate any critical state of the flow. Figure 2 clearly shows that the curve does not present any point of intersection with the first bisector, meaning that the pair of parameters S_0 = 0.002, D = 0.5m does not generate any critical state of the flow. The diameter D = 0.5m will be the seat of a critical flow for a different slope S_0.

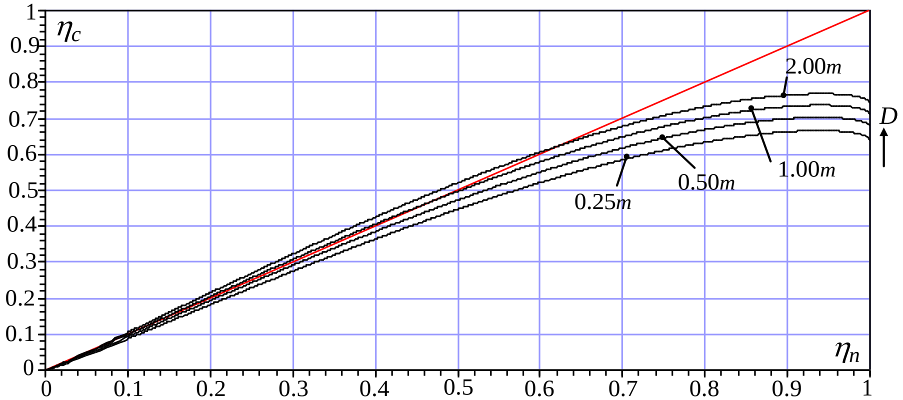

Figure 1: Variation of the relative critical depth η_c with the relative normal depth η_n in a partially filled smooth circular conduit of various diameters *D*, for the slope $S_0 = 0.002$, $\eta_{n,\max}$ = 0.939. Red curve: First bisector corresponding to $\eta_n = \eta_c$

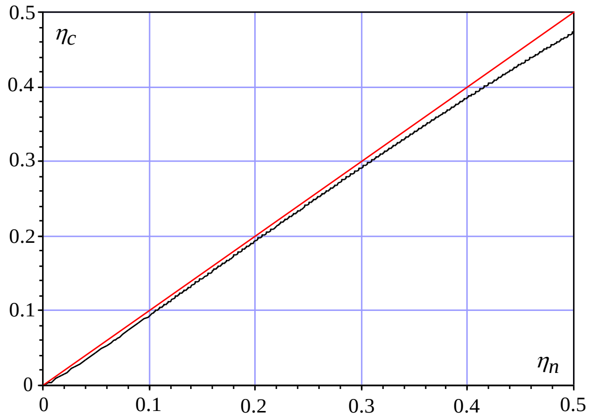

Figure 2: Variation of the relative critical depth η_c with the relative normal depth η_n in a partially filled smooth circular conduit of diameter *D* = 0.5m, for the slope $S_0 = 0.002$. Red curve: First bisector corresponding to $\eta_n = \eta_c$

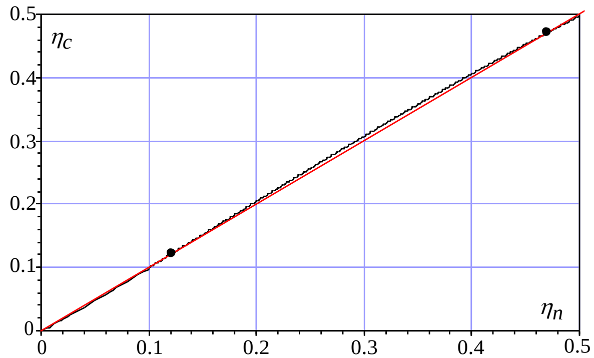

Figure 3: Variation of the relative critical depth η_c with the relative normal depth η_n in a partially filled smooth circular conduit of diameter *D* = 1m, for the slope $S_0 = 0.002$. Red curve: First bisector corresponding to $\eta_n = \eta_c$, (•) Critical flow

Figure 3 shows that for the pair of parameters S_0 = 0.002 and D = 1m, there are two critical states of the flow. The first occurs at the filling rate $\eta = 12.2\%$, while the second occurs at the filling rate $\eta = 47\%$.

It is to be concluded, according to figure 1, that for a partially filled smooth circular conduit with a given slope S_0, there is a diameter D_1, the smallest, for which there is only one critical state of the flow. This corresponds to the curve tangent to the first bisector of figure 1. All diameters D greater than D_1 will be the site of two critical flow states, while diameters smaller than D_1 will not generate any critical flow. Using Eq.(6), it was possible to determine the pair of values (D_1 ; S_0).These are reported in table 1 and have been represented graphically in Fig.4.

Table 1: Values of (D_1 ; $S_{0;1}$) and corresponding relative critical depth, according to Eq.(7).

D_1(m)	S_0	η_C
0.25	0.002718528	0.301261184
0.35	0.002457198	0.292927912
0.50	0.002217658	0.284548538
0.60	0.002107822	0.280429502
0.80	0.001949519	0.274142118
1.00	0.00183813	0.269449284
1.20	0.001753889	0.265713456
1.50	0.001658065	0.261278701
1.80	0.001585149	0.257819483
2.00	0.00154503	0.255749656
2.50	0.001464827	0.2516108441
2.75	0.001432439	0.249882737
3.00	0.001403708	0.248374690

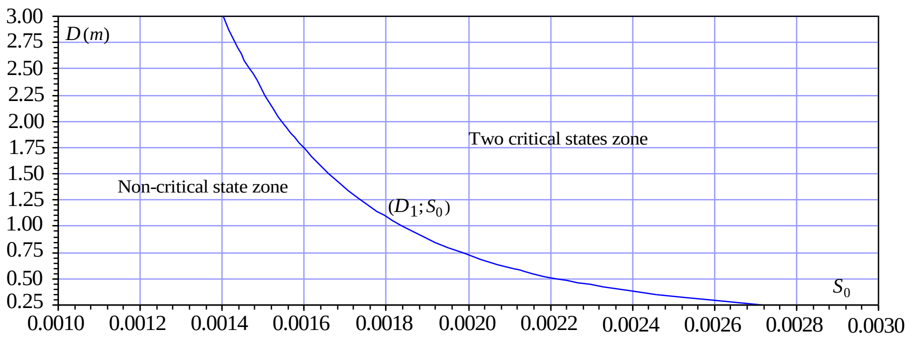

Figure 4: Variation of the smallest diameter D_1 of a smooth circular conduit generating one critical state of the flow with respect to the slope S_0.

The blue curve in Fig.4 delimits two zones. The first one is a zone where the pair of values (D; S_0) generates two critical states of the flow. The second one is a zone where the pair of values (D; S_0) generates no critical state of the flow.

The blue curve in Fig.4 can be represented by a simple equation which results in a maximum relative error of 1.5%. This is expressed as:

$$S_0 = \left(\frac{4.5 \times 10^{-11}}{D} \right)^{0.264} \tag{12}$$

Diameter D must be expressed in meters. Eq.(12) is valid in the range $0.25m \le D \le 3m$.

On the other hand, the larger the diameter, the more the first critical state occurs at shallower depths. At the opposite, the second critical flow occurs at greater depths. This case is illustrated in Fig. 5 for the smooth circular conduit of diameter D = 2m. The first critical state occurs at the 5% filling rate, while the second critical state occurs at the 62.6% filling rate.

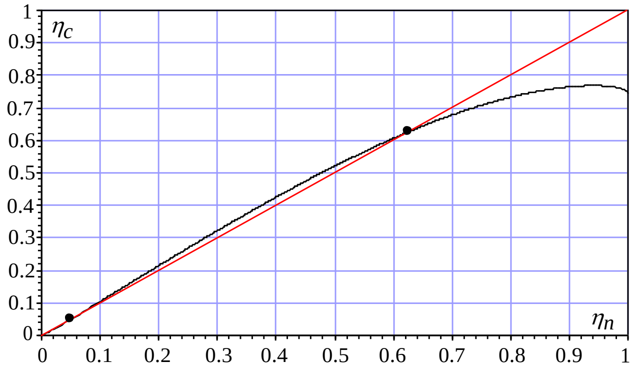

Figure 5: Variation of the relative critical depth η_c with the relative normal depth η_n in a partially filled smooth circular conduit of diameter D = 2m, for the slope S_0 = 0.002. Red curve: First bisector corresponding to $\eta_n = \eta_c$, (•) Critical flow, $\eta_{n,max}$=0.939

CONCLUSIONS

The main objective of the study was to observe the influence of the diameter D of a smooth circular pipe on the behaviour of the critical states of the flow, the existence of which was demonstrated during the first part of the study. It has been shown that certain diameters, associated with conduit slopes S_0, could generate two critical states of the flow or a single critical state, or even no critical state. From the general equation which governs the critical flow in the circular conduit [Eq.(6)], the pairs of values (D; S_0) which generate a single critical state of the flow have been calculated. This allowed plotting a graph [Fig.(4)] from which the user can predict whether or not the conduit would be the seat of a critical state of the flow, provided D and S_0 are given. An approximate relationship has been proposed for the calculation of the slope S_0 which generates a single critical state of the flow, when the diameter D of the conduit is given. This relation causes a maximum relative error of 1.5% and can be used to easily know the order of magnitude of S_0.

The study also showed that the more the diameter of the conduit increases, the more the first critical state of the flow occurs at low filling rates. In contrast, the second critical state moves away from the first and manifests itself at the greatest filling rates.

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