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CRITICAL FLOW IN A RECTANGULAR-SHAPED CHANNEL

ECOULEMENT CRITIQUE DANS UN CANAL RECTANGULAIRE

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ABSTRACT

Using the rational relationship, established by the authors in a previous study that links the properties of critical flow and normal flows, the present investigation is related to the critical state of the flow in a rectangular-shaped channel. The main objective is to observe the behaviour of this type of flow under the variation of some flow parameters such as the slope S_0 and the width b of the channel. The relation which governs the critical flow in this form of channel is an implicit relationship for the relative critical depth $\eta_c = y_c/b$ where y_c is the critical flow depth. It consists of four dimensionless terms which are the relative critical depth defined above, the relative roughness ε/b where ε is the absolute roughness, the channel slope S_0, and the shear Reynolds number R^*_{nc} characterizing the flow in a very wide rectangular-shaped channel. To simplify the calculation, the example of the smooth rectangular-shaped channel ($\varepsilon \rightarrow 0$) of width b = 1m is first considered to observe the influence of the slope S_0. It was observed that, for the chosen channel, the slope S_0 = 0.0020949472 generates a single critical state of the flow. All the slopes S_0 greater than 0.0020949472 generate two critical flow states corresponding to two different flow rates. Finally, slopes less than 0.0020949472 do not generate any critical state of the flow in the channel. On the other hand, it has also been demonstrated that for a given slope S_0, there is a particular width b_1 of the channel which generates a single critical state of the flow. Widths b less than b_1 do not generate any critical state in the channel, while widths b greater than b_1 engender two critical states of the flow.

Keywords: Critical flow, channel slope, channel width, rectangular channel, relative critical depth.

RESUME

En utilisant la relation rationnelle, établie par les auteurs dans une étude précédente qui lie les propriétés des écoulements critique et normal, la présente enquête est liée à l'état critique de l'écoulement dans un canal de forme rectangulaire. L'objectif principal est d'observer le comportement de ce type d'écoulement sous la variation de certains paramètres d'écoulement tels que la pente S_0 et la largeur b du canal. La relation qui régit l'écoulement critique dans cette forme de canal est une relation implicite vis-à-vis de la profondeur critique relative $\eta_c = y_c/b$ où yc est la profondeur critique d'écoulement. La relation obtenue se compose de quatre termes sans dimension qui sont la profondeur critique relative définie ci-dessus, la rugosité relative ε/b où ε est la rugosité absolue, la pente du canal S_0 et le nombre de Reynolds de cisaillement R^*_{nc} caractérisant l'écoulement dans un très large canal de forme rectangulaire. Pour simplifier le calcul, l'exemple du canal lisse rectangulaire ($\varepsilon \to 0$) de largeur b = 1m est d'abord considéré pour observer l'influence de la pente S_0. Il a été observé que, pour le canal choisi, la pente S_0 = 0,0020949472 génère un seul état critique de l'écoulement. Toutes les pentes S_0 supérieures à 0,0020949472 génèrent deux états d'écoulement critiques correspondant à deux débits différents. Enfin, les pentes inférieures à 0,0020949472 ne génèrent aucun état critique de l'écoulement dans le canal. D'autre part, il a également été démontré que pour une pente S_0 donnée, il existe une largeur b_1 particulière du canal qui génère un seul état critique de l'écoulement. Les largeurs b inférieures à b_1 ne génèrent aucun état critique dans le canal, tandis que les largeurs b supérieures à b_1 engendrent deux états critiques de l'écoulement.

Mots clés : Ecoulement critique, pente de canal, largeur de canal, canal rectangulaire, profondeur critique relative.

INTRODUCTION

The rectangular-shaped channel is one of the most widely used structures in practice due to its simple geometric shape and easy implementation. It is an open-channel with a rectangular cross-section. By keeping constant the water area A of a rectangular-shaped channel of width b, it is possible to minimize the wetted perimeter P. This is done by setting the derivative of the wetted perimeter with respect to the flow depth y equal to zero, i.e. $dP/dy = 0$ (Chow, 1959). The calculations show that the final result is $y = b / 2$. This means that the maximum flow efficiency is achieved when the flow depth y is equal to one half the channel width b. This corresponds to an optimal section in which a semi-circle is perfectly inscribed.

In practice, the flow in an open-channel is non-uniform, i.e. the parameters of the flow such as velocity and depth vary along the flow path (Chaudhry, 2008). However, the open- channels are designed under the assumption of a uniform flow for which flow parameters, such as the depth called normal depth, are constant in space and time. In general, there are three main problems encountered in the study of open-channels flow namely, discharge calculation, flow depth computation, and the sizing of the channel. To do this, the so-called uniform flow equations are used such as Manning's relation (Manning, 1891) which remains the most popular today. Applying uniform flow relationships to solve the flow depth problem often leads to an implicit equation. This is also the case with Manning's formula. For this reason, research workers proposed graphical solutions (Chow, 1959) or approximate analytical solutions (Vatankha and Easa, 2011; Shang et al., 2019). The proposed approximate formulas are very precise but unfortunately, they cannot be applied in their current form. The main reason is that these formulas contain Manning's n resistance coefficient, knowing that the latter itself depends on the flow depth sought. Indeed, it has been demonstrated in the distant and recent past that Manning's coefficient n strongly depends on the flow depth (Camp, 1946; Achour and Bedjaoui, 2006; Achour, 2014).

In open-channels, also occurs a particular flow which is the critical flow of depth denoted y_c. It is a particular depth because it meets the criticality criterion resulting from the equality $F = 1$, where F is the Froude number (Subramanya, 2009; Chaudhry, 2008; Moglen, 2015). With the exception of the criticality criterion, the literature is not provided with data and research on critical flow. Although it is particular, the critical depth is a uniform depth that should depend on the flow characteristics as well as the geometry of the channel. These characteristics are, in particular, the slope S_0 of the channel, the absolute roughness ε which characterizes the state of the internal wall of the channel, and the kinematic viscosity ν of the flowing liquid.

In a recent study, Achour and Amara (2020) investigated the change in critical depth in a partially filled smooth conduit taken as an example, based on both the dimensionally consistent uniform flow relationship (Achour and Bedjaoui, 2006) and the criticality criterion. It emerges from this study that, for a given conduit, there are two critical states of the flow for two different flow rates, depending on the slope S_0 of the conduit. One occurs at shallow depths, the other occurs at greater depths. Besides, some slopes do not generate a critical state of the flow, while others generate a single critical state.

The main objective of the present study is to know whether in the rectangular-shaped channel the same observations can be made.

GEOMETRIC PROPERTIES

For a rectangular-shaped channel, the geometric properties are as follows:

$$A = by \tag{1}$$

Where A is the water area, b is the width of the channel, and y is the flow depth. Eq.(1) can be rewritten as:

$$A = b^2\eta \tag{2}$$

$\eta = y/b$ is the relative depth or the aspect ratio.

$$T = b \tag{3}$$

Where T is the top width at the surface water.

$$P = b + 2y \tag{4}$$

Or:

$$P = b(1+2\eta) \tag{5}$$

Where P is the wetted perimeter.

$$R_h = A/P \tag{6}$$

Where R_h is the hydraulic radius.

Taking into account Eqs.(2) and (5), Eq. (6) becomes:

$$R_h = \frac{b^2\eta}{b(1+2\eta)} = \frac{b\eta}{(1+2\eta)}$$

Or:

$$R_h = \frac{b}{(\eta^{-1}+2)} \tag{7}$$

Eq.(7) can be written as:

$$R_h = \frac{b}{\varphi(\eta)} \tag{8}$$

Where:

$$\varphi(\eta) = (\eta^{-1}+2) \tag{9}$$

The shape of the channel section can be classified between very wide and very narrow through a shape factor introduced by Vedernikov in the years 1945, and reported by Chow (1959).The shape factor of a channel section is as:

$$\gamma = 1 - R_h \frac{dP}{dA} \tag{10}$$

Where *dP/dA* is the derivative of the wetted perimeter *P* with respect to the water area *A*. From Eqs.(1) and (4), one may write $dA = bdy$ and $dP = 2dy$, resulting in what follows:

$$\frac{dP}{dA} = \frac{2}{b} \tag{11}$$

Taking into account Eqs.(7) and (11), Eq.(10) becomes:

$$\gamma = 1 - \frac{2}{(\eta^{-1} + 2)} \tag{12}$$

For the case of a very narrow rectangular-shaped channel, one may write $b \to 0$ or $\eta^{-1} \to 0$ corresponding to $\varphi(\eta) = 2$. Thus, Eqs.(8) and (12) are reduced respectively to:

$$R_h = \frac{b}{2} \tag{13}$$

$$\gamma = 0 \tag{14}$$

Thus, for the very narrow rectangular-shaped channel, the hydraulic radius is equal to one half the width of the channel corresponding to a shape factor equal to zero. For the case of a very wide rectangular channel, corresponding to $b \to \infty$ or $\eta^{-1} \to \infty$, Eq.(12) gives:

$$\gamma = 1 \tag{15}$$

AVAILABLE FUNDAMENTAL RELATIONSHIPS

In their recent study, Achour and Amara (2020) highlighted the fundamental relationship that links the characteristics of critical and normal flows in a channel or conduit of any shape. This is expressed as:

$$\frac{A_c^{3/2}}{\sqrt{T_c}} = -4\sqrt{2}\,\frac{A_n^{3/2}}{P_n^{1/2}}\sqrt{S_0}\,\log\left(\frac{\varepsilon}{14.8R_{h,n}} + \frac{10.04}{R^*}\right) \tag{16}$$

$$R^* = 32\sqrt{2}\,\frac{\sqrt{gR_{h,n}^3 S_0}}{\nu} \tag{17}$$

Where *A* is the water area, *P* is the wetted perimeter, *T* is the top width at the water surface, R_h is the hydraulic radius, S_0 is the slope of the channel, ν is the kinematic viscosity of the flowing liquid, ε is the absolute roughness, and R^* is the shear Reynolds number.

By replacing the subscript "*n*" by the subscript "*c*" in Eq.(16), Achour et Amara (2020) show that:

$$\frac{\sqrt{P_c}}{\sqrt{T_c}} = -4\sqrt{2}\sqrt{S_0}\log\left(\frac{\varepsilon}{14.8R_{h,c}} + \frac{10.04}{R^*}\right) \tag{18}$$

This is the general relationship which governs the critical flow in a channel or conduit of a given shape.

SIMULTANEOUS VARIATION OF THE CRITICAL AND NORMAL DEPTHS FOR A GIVEN RECTANGULAR-SHAPED CANAL

Applying Eq.(16) to the rectangular-shaped channel, results in:

$$\frac{b^3\eta_c^{3/2}}{\sqrt{b}} = -4\sqrt{2}\,\frac{b^3\eta_n^{3/2}}{b^{1/2}(1+2\eta_n)^{1/2}}\sqrt{S_0}\log\left[\frac{\varepsilon/b}{14.8}(\eta_n^{-1}+2) + \frac{10.04}{R^*}\right]$$

Or, after some simplifications:

$$\eta_c^{3/2} = -4\sqrt{2}\,\frac{\eta_n^{3/2}}{(1+2\eta_n)^{1/2}}\sqrt{S_0}\log\left[\frac{\varepsilon/b}{14.8}(\eta_n^{-1}+2) + \frac{10.04}{R^*}\right] \tag{19}$$

According to Eq.(17), the shear Reynolds number R^* is expressed as :

$$R^* = 32\sqrt{2}\,\frac{\sqrt{gb^3S_0}}{\nu}(\eta_n^{-1}+2)^{-3/2} \tag{20}$$

On the other hand, for the very narrow rectangular-shaped channel, one may write $\varphi(0) = 2$, according to Eq.(9), resulting in:

$$[\varphi(0)]^{-3/2} = 2^{-3/2} = 1/(2\sqrt{2})$$

Consequently, according to Eq.(20), the shear Reynolds number for a very narrow rectangular-shaped channel can be written as:

$$R^*_{nc} = 16\,\frac{\sqrt{gb^3S_0}}{\nu} \tag{21}$$

The subscript « *nc* » denotes the very narrow rectangular-shaped channel. Thus, Eq.(20) becomes :

$$R^{*}=2\sqrt{2}R_{nc}^{*}(\eta_{n}^{-1}+2)^{-3/2} \quad (22)$$

Finally, Eq.(19) is definitely written in the following form:

$$\eta_{c}^{3/2}=-4\sqrt{2}\,\frac{\eta_{n}^{3/2}}{(1+2\eta_{n})^{1/2}}\sqrt{S_{0}}\log\left[\frac{\varepsilon/b}{14.8}(\eta_{n}^{-1}+2)+\frac{3.549676(\eta_{n}^{-1}+2)^{3/2}}{R_{nc}^{*}}\right] \quad (23)$$

This is the general relationship that links the characteristics of critical and normal flows in a rectangular-shaped channel, and contains all the parameters that influence the flow.

Eq. (23) is shown in fig. 1 for a smooth rectangular-shaped channel ($\varepsilon \to 0$) with a slope S_0 = 0.002 and a width b = 1m.

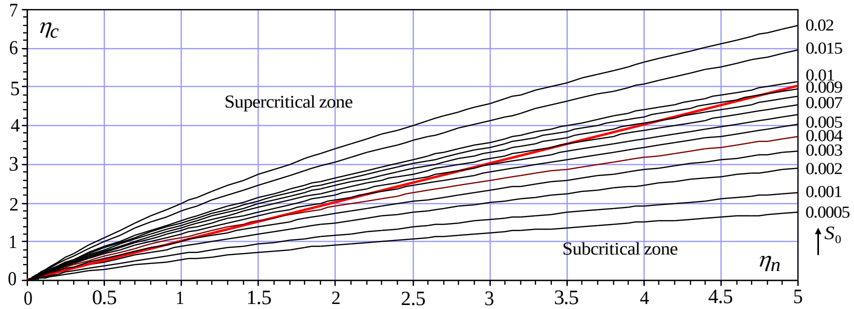

Figure 1: Variation of the relative critical depth versus the relative normal depth in a smooth rectangular-shaped channel according to Eq.(23). *b* = 1m, S_0=0.002, ν = 10^{-6} m^2/s, Red curve: First bisector corresponding to $\eta_n = \eta_c$

As can be seen in Fig. 1, all the curves take their origin in the subcritical flow zone. Moving along a given curve, from left to right, results from the increase of the discharge. There are certain curves which intersect the first bisector at two points, meaning that the channel can be the seat of two critical depths at different flow rates. However, it is worth noting that the first critical state can be observed at very shallow depths. This critical state of the flow does well and truly exists theoretically, but it is almost imperceptible in the figure. As an example, let us take the case of the strong slope S_0 = 0.02, keeping the width of the channel at b = 1m. For this case, the fundamental Eq. (23) allowed plotting Fig. 2. This shows the variation of η_c as a function of η_n. One may thus observe that the curve intersects the first bisector, represented by the red curve, at a point such that:

$$\eta_{n}=\eta_{c}=0.0002\,,$$

meaning that:

$$y_{c}=0.0002m=0.2mm$$

This is the first critical state of the flow appearing, as it was mentioned earlier, at very shallow depths.

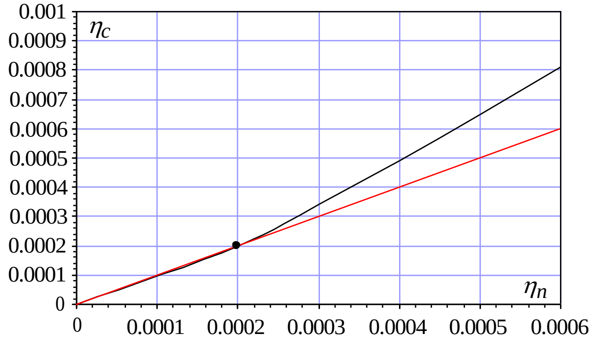

Figure 2: η_c versus η_n for a smooth rectangular channel of a width *b* = 1m and a slope S_0 = 0.02, (•) critical flow, $\eta_n = \eta_c$ =0.0002

The same observation can be made for a lower slope S_0 of the conduit. Let us take the example of the slope $S_0 = 0.005$, always keeping the width of the channel at b = 1m. The calculations performed using Eq. (23) are graphically represented in Fig. 3.

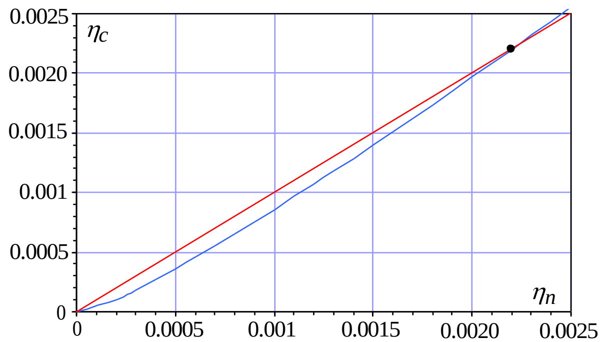

Figure 3: η_c versus η_n for a smooth rectangular-shaped channel of a width *b* = 1m and a slope S_0 = 0.005, (•) critical flow, $\eta_n = \eta_c$ =0.0022

The curve $\eta_c(\eta_n)$, shown in blue color, intersects the first bisector, represented in red color, at the point:

$$\eta_n = \eta_c = 0.0022 \,,$$

implying that:

$$y_c = 0.0022m = 2.2mm$$

This is a very low critical flow level, as in the previous case even if it is not of the same magnitude.

One may conclude that, for a given rectangular-shaped channel, the more the slope S_0 increases, the more the two critical states move away from each other. The first critical state occurs at shallow depths and the second critical state is observed at greater depths.

These critical states occurring at very shallow depths, from the order of a millimeter or even of a centimeter, are unlikely to persist in practice, as a small energy fluctuation will shift the flow into a supercritical flow regime as foreseen by Fig. 1. It is indeed recognized that the critical flow is unstable, even more so when the critical depths are very shallow. However, the value of the two relative critical depths does not depend only on the slope S_0; it also depends on the width b of the channel. The influence of the width b will be observed in one of the following sections. Referring to Fig. 1, one may predict that there exists a particular slope $S_{0;1}$ such that the curve η_c versus η_n is tangent to the first bisector at a single point, meaning that, for the corresponding pair of values (b_1; $S_{0;1}$), there is only one critical state of flow in the channel. This feature appears, for a given channel, as and well the slope S_0 increases in the subcritical zone until the curve becomes tangent to the first bisector at a single point. The following slope, slightly above $S_{0;1}$, will generate on the first bisector two points of intersection close to each other. By increasing the slope S_0 even more, these two points will move away from each other along the first bisector.

Fig.4 illustrates the case of a single critical flow state in a smooth rectangular-shaped channel of a width $b = b_1 = 1$m and a slope $S_0 = S_{0;1} = 0.0020949472$ which generate the relative critical depth $\eta_n = \eta_c = 0.14815$.

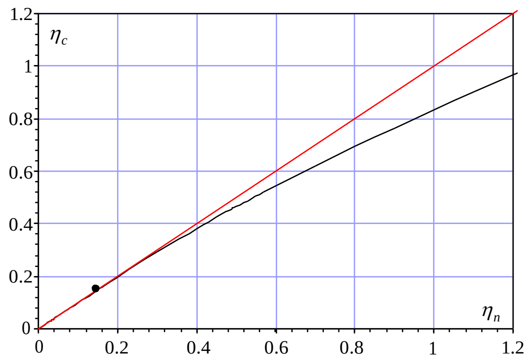

Figure 4: η_c versus η_n for a smooth rectangular-shaped channel of a width b = 1m and a slope S_0 = 0.0020949472, (•) single critical flow state, $\eta_n = \eta_c$ =0.14815

On the other hand, in the subcritical zone indicated in Fig.1, there are curves which have no point of intersection with the first bisector. This means that for certain values of the pair of parameters (b; S_0), the channel is not the seat of any critical flow. All the slopes S_0 less than $S_{0;1}$ do not generate any critical state of the flow. This case is illustrated in Fig.5 for a smooth rectangular-shaped channel of a width b = 1m and a slope S_0 = 0.001.

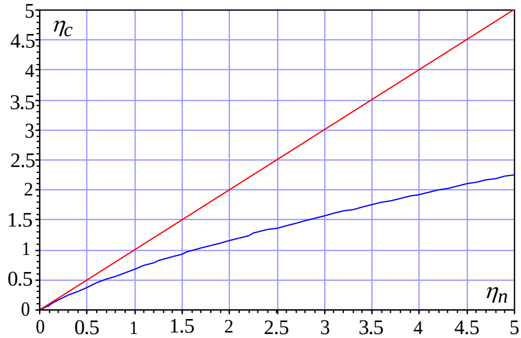

Figure 5: η_c versus η_n for a smooth rectangular-shaped channel of a width b = 1m and a slope S_0 = 0.001, for which there is no critical flow state in the channel

CRITICAL STATE OF FLOW

The general relationship which governs the critical flow in a rectangular-shaped channel can be deduced from Eq.(23) by replacing η_n by η_c. The final result is:

$$(1+2\eta_c)^{1/2} = -4\sqrt{2}\sqrt{S_0}\log\left[\frac{\varepsilon/b}{14.8}(\eta_c^{-1}+2)+\frac{3.549676(\eta_c^{-1}+2)^{3/2}}{R_{nc}^*}\right] \quad (24)$$

R_{nc}^* is always expressed by Eq.(21), i.e. :

$$R_{nc}^* = 16\frac{\sqrt{gb^3S_0}}{\nu} \quad (21)$$

As it can be seen, Eq. (24) is implicit with respect to η_c. It is implicit but it is complete, containing all the parameters of the flow. Furthermore, it is presented in dimensionless terms. The constant 3.549676, appearing in the last term of the hand-right side of the equation, can be rounded off to 3.55 without affecting the calculation. Eq.(24) shows that the relative critical depth η_c depends on three dimensionless parameters namely, the conduit slope S_0, the relative roughness ε/b, and the shear Reynolds number R^*_{nc}.

For the smooth rectangular-shaped channel ($\varepsilon \to 0$), Eq.(24) allowed plotting Fig. (6), for the width b = 1m. It shows a clear overview of the behaviour of the flow in the considered channel.

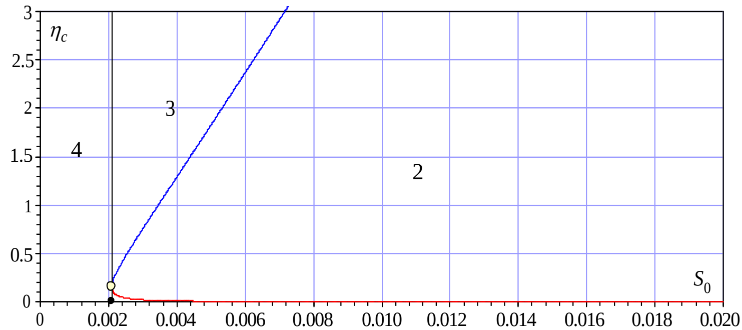

Figure 6: Variation of the relative critical depth η_c versus the slope S_0 for the smooth rectangular-shaped channel of width b =1m. (•) Smallest slope that generates a single critical state of the flow, i.e. S_0 = 0.0020949472, (○)η_c = 0.1481441

Fig. (6) reveals four zones for the flow regime (1 to 4) and two curves for the critical state of the flow (red and blue curves). Zone 1, which is below the red curve, is not clearly visible in Fig. (6). It has therefore been shown on a larger scale in Fig. (7). Zones 1 to 4 can be interpreted as follows:

Zone 1: It is an area of subcritical flow. Whatever the slope S_0, the flow originates in this zone.

Zone 2: This is the area of supercritical flow. The flow, which was subcritical in zone 1, becomes critical when intersecting the red curve, and then ends in supercritical zone 2.

Zone 3: This is the subcritical flow zone. The flow, which was supercritical in zone 2, becomes critical again on the blue curve then resumes its subcritical character in zone 3.

Zone 4: It is a subcritical zone of the flow. Its peculiarity lies in the fact that the slopes are weak generating no critical state of the flow. In this zone, the slopes S_0 are lower than the slope limit S_0 = 0.0020949472 which generates a single critical state of the flow corresponding to η_c = 0.1481441 or y_c = 0.148m.

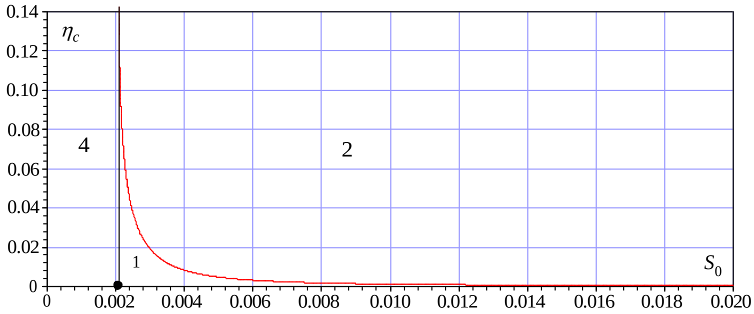

Figure 7: Overview of zone 1 with the red curve as the first critical state of the flow

INFLUENCE OF THE WIDTH *b*

To observe the influence of the width *b* of the channel, let's use Eq. (23) along with Eq.(21) and apply them to the smooth channel for which ε approaches zero, to simplify the calculations. Let us arbitrarily choose the slope S_0 = 0.002 as well as the kinematic viscosity ν = 10^{-6} m^2/s. Once these parameters are set, vary the width *b* of the channel and calculate the relative critical depth as a function of the relative normal depth for each of the chosen values of the width *b*.

For the smooth rectangular-shaped channel, Eq.(23) is reduced to:

$$\eta_c^{3/2} = -4\sqrt{2}\,\frac{\eta_n^{3/2}}{(1+2\eta_n)^{1/2}}\sqrt{S_0}\,\log\left[\frac{3.549676\,(\eta_n^{-1}+2)^{3/2}}{R_{nc}^{*}}\right] \qquad (25)$$

The shear Reynolds number R_{nc}^{*} is always governed by Eq.(21).

By adopting the procedure described above, Eq. (25) allowed drawing Fig. (8)

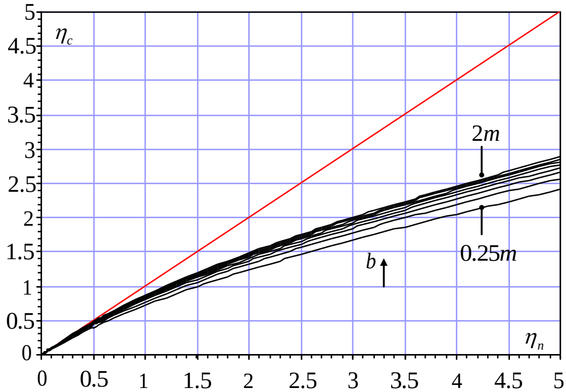

Figure 8: η_c versus η_n for a smooth rectangular-shaped channel for various widths *b* and a fixed slope S_0 = 0.002, according to Eq.(25)

As can be seen in Fig.(8), the curves are very close to each other despite the wide range of values chosen for *b*. Certain curves have no point of intersection with the first bisector which means that, for the chosen slope, there are widths *b* which do not generate any critical state of the flow. On the other hand, there are curves that have two points of intersection with the first bisector which means that, for the chosen slope, there are widths *b* of the channel which generate two critical states of the flow. More so, from these findings, it is logical to assume that there is a curve that is tangent to the first bisector at a single point. In other words, there exists, for the chosen slope, a channel width *b* that generates a single critical state of the flow. Fig. (9) shows the details of these three configurations.

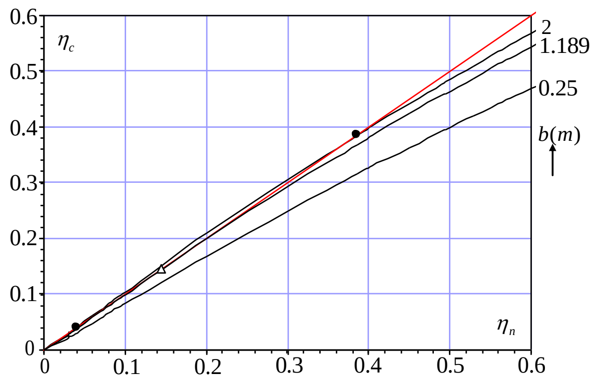

Figure 9: Influence of the width *b* of a smooth rectangular-shaped channel with slope S_0 = 0.002 on the behaviour of the critical flow, (•) two critical states of flow, (Δ) single critical state of flow

With respect to Fig. (9), one may conclude, for the chosen slope S_0 = 0.002, what follows:

1. The width of the channel b = 0.25 m does not generate any critical state of the flow because the corresponding curve has no point of intersection with the first bisector corresponding to $\eta_n = \eta_c$.
2. The width b = 1.189 m engender a single critical flow state of relative depth η_c = 0.145 corresponding to a critical depth y_c = 0.172m. This point is highlighted by the symbol (Δ) in Fig. (9).
3. The width b = 2 m gives rise to two critical flow states. The first one occurs at a shallow depth such that η_c = 0.04corresponding to a critical depth y_c = 0.08m. The second one takes place at a greater depth of relative value η_c = 0.385 corresponding to a critical depth y_c = 0.77m. These points are represented by the symbol (•) in Fig. (9).

Using Eq. (25), it was possible to compute the pair of values (b_1; $S_{o;1}$) which generates a single critical state of the flow in a smooth rectangular-shaped channel. These values are reported in table 1 and plotted in Fig.10 as well.

Table 1: Values of b_1 and $S_{0;1}$ according to Eq.(25)

b_1 (m)	$S_{0;1}$
0.25	0.0031419304
0.35	0.0028298403
0.5	0.0025444663
0.75	0.0022667617
1.00	0.0020949472
1.50	0.0018824406
2.00	0.0017496846
2.50	0.0016556530
3.00	0.0015840468

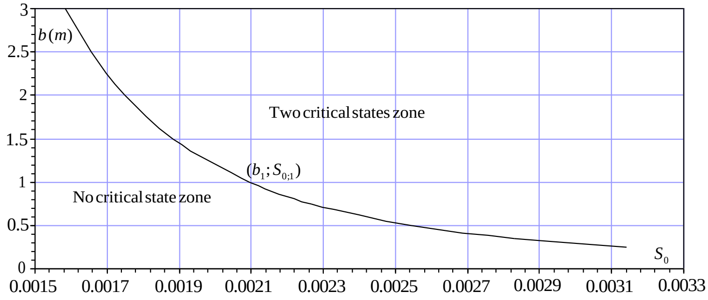

Figure 10: Variation of the smallest width *b* of a smooth rectangular-shaped channel generating one critical state of the flow with respect to the slope S_0

Having the width b of the channel as well as the slope S_0, the graph of Fig. (8) allows the user to know if the channel will be the seat of a single critical state of the flow, of two critical states or even no critical state.

CONCLUSIONS

The present study was able to establish the general relationship that governs the critical flow in a rectangular-shaped channel [Eq.(24)]. This relation is made up of four dimensionless terms representing the set of parameters which influence the flow. Although it is implicit for the relative critical depth, it is nonetheless complete and rational.

Using the relationship that links the critical and normal flows parameters [Eq.(23)], it has been demonstrated the existence of two critical states of the flow in a rectangular-shaped channel of both well defined slope S_0 and width b, obtained for two different flow rates. The example of the smooth rectangular-shaped channel of width b = 1m was considered in order to facilitate the calculations. For this channel, it appeared that the slope $S_{0;1}$ = 0.0020949472 represents the smallest slope S_0 which generates a single critical state of the flow. Slopes less than $S_{0;1}$ do not generate any critical state of the flow, while slopes greater than $S_{0;1}$ give rise to two critical states of the flow. The first critical state appears at shallow depths, while the second is observed at greater depths. The more the slope S_0 increases, the more the two critical states move away from each other. In this condition, the first critical state then occupies an extremely small sub-critical zone where the critical depth is so shallow that it cannot perdure and maintained itself in practice, especially since the critical flow is unstable. This is the case, for the considered channel of width b = 1m, where the first critical state is of depth 2.2 mm achieved for the slope S_0 = 0.005. In general, it has been observed that, in the case of the considered canal, the critical depth varies between 0 and 14.8 cm for slopes greater than $S_{0;1}$.

The study also aimed to observe the influence of the width b of the channel. As for the slope S_0, there is a width b_1 that generates a single critical state of the flow, for a given slope. All widths less than b_1 do not generate any critical state of the flow. On the other hand, two critical states of the flow are observed when the width b of the channel is greater than b_1. These three configurations were calculated and represented graphically for the slope S_0 = 0.002 [Fig. (9)]. Further, a graph [Fig.(10)] was drawn showing the variation of the pair of values (b_1; $S_{0;1}$). This is a single curve separating the no critical state zone from the two critical states zone, valid for the smooth rectangular-shaped channel. When the user has the pair of values (b; S_0), the graph lets know if the channel will be the seat of a single critical state of flow, or two critical states or no critical state of the flow.

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