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TECHNICAL NOTE

MANNING'S ROUGHNESS COEFFICIENT IN A TRAPEZOIDAL-SHAPED CHANNEL

COEFFICIENT DE RUGOSITE DE MANNING DANS UN CANAL DE FORME TRAPEZOIDALE

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ABSTRACT

The study proposes to establish the relation which governs the Manning's *n* roughness coefficient in a trapezoidal-shaped channel. It appears that *n* depends on the depth of the flow, the bottom width of the channel, the slope of the channel, the absolute roughness characterizing the state of the internal wall of the channel, the side slope, the acceleration due to gravity, and the kinematic viscosity of the flowing liquid. The coefficient *n* is therefore not constant for a given trapezoidal canal, the bottom width of which as well as the side slope are known. It does not depend solely on the material constituting the channel as suggested in the literature. Considering the cases of the smooth and rougher trapezoidal canal, with a given side slope, the curves of variation of n have been drawn. The defined shear Reynolds number as a function of the bottom width, the slope of the channel and the kinematic viscosity plays an important role in the variation of *n* up to a lower limit value of the relative roughness beyond which it has no longer influence. The rough turbulent flow regime is then achieved. For shallow depths, the coefficient *n* takes on large values, which is in mathematical accordance with Manning's equation.

Keywords: Manning's coefficient, relative depth, trapezoidal-shaped channel, free surface flow, open channel flow.

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RESUME

L’étude propose d’établir la relation qui régit le coefficient de rugosité *n* de Manning dans un canal de forme trapézoïdale. Il apparaît que *n* dépend de la profondeur de l'écoulement, de la largeur du fond du canal, de la pente du canal, de la rugosité absolue caractérisant l'état de la paroi interne du canal, de la pente de la paroi latérale, de l'accélération de la pesanteur, et de la viscosité cinématique du liquide en écoulement. Le coefficient *n* n'est donc pas constant pour un canal trapézoïdal donné dont la largeur du fond ainsi que la pente de la paroi latérale sont connues. Elle ne dépend pas uniquement du matériau constituant le canal comme le suggère la littérature. En considérant les cas du canal trapézoïdal lisse et plus rugueux, avec une pente latérale donnée, les courbes de variation de *n* ont été tracées. Le nombre de Reynolds de cisaillement défini en fonction de la largeur du fond, de la pente du canal et de la viscosité cinématique joue un rôle important dans la variation de *n* jusqu'à une valeur limite inférieure de la rugosité relative au-delà de laquelle il n'a plus d'influence. Le régime d’écoulement turbulent rugueux est alors atteint. Pour les faibles profondeurs, le coefficient *n* prend de grandes valeurs, ce qui est en accord mathématique avec l'équation de Manning.

Mots clés : Coefficient de Manning, profondeur relative, canal trapezoidal, écoulement à surface libre, écoulement dans les canaux ouverts

INTRODUCTION

The Manning’s *n* coefficient represents the friction applied to the flow by the inner wall of a channel or a conduit. Manning’s *n* values are selected from tables that can be found in literature, according to the material that compose the channel (Chow, 1959; Henderson, 1966; Streeter, 1971). The coefficient *n* is an empirical derived coefficient which depends in particular on the surface roughness. The rougher the channel, the greater the coefficient *n*. Conversely, the smoother the channel, the smaller *n*. In the past, Manning's *n* coefficient was expected to be a constant for a given channel of well-defined roughness. But studies which followed this time clearly showed that the coefficient *n* is not a constant and that it depends on several parameters such as the depth of the flow in particular (Camp, 1946). This observation has been confirmed in the recent past with regard to different shapes of man-made canals (Achour and Bedjaoui, 2006; Achour, 2014; Achour and Amara, 2020).

For a correct design of hydraulic systems, the designer should have an appropriate value of this coefficient. A slight variation in this parameter can significantly affect discharge, depth, and velocity computations.

Manning’s *n* laboratory values are systematically corrected before using them in actual installed conditions. The correction factor, called design factor, could vary between 20% and 30% (ACPA, 2000; 2012). This correction is justified by the fact that the test channel is straight without bend, and the flow does not contain debris or any obstruction contrary to reality. For some pipe material, recommended n design values can be selected from tables given by many institutions and authors such as the University of Minnesota (1950), Barfuss and Tullis (1989), the American concrete pipe association (ACPA, 2000), and the US department of transportation (2012).

The main purpose of this technical note is to translate through an appropriate relationship the link between Manning's coefficient n, the geometry of a trapezoidal-shaped channel and the flow parameters such as the flow depth, the absolute roughness and the slope of the channel. To do this, the general Manning's n relationship, valid for any shape of channel, is apply to the trapezoidal-shaped open channel. The resulting relationship is presented in dimensionless terms and the emanating graphs, for the case of a smooth channel and a rougher channel, allow interesting conclusions.

GEOMETRIC PROPERTIES

Fig.1 is a schematic representation of the Trapezoidal-shaped open channel and the definition of the geometric characteristics of the channel as well the hydraulic parameters of the flow considered herein.

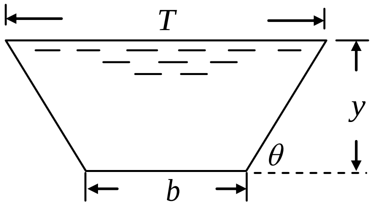

Figure 1: Definition of the trapezoidal-shaped open channel

The water area A is expressed as:

$$A = by + my^2 \tag{1}$$

b is the bottom width, m is the side slope computed as horizontal distance divided by vertical distance i.e. $m = \text{cotg}\,\theta$ where θ is the angle formed by the side wall of the channel with respect to horizontal, and y is the flow depth measured perpendicular to the bottom of the channel. Defining the aspect ratio or the relative depth as $\eta = y/b$, Eq.(1) can be rewritten as:

$$A = b^2\eta\left(1 + m\eta\right) \tag{2}$$

The wetted perimeter P is as:

$$P = b + 2y\sqrt{1 + m^2} \tag{3}$$

Note that the quantity [y (1 + m^2)$^{1/2}$] is called slant length.

Eq.(3) can be rewritten as follows:

$$P = b\left(1 + 2\eta\sqrt{1+m^2}\right) \tag{4}$$

Thus, the hydraulic radius $R_h = A/P$ is:

$$R_h = \frac{b\eta\left(1+m\eta\right)}{\left(1+2\eta\sqrt{1+m^2}\right)} \tag{5}$$

Eq.(5) can be written as:

$$R_h = b\,\varphi(m;\eta) \tag{6}$$

Where $\varphi(m;\eta)$ is as:

$$\varphi(m;\eta) = \frac{\eta\left(1+m\eta\right)}{\left(1+2\eta\sqrt{1+m^2}\right)} \tag{7}$$

MANNING'S COEFFICIENT RELATIONSHIP

It has been evidenced in the past and confirmed in a recent study that Manning's n coefficient is governed by the following general relationship, valid for all known channel shapes (Achour and Bedjaoui, 2006; Achour and Amara, 2020):

$$\frac{n\sqrt{g}}{R_h^{1/6}} = \frac{1}{4\sqrt{2}}\left[-\log\left(\frac{\varepsilon}{14.8R_h} + \frac{10.04}{R^*}\right)\right]^{-1} \tag{8}$$

Where g is the acceleration due to gravity, ε is the absolute roughness characterizing the inner wall of the channel, R_h is the hydraulic radius, R^* is the shear Reynolds number expressed as:

$$R^* = 32\sqrt{2}\,\frac{\sqrt{gR_h^3 S_0}}{\nu} \tag{9}$$

Where S_0 is the bottom slope of the channel, and ν is the kinematic viscosity of the flowing liquid.

It emerges from Eq.(8) along with Eq.(9) that Manning's n coefficient depend on five variables namely, the slope S_0, the absolute roughness ε, the acceleration due to gravity

g, the hydraulic radius R_h, and the kinematic viscosity ν. It is worth noting that the hydraulic radius R_h, is itself depending on the relative depth η, the bottom width b of the channel and the side slope m, in accordance with Eq.(5).

Taking into account Eqs.(5) and (9), Eqs.(8) is reduced to:

$$\frac{n\sqrt{g}}{b^{1/6}} = \frac{\varphi^{1/6}}{4\sqrt{2}}\left[-\log\left(\frac{\varepsilon/b}{14.8\varphi} + \frac{0.22185475}{R_0^*\varphi^{3/2}}\right)\right]^{-1} \tag{10}$$

Where :

$$R_0^* = \frac{\sqrt{gb^3S_0}}{\nu} \tag{11}$$

R_0^* is also a shear Reynolds number.

The constant containing in the last term of the right-hand side of Eq.(10) can be rounded off to 0.222 without affecting the calculations.

Eq. (10) is presented in four dimensionless parameters, i.e.:

$$n\sqrt{g}\,/\,b^{1/6},\ \varphi,\ \varepsilon/b, \text{ and } R_0^*$$

In order to observe the variation in Manning's coefficient n, let us first consider a smooth trapezoidal-shaped canal with side slope $m = 1/\sqrt{3}$ corresponding to $\theta = 60°$. Let us vary η for a fixed value of the shear Reynolds number R^*_0. For the known values of η and m, the function φ is well defined in accordance with Eq.(7). According to this procedure, Eq.(10) has been plotted in the following Figure 2.

Fig. 2 shows that for low values of R^*_0 around 10^4, the coefficient n is practically constant when the relative depth η exceeds 0.4. In this case, when the relative depth η varies while remaining greater than 0.4, it then has no influence on the variation of n. The curves clearly show that the coefficient n varies as a function of both η and R^*_0. The more R^*_0 increases, the more the coefficient n decreases. Around shallow depths, i.e. $\eta \leq 0.2$, the coefficient n undergoes a strong variation, greater than that observed at greater depths. All the curves in Fig.2 show that n takes large values for shallow depths. It tends towards infinity when n tends towards zero. This observation is justified by the fact that when n tends towards zero, the hydraulic radius also tends towards zero and therefore n should tend towards infinity for the discharge to be zero, according to Manning's equation. This is therefore mathematically justified by the form of Manning's equation, but the physical meaning is another issue that cannot be explained herein. The two curves corresponding to $R^*_0 = 10^6$ and 10^7 are no exception to this observation, contrary to what appears in Fig. 1. In fact, these two curves have an inflection point in the interval $0 \leq \eta \leq 0.05$ which does not appear in Fig.2 due to the scale. For all the other curves, the inflection point appears in the range $0 \leq \eta \leq 0.2$. For practical values of R^*_0, i.e. R^*_0 greater than 10^4, the curves in Fig.2 reveal that the coefficient n decreases as η decreases and undergoes a large increase beyond the inflection point. The more R^*_0 increases and the more the inflection point occurs at shallower depths.

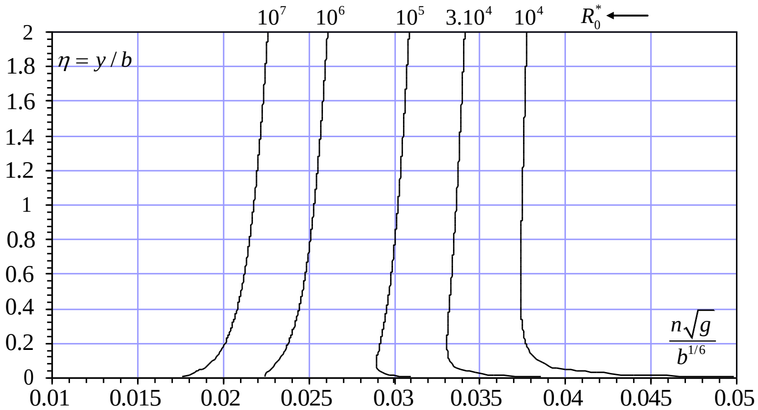

Figure 2: Variation of the relative depth $\eta = y/b$ versus $n\sqrt{g}\,/\,b^{1/6}$ for various R_0^* according to Eq.(10), for a smooth trapezoidal-shaped channel of side slope $m = 1/\sqrt{3}$

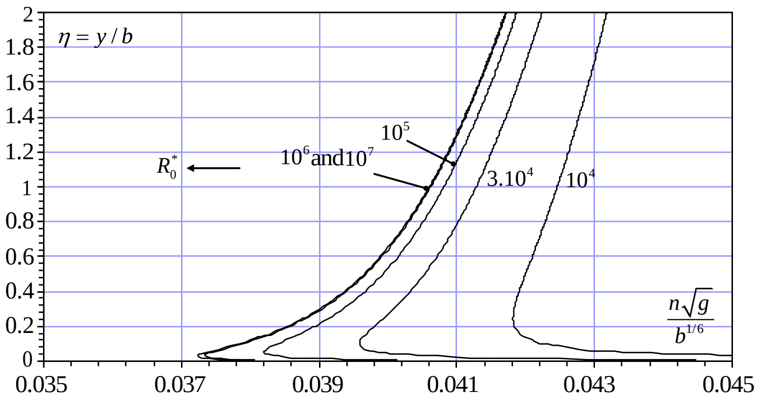

Figure 3: Variation of the relative depth $\eta = y/b$ versus $n\sqrt{g}\,/\,b^{1/6}$ for various R_0^* according to Eq.(10), for trapezoidal-shaped channel of side slope $m = 1/\sqrt{3}$ and relative roughness $\varepsilon / b = 0.001$

Let us consider the second case of a trapezoidal-shaped canal rougher than the previous one. Let us assume that the relative roughness is $\varepsilon / b = 0.001$ and the side slope is again

$m = 1/\sqrt{3}$. By adopting the same approach described previously, observe the variation of n by plotting Eq. (10) for these data. The obtained curves can be seen in Fig. 3.

Compared to Fig. 2, Fig. 3 shows curves of the same shape but with much more pronounced concavities. The coefficient n undergoes a greater variation as a function of η, for a given Reynolds number R^*_0, taking on larger values. These are justified by the fact that the relative roughness is greater. It should also be noted that the inflection points get closer to shallow depths as R^*_0 increases. The most striking observation resides in the fact that the curves corresponding to $R^*_0 = 10^6$ and $R^*_0 = 10^7$ are intermingled, i.e. superimposed, which means that the rough turbulent flow is achieved, for which the Reynolds number R^*_0 no longer influences the variation of n. Some might conclude that there is a lower limit value of the relative roughness ε / b beyond which the variation of the coefficient n is not influenced by the Reynolds number R^*_0. The variation of n will depend only on the relative depth η and the relative roughness ε / b for a given channel. The non-influence of the shear Reynolds number R^*_0 occurs at large values thereof which amounts to writing mathematically that $R^*_0 \to \infty$. In this case, Eq.(10) is reduced to:

$$\frac{n\sqrt{g}}{b^{1/6}} = \frac{\varphi^{1/6}}{4\sqrt{2}}\left[-\log\left(\frac{\varepsilon / b}{14.8\varphi}\right)\right]^{-1} \tag{12}$$

CONCLUSIONS

The general relationship which governs the Manning's n resistance coefficient [Eq.(8)] was applied to the case of a trapezoidal-shaped open channel defined by the bottom width b, the side slope m, the absolute roughness ε, and the slope S_0. The resulting relation was consisted of by four dimensionless parameters encompassing all flow variables and the geometry of the channel as well [Eq.(10)]. The graphic representation of this dimensionless relationship, for the case of the smooth trapezoidal canal and of a rougher one, showed that the coefficient n is influenced by the relative roughness, the depth of the flow and especially the shear Reynolds number. This depends on the bottom width b of the channel, the slope S_0 and the kinematic viscosity ν of the flowing liquid [Eq.(11)]. It has been observed that n decreases with the decrease of the depth until the latter reaches a certain limit from which n takes large values, i.e. n increases sharply from a certain limit as the depth continues to decrease. This limit is materialized by an infection point of the curves in the vicinity of shallow depths. The large values of n associated with shallow depths are in mathematical conformity with Manning's equation. It has also been observed that there is a limiting value of the relative roughness beyond which the shear Reynolds number has no influence on the variation of n. In this case, n is governed by [Eq.(12)].

REFERENCES

ACHOUR B. (2014). Canal rectangulaire en charge et à surface libre, Chapitre II, Cours et exercices d'application, Pressurized and free surface flow rectangular channel, Chapter II, Application courses and exercises, Editions Al-Djazair, Algiers, In French, 63p.

ACHOUR B., AMARA L. (2020). Proper relationship of Manning's Coefficient in a Partially Filled Circular Pipe, Larhyss Journal, No 42, pp.107-119.

ACHOUR B., BEDJAOUI A. (2006). Discussion to "Exact solution for normal depth problem, by SWAMME P.K. and RATHIE P.N., Journal of Hydraulic Research, Vol.44, Issue 5, pp.715-717.

ACPA (The American Concrete Pipe Association). (2000). Concrete Pipe Design Manual, 561p.

ACPA (The American Concrete Pipe Association). (2012). Manning's n Values-History of Research, USA, 12p.

BARFUSS S., TULLIS J.P. (1989). Friction Factor Test on High Density Polyethylene Pipe, Hydraulic Report, No 208, Utah Water Research Hydraulic Report, Utah State University, Logan, Utah, USA.

CAMP T.R. (1946). Design of Sewers to Facilitate Flow, Sewage Works Journal, Vol.18, Issue 1, pp.3-16.

CHOW V.T. (1959). Open-channel Hydraulics, McGraw-Hill, New York, USA, 680p.

HENDERSON F. M. (1966). Open channel flow, MacMillan Publishing Co. Inc., New-York.

STREETER V.L. (1971). Fluid Mechanics, McGraw-Hill, USA, 751p.

U.S. Department of Transportation (2012). Hydraulic Design of Highway Culverts, Federal Highway Administration's FHWA, 3rd Edition, Publication No FHWA-HIF-12-026, Hydraulic Design Series No 5, USA, 323p.

University of Minnesota (1950). Hydraulic Tests on Concrete Culvert Pipes, St. Anthony Falls Hydraulic Laboratory, Technical paper No 4, Series B, USA.