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THEORETICAL DISCHARGE COEFFICIENT RELATIONSHIP FOR CONTRACTED AND SUPPRESSED RECTANGULAR WEIRS

RELATION THEORIQUE DU COEFFICIENT DE DEBIT DES DEVERSOIRS RECTANGULAIRES AVEC ET SANS CONTRACTION LATERALE

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ABSTRACT

The rectangular weirs with and without lateral contraction are theoretically examined, more particularly their discharge coefficient. Under the realistic assumption of a critical state at the location of the weir, the application of the energy equation between two judiciously chosen sections, while taking into account the effect of the approaching flow velocity, leads to a third degree equation. The analytical solution of the equation shows that the discharge coefficient is a function of both the relative height of the weir and the rate of contraction. This is what the relationships drawn from the experiment reveal. It was easy to deduce the discharge coefficient of the rectangular weir without lateral contraction by writing that the contraction coefficient is equal to unity. The theoretical relationship of the discharge coefficient is compared to the experimental tests abstracted from the literature and is corrected consequently to be in conformity. Also, a comparison is made with the recognized experimental relationships proposed by some research workers and good agreement is observed.

Keywords: Rectangular weir, contracted weir, suppressed weir, discharge coefficient, lateral contraction.

RESUME

Les déversoirs rectangulaires avec et sans contraction latérale sont théoriquement examinés, plus particulièrement leur coefficient débit. Sous l'hypothèse réaliste d'un état critique à l'endroit du déversoir, l'application de l'équation de l'énergie entre deux sections judicieusement choisies, tout en tenant compte de l'effet de la vitesse d'approche de l'écoulement, mène à une équation du troisième degré. La solution analytique de l'équation montre que le coefficient de débit est à la fois fonction de la hauteur relative du déversoir et du taux de contraction. C'est ce que révèle les relations tirées de l'expérimentation. Il a été aisé de déduire le coefficient de débit du déversoir rectangulaire sans contraction latérale en écrivant que le coefficient de contraction est égal à l'unité. La relation théorique du coefficient de débit est comparée aux essais expérimentaux extraits de la littérature ainsi qu'aux relations expérimentales reconnues proposées par certains chercheurs. Cette comparaison a abouti à la correction de la relation théorique pour être conforme aux résultats expérimentaux donnés par la littérature.

Mots clés : Déversoir rectangulaire, déversoir contracté, déversoir sans contraction, coefficient de débit, contraction latérale.

INTRODUCTION

Weirs are constructed as an obstruction to flow of water. These are commonly used to measure the volumetric rate of water flow (Achour et al., 2003; Bos, 1976). Weirs are typically classified as being either sharp-crested or broad-crested. Some weirs are based on the effect of their sides on the emerging nappe. This is the case of weirs with end contraction, called contracted weirs. They have a central opening which can be rectangular (rectangular weir), triangular (triangular weir) or trapezoidal shape (trapezoidal weir).

The suppressed weir is a weir without end contraction such that the crest is running all the way across the width of the channel so the head loss will be negligible.

Weirs generally have a certain height, denoted P, the primary purpose of which is to raise the upstream water level and make it tranquil and undisturbed corresponding to a subcritical flow. Depending on the height P of the weir, the approach velocity of the flow can be negligible.

Basically, a weir forces water to flow through a critical state. Installing a weir in an open channel causes critical depth to form over the weir. Since there is a unique relationship between the critical depth and discharge, a weir can be designed as a flow-measuring device. As the flow passes over the crest it achieves critical velocity where the critical depth of flow is two-thirds of the upstream height of the water above the rectangular weir crest. The critical velocity is directly related to the critical depth, so measurement of the upstream height of water behind the weir can be used to determine the discharge in the channel. The derivation of these relationships from the Bernoulli equation is fairly simple and is well explained in the literature (Henderson, 1966; Bos, 1989), by making some assumptions with regard to head loss and pressure distribution of the flow passing over the weir.

For this reason, the experimental flow rate is not equal to the theoretical one and a discharge coefficient, denoted C_d representing the ratio of the two flow rates, must be determined. The discharge coefficient C_d needs to be determined experimentally for each weir to account for errors in estimating the flow rate that is due to these assumptions.

A rectangular weir is a standard shape of the weir. The top edge of the weir may be sharp-crested or narrow crested. It is generally suitable for larger flowing channels.

The sharp-crested weir is characterized by a very sharp crest such that the water will springs clear of the crest. The weir plate is beveled at the crest edges to obtain the necessary thickness. The weir plate should be made of smooth metal that is free from rust and nicks. Flow over the sharp-crested weir is similar to the rectangular weir.

The purpose of this study is to examine from a theoretical point of view both the rectangular contracted weir and the suppressed one. The rectangular contracted weir has a rectangular opening of width b, the sides of which are straight from top to bottom. A contracted weir means that the ditch or channel of width B leading to the weir is wider than the opening of the weir. The ratio $\beta = b/B$, denoted contraction rate, is then less than unity, i.e. $\beta < 1$. This is sometimes called an unsuppressed rectangular weir. The suppressed weir is a special case of the previous one, for which $\beta = 1$. The present study proposes to determine from a theoretical point of view the discharge coefficient relationship of these two devices assuming a critical sate of the flow over the weir. Note that the theoretical development takes into account the effect of the approach velocity of the flow. Due to the previously mentioned assumptions, the theoretical relationship of the discharge coefficient must be corrected. This was done based on the experimental data available in the literature (Ramamurthy et al., 1987; Kandaswamy, 1957). Then a comparison was made with the experimental relationships established by some authors and whose reliability is recognized, such as those of the SIA (1924), Rehbock (1929), and kindsvater-Carter (1959).

DESCRPTION OF THE DEVICE

Fig. 1 describes the studied device representing a contracted weir. The central opening is of width b while the channel is of width B. The ratio $\beta = b/B$ is the contraction rate.

The weir is also characterized by a height P above which the upstream depth h_1 is measured. When the ratio $\beta = b/B$ is equal to 1, i.e. $b = B$, one obtains the rectilinear weir called suppressed weir which extends over the entire width B of the channel. The discharge flowing through the channel is noted Q.

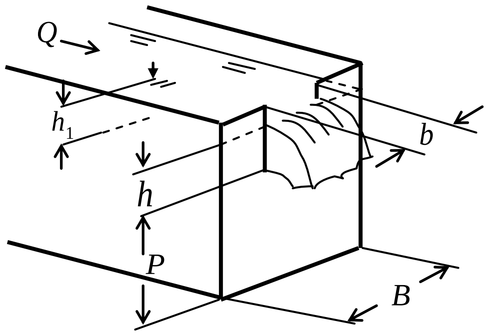

Figure 1: Definition sketch of the studied contracted weir

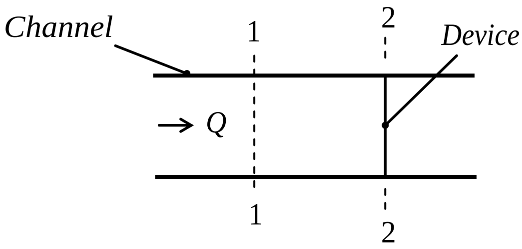

Figure 2: Plan view of the channel and the device

THEORETICAL DEPTH-DISCHARGE RELATIONSHIP

The critical depth in the rectangular cross-section 1-1 (Fig. 2) is written as:

$$h_{1c} = \left(\frac{Q^2}{g\,B^2}\right)^{1/3} \quad (1)$$

where the subscript « c » denotes the critical conditions.

On the other hand, the critical depth in the rectangular cross-section 2-2 (Fig. 2) is as:

$$h_{2c} = \left(\frac{Q^2}{g b^2}\right)^{1/3} \quad (2)$$

The ratio of relations (1) and (2) gives:

$$\frac{h_{1c}}{h_{2c}} = (b / B)^{2/3} \quad (3)$$

Resulting in:

$$h_{2c} = \beta^{-2/3} h_{1c} \tag{4}$$

where: $\beta = b / B$

Assume that there is no head loss between sections 1-1 and 2-2. Equal total heads between sections 1-1 and 2-2 translates into:

$$H_1 = H_2 = \frac{3}{2} h_{2c} \tag{5}$$

Combining Eqs. (4) and (5) results in:

$$H_1 = \frac{3}{2} h_{1c} (B / b)^{2/3} \tag{6}$$

Hence:

$$\frac{H_1}{h_{1c}} = \frac{3}{2} \beta^{-2/3} \tag{7}$$

The total head H_1 can be written as:

$$H_1 = h_1 + \frac{Q^2}{2gB^2 \left(h_1 + P\right)^2} \tag{8}$$

Implying that:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 \left(h_1 + P\right)^2 h_{1c}} \tag{9}$$

Eq. (9) can be written as:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{Q^2}{2gB^2 h_1^2 \left(1 + P / h_1\right)^2 h_{1c}} \tag{9a}$$

Eq. (1) allows writing that:

$$\frac{Q^2}{gB^2} = h_{1c}^3 \tag{10}$$

Combining Eqs. (9a) and (10) yields:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2} \tag{11}$$

Eqs. (7) and (11) give what follows:

$$\frac{H_1}{h_{1c}} = \frac{h_1}{h_{1c}} + \frac{1}{2(h_1/h_{1c})^2(1+P/h_1)^2} = \frac{3}{2}\beta^{-2/3} \tag{12}$$

Let us adopt the following non-dimensional parameter

$$h_1/h_{1c} = h_1^* \tag{13}$$

Inserting Eq. (13) into Eq. (12) results in:

$$h_1^* + \frac{1}{2h_1^{*2}(1+P/h_1)^2} = \frac{3}{2}\beta^{-2/3} \tag{14}$$

Note that the flow in the section 1-1 is subcritical, meaning that $h_1 > h_{1c}$ or $h_1^* > 1$.

Eq. (14) can be rewritten as:

$$h_1^{*3} - \frac{3}{2}\beta^{-2/3}h_1^{*2} + \frac{1}{2(1+P/h_1)^2} = 0 \tag{15}$$

It is a third order equation of the form:

$$z^3 + az^2 + bz + c = 0 \tag{16}$$

Where: $a = -\frac{3}{2}\beta^{-2/3}$; $b = 0$; $c = \frac{1}{2(1+P/h_1)^2}$

To find the solutions of equation (15), use the method described by Spiegel (1974). Let us assume the following parameters:

$$H = \frac{3b-a^2}{9} = -\frac{9\beta^{-4/3}}{4\times 9} = -\frac{\beta^{-4/3}}{4} \tag{17}$$

$$R = \frac{9ab-27c-2a^3}{54} = \frac{-27c-2a^3}{54} = \frac{\beta^{-2}}{8} - \frac{1}{4\left(1+P/h_1\right)^2}$$

Thus:

$$R = \frac{\beta^{-2}}{8} - \frac{1}{4\left(1+P/h_1\right)^2} \tag{18}$$

Let's find the angle α such that:

$$\cos\alpha = +\frac{R}{\sqrt{-H^3}} = \frac{\dfrac{\beta^{-2}}{8} - \dfrac{1}{4\left(1+P/h_1\right)^2}}{\sqrt{\dfrac{(B/b)^4}{64}}} = \frac{\beta^{-2} - \dfrac{2}{\left(1+P/h_1\right)^2}}{\beta^{-2}} \tag{19}$$

That is:

$$\alpha = \cos^{-1}\left[1 - 2\beta^2\left(1+P/h_1\right)^{-2}\right] \tag{20}$$

The discriminant of equation (16) is expressed as:

$$\Delta = H^3 + R^2 \tag{21}$$

Inserting Eqs. (17) and (18) into Eq. (21) and rearranging, one may obtain:

$$\Delta = \frac{\left[1-\beta^{-1}(1+P/h_1)\right]\left[1+\beta^{-1}(1+P/h_1)\right]}{16(1+P/h_1)^4} \tag{22}$$

The study is focussed on cases where the ratio *b/ B* is less than 1, which means that Δ is negative according to Eq. (22). In this case, Eq. (16) has three real roots which are given as:

$$z_1 = 2\sqrt{-H}\cos(\alpha/3) - a/3 \tag{23}$$

$$z_2 = 2\sqrt{-H}\cos(\alpha/3 + 240°) - a/3 \tag{24}$$

$$z_3 = 2\sqrt{-H}\cos(\alpha/3 + 120°) - a/3 \tag{25}$$

Whence:

$$z_1 = \beta^{-2/3}\left[\cos\left(\frac{1}{3}\cos^{-1}\left[1-2\beta^2\left(1+P/h_1\right)^{-2}\right]\right) + \frac{1}{2}\right] = h^*_{1;1} \tag{26}$$

$$z_2 = \beta^{-2/3}\left[\cos\left(\frac{1}{3}\cos^{-1}\left[1-2\beta^2\left(1+P/h_1\right)^{-2}\right] + 240°\right) + \frac{1}{2}\right] = h^*_{1;2} \tag{27}$$

$$z_3 = \beta^{-2/3}\left[\cos\left(\frac{1}{3}\cos^{-1}\left[1-2\beta^2\left(1+P/h_1\right)^{-2}\right]+120°\right)+\frac{1}{2}\right] = h^*_{1;3} \qquad (28)$$

For a given configuration indicated in Figure 2, there is obviously only one solution to choose among the three solutions given by equations (26), (27), and (28). It is the one that corresponds to $h_1^* > 1$.

Eq. (1) allows writing that:

$$Q = \sqrt{g}Bh_{1c}^{3/2} \qquad (29)$$

Taking into account Eq. (13), Eq. (29) becomes:

$$Q = \sqrt{g}B\frac{h_1^{3/2}}{h_1^{*3/2}} \qquad (30)$$

Eq. (30) can be rewritten as:

$$Q = C_d\sqrt{2g}Bh_1^{3/2} \qquad (31)$$

Eq. (31) is the theoretical depth-discharge relationship for the studied device, where C_d is the discharge coefficient expressed as:

$$C_d = \frac{1}{\sqrt{2}\,h_1^{*3/2}} \qquad (32)$$

The upstream depth h_1 of the flow is measured by a simple point gauge reading at the inlet of the device.

For $P = 0$ and $\beta = 1$, the real root of Eq. (15) is $h_1^* = 1$. Inserting this value into Eq. (32) results in $C_d = \sqrt{2}/2 \approx 0.707$.

As the dimensionless parameter h_1^* is governed by Eqs. (26), (27), and (28), on may conclude that the discharge coefficient C_d depends on both the *B/b* ratio and the relative height P/h_1. Eqs. (26), (27), and (28), along with Eq. (32), allow computing the discharge coefficient C_d some values of which are given in tables 1.

For the device under study, numerous tests were carried out by SIA (Castex, 1969). These tests showed that the flow rate Q could be calculated by applying the following equation:

$$Q = \frac{2}{3}\mu b\sqrt{2g}\,h_1^{3/2} \qquad (33)$$

where μ is the discharge coefficient expressed as:

$$\mu = \left[0.578 + 0.037\beta^2 + \frac{0.003615 - 0.0030\beta^2}{h_1 + 0.0016}\right]\left[1 + 0.5\beta^4\left(\frac{1}{1 + P/h_1}\right)^2\right] \quad (34)$$

In relation (34), the contraction rate is such that $\beta < 1$. β must be less than 0.80 and greater than 0.30, i.e. $0.30 < \beta < 0.80$. The relation (34) is composed of four terms: a) the first term is represented by the constant 0.578. b) the second term is represented by the contraction rate β. c) the third term takes into account the effect of surface tension, proportional to $1/h_1$ (h_1 is expressed in meters). d) The fourth term takes into account the effect of the approach velocity of the flow, through the P/h_1 ratio.

Eq. (33) can be rewritten as:

$$Q = \frac{2}{3}\mu B \frac{b}{B}\sqrt{2g}\, h_1^{3/2} = \frac{2}{3}\mu B\beta\sqrt{2g}\, h_1^{3/2} \quad (33a)$$

Comparing Eqs. (31) and (33a) one may write:

$$C_d = \frac{2}{3}\mu\beta \quad (35)$$

Inserting Eq. (32) into Eq. (35) results in:

$$\frac{1}{\sqrt{2}\, h_1^{*3/2}} = \frac{2}{3}\mu\beta \quad (36)$$

That is:

$$\mu_{Th} = \frac{3\sqrt{2}}{4\beta h_1^{*3/2}} \quad (36a)$$

where the subscript "*Th*" denotes "*Theoretical*".

Table 1: Some values of μ_{Exp} and μ_{Th} as a function of the contraction rate β

β^{-1}	h_1	*P*	P/h_1	μ_{Exp} **SIA Eq. (34)**	μ_{Th} **Eq. (36a)**	$\Delta\mu/\mu$ **%**
1.4	0.20	0.45	2.25	0.61469907	0.58369614	5.31148441
1.4	0.25	0.45	1.8	0.61520855	0.58597221	4.98937288
1.4	0.28	0.45	1.60714286	0.61585037	0.58734555	4.85315927
1.4	0.30	0.45	1.5	0.61636234	0.58825726	4.77768629
1.4	0.32	0.45	1.40625	0.61692174	0.58916304	4.71154869
1.4	0.35	0.45	1.28571429	0.61782313	0.59050656	4.62595443
1.4	0.38	0.45	1.18421053	0.61877254	0.59182773	4.55281235
1.4	0.40	0.45	1.125	0.61942119	0.59269436	4.50937774
1.4	0.45	0.45	1	0.62106481	0.59480641	4.41461271
1.8	0.25	0.70	2.8	0.60208709	0.58012116	3.78643668
1.8	0.30	0.70	2.33333333	0.60090066	0.58096265	3.43189194
1.8	0.35	0.70	2	0.60022767	0.58182435	3.16303736
1.8	0.40	0.70	1.75	0.59987009	0.58269202	2.94805447
1.8	0.45	0.70	1.55555556	0.5997164	0.5835557	2.7693493
1.8	0.50	0.70	1.4	0.59969904	0.58440848	2.61641631
1.8	0.55	0.70	1.27272727	0.59977488	0.58524559	2.48259746
1.8	0.60	0.70	1.16666667	0.59991524	0.58606382	2.36346599
1.8	0.65	0.70	1.07692308	0.60010044	0.5868611	2.25595944

Tables 1 and 2 show on one hand that μ_{Exp} is greater than μ_{Th} and the deviation between them varies in the range [2%;5%] on the other hand. However, calculations have shown that this deviation can reach 6% depending on the β value. Thus, Eq. (36) must be corrected for the effect of β.

Eq. (36a) can be written as:

$$\mu_{Th} = \alpha \frac{3\sqrt{2}}{4\beta h_1^{*3/2}} \tag{36b}$$

where α is the correction factor. Eq. (36b) can also be rewritten as:

$$\mu_{Th} = \frac{\zeta}{\beta h_1^{*3/2}} \tag{36c}$$

where ζ is expressed as:

$$\zeta = \alpha \frac{3\sqrt{2}}{4} \tag{37}$$

Table 2 gives values of ζ as a function of *β*.

Table 2: Values of ζ of Eq. (37) as a function of the contraction rate β

β	ζ	β	ζ
0.20	1.08420683	0.62	1.10075313
0.22	1.08420683	0.64	1.10255625
0.24	1.08473716	0.66	1.10446544
0.26	1.08494929	0.68	1.10658676
0.28	1.08526749	0.70	1.10892021
0.30	1.08569175	0.72	1.11135973
0.32	1.08611602	0.74	1.11401138
0.34	1.08654028	0.76	1.11676909
0.36	1.08707061	0.78	1.11984501
0.38	1.08760094	0.80	1.12302699
0.40	1.08823734	0.82	1.1264211
0.42	1.08887373	0.84	1.13013341
0.44	1.08972226	0.86	1.13405786
0.46	1.09046472	0.88	1.13819443
0.48	1.09141932	0.90	1.1426492
0.50	1.09247998		
0.52	1.09354064		
0.54	1.09470736		
0.56	1.09608622		
0.58	1.09746508		
0.60	1.09905607		

Eq. (36b) is not only in accordance with Eq. (34) of SIA but also with the Kindsvater-Carter relationships (K-C) rightly recalled by Castex (1969) for some values of the contraction rate β .

EXAMPLE

Consider the device of Fig. 1 with the following data:

$\beta = 0.40$; $h_1 = 0.40m$, and $P = 0.60m$

Compute and compare the discharge coefficients μ_{Th} [Eq. (36c), μ_{Exp} [SIA, Eq. (34)], and μ_{K-C} (Kindsvater-Carter).

SOLUTION

According to the given data, on may write :

$P/h_1 = 0.60/0.40 = 1.5$

The root of h_1^* is given by Eq. (26) as being:

$$z_1 = \beta^{-2/3}\left[\cos\left(\frac{1}{3}\cos^{-1}\left[1-2\beta^2\left(1+P/h_1\right)^{-2}\right]\right)+\frac{1}{2}\right] = h_{1;1}^*$$

$$h_1^* = (0.40)^{-2/3}\times\left[\cos\left(\frac{1}{3}\times\cos^{-1}\left[1-2\times(0.40)^2\times\left(1+1.5\right)^{-2}\right]\right)+\frac{1}{2}\right] = 2,75246404$$

For $\beta = 0.40$, table 2 gives: ζ =1.08823734

Thus, Eq. (36c) gives μ_{Th} as:

$$\mu_{Th} = \frac{1.08823734}{0.40\times 2,75246404^{3/2}} = 0,59577346$$

Applying SIA Eq. (34), the discharge coefficient μ_{Exp} is as:

$$\mu_{Exp} = \left[0.578+0.037\beta^2+\frac{0.003615-0.0030\beta^2}{h_1+0.0016}\right]\left[1+0.5\beta^4\left(\frac{1}{1+P/h_1}\right)^2\right]$$

$$\mu_{Exp} = \left[0.578+0.037\times 0.40^2+\frac{0.003615-0.0030\times 0.40^2}{0.40+0.0016}\right]\times$$
$$\left[1+0.5\times 0.40^4\times\left(\frac{1}{1+1.5}\right)^2\right] = 0.59293813$$

Thus, the deviation between μ_{Th} and μ_{Exp} is:

$$\Delta\mu/\mu\,(\%) = 100\times\left|\frac{\mu_{Exp}-\mu_{Th}}{\mu_{Exp}}\right| = 100\times\left|\frac{0.59293813-0.59577346}{0.59293813}\right| \approx 0.478$$

For $\beta = 0.40$, Kindsvater-Carter proposed the following relationship for the discharge coefficient:

$$\mu_{K-C} = 0.591 + 0.0058\frac{h_1}{P}$$

That is:

$$\mu_{K-C} = 0.591 + 0.0058 \times \frac{0.40}{0.60} = 0.59486667$$

Thus, the deviation between μ_{Th} and μ_{K-C} is:

$$\Delta\mu / \mu\,(\%) = 100 \times \left|\frac{\mu_{K-C} - \mu_{Th}}{\mu_{K-C}}\right| = 100 \times \left|\frac{0.59486667 - 0.59577346}{0.59486667}\right| \approx 0.152$$

STUDY OF THE SUPPRESSED WEIR

As mentioned earlier, suppressed weir is obtained for $\beta = 1$, i.e. $B = b$. The device extends over the entire width of the channel (Fig. 1). It is characterized only by its height P, its width being that of the channel. The flow rate is expressed by Eq. (33) where the discharge coefficient μ has been investigated by several research workers. The discharge coefficient has been determined experimentally and relationships have been proposed. The main ones are listed in the next section with which the theoretical formula will be compared and possibly corrected. The theoretical discharge coefficient relationship will also be adjusted to the experimental tests available in the literature (Ramamurthy et al., 1987; Kandaswamy, 1957).

Available formulas for the discharge coefficient

Bazin's formula (1888)

According to Bazin, the flow rate is given by the following relation, identical to Eq. (33):

$$Q = \frac{2}{3}\mu_B B\sqrt{2g}\,h_1^{3/2} \quad (33a)$$

where the subscript "*B*" denotes "Bazin". The discharge coefficient μ_B is expressed as:

$$\mu_B = \frac{3}{2}\left[0.405 + \frac{0.003}{h_1}\right]\left[1 + \frac{0.55}{\left(1 + P/h_1\right)^2}\right] \quad (38)$$

The following inequalities represent the scope of Eq. (38), i.e. the field of its applicability: $0.10m < h_1 < 0.60m$; $0.20m < P < 2m$

SIA formula (1924)

As for Bazin, the flow rate formula proposed by the SIA is such that:

$$Q = \frac{2}{3}\mu_{SIA} B\sqrt{2g}h_1^{3/2} \tag{33a}$$

where μ_{SIA} is given by Eq. (34) for $\beta = 1$ resulting in:

$$\mu_{SIA} = \left[0.615 + \frac{0.000615}{h_1 + 0.0016}\right]\left[1 + \frac{0.5}{\left(1 + P/h_1\right)^2}\right] \tag{34a}$$

Rehbock formula (1929)

The flow rate formula proposed by Rehbock differs somewhat from Eq. (33). Rehbock suggests adding a constant to the measured depth h1 in order to account for the effect of surface tension. The discharge formula is expressed as:

$$Q = \frac{2}{3}\mu_R B\sqrt{2g}\left(h_1 + 0.0011\right)^{3/2} \tag{39}$$

where the subscript "*R*" denotes "Rehbock". Eq. (39) can be rewritten as:

$$Q = \frac{2}{3}\mu_R B\sqrt{2g}h_1^{3/2}\left(1 + \frac{0.0011}{h_1}\right)^{3/2} \tag{39a}$$

Thus, one may write:

$$\mu_0 = \mu_R\left(1 + \frac{0.0011}{h_1}\right)^{3/2} \tag{40}$$

Rehbock gives μ_R as:

$$\mu_R = 0.611 + \frac{0.08\,h_1}{P} \tag{41}$$

Hence:

$$\mu_0 = \left(0.611 + \frac{0.08\,h_1}{P}\right)\left(1 + \frac{0.0011}{h_1}\right)^{3/2} \tag{42}$$

Rehbock's formula should be used within the following usage limits:

$0.03m < h_1 < 0.75m$; $P > 0.10$m; $h_1 / P < 1$.

Some authors claim that Eq. (41) is applicable for values of h_1 / P up to 5. These same authors have proposed relations applicable for values of h_1 / P greater than 15, but do not say in what interest. Eq. (41) has been proposed to measure flow rate with reasonable weir dimensions as mentioned above. If we consider in practice a value of P of 0.70m, a ratio h_1 / P = 15 would result in a flow depth above the weir crest such that h_1 =15 x 0.70 = 10.5m. Conversely, assuming a practical value h_1 = 0.40m, a ratio h_1 / P = 15 would result in a weir height P = 0.40/15 = 2.7 cm. So, one can reasonably wonder in which practical cases can these two examples be encountered. They are not found either in municipal facilities, nor in nature, or in industry. As a result, the relationships proposed by these authors will not be cited herein because they are of no practical interest.

Correction of the theoretical discharge coefficient relationship

For $\beta = 1$, Eq. (15) becomes:

$$h_1^{*3} - \frac{3}{2}h_1^{*2} + \frac{1}{2(1+P/h_1)^2} = 0 \tag{43}$$

Eqs. (26), (27), and (28) are expressed as:

$$z_1 = \left[\cos\left(\frac{1}{3}\cos^{-1}\left[1-\left(1+P/h_1\right)^{-2}\right]\right) + \frac{1}{2}\right] = h^*_{1;1} \tag{44}$$

$$z_2 = \left[\cos\left(\frac{1}{3}\cos^{-1}\left[1-\left(1+P/h_1\right)^{-2}\right] + 240°\right) + \frac{1}{2}\right] = h^*_{1;2} \tag{45}$$

$$z_3 = \left[\cos\left(\frac{1}{3}\cos^{-1}\left[1-\left(1+P/h_1\right)^{-2}\right] + 120°\right) + \frac{1}{2}\right] = h^*_{1;3} \tag{46}$$

For $\beta = 1$, Eq. (36a) reduces to:

$$\mu_{Th} = \frac{3\sqrt{2}}{4\,h_1^{*3/2}} \tag{47}$$

Eq. (47) must be corrected for the effect of h_1 / P as:

$$\mu_{Th} = \frac{f(h_1 / P)}{h_1^{*3/2}} \tag{48}$$

The correction should be done so that Eq. (48) would be in agreement with available experimental data (Ramamurthy et al., 1987; Kandaswamy, 1957) and with the experimental relationships (38), (34a), and (42) which are recognized for their reliability because they result from an intense experimental program.

Calculations show that μ_{Th} expressed by Eq. (48) can be rationally written as:

$$\mu_{Th} = \frac{1.1244 + 0.0768(h_1 / P)}{h_1^{*3/2}} \tag{49}$$

Eq. (49) is in good agreement with both experimental data and the relationships mentioned above. The deviation between Eq. (49) and Rehbock formula is less than 1% for the large values of P / h_1 ratio, while for low values of P / h_1 ratio the deviation does not exceed 0.3%. Eq. (49) would also be in accordance with the SIA formula [Eq. (34a)] with a deviation not exceeding 0.25% in all cases. Regarding Bazin's formula, the calculation showed that the deviation is less than 1%.

Note that for low values of h_1 / P ratio, i.e. $h_1 \to 0$ or $P \to \infty$, Eq. (43) gives:

$$h_1^* = 3/2$$

Inserting this value into Eq. (49) results in:

$$\mu_{Th} = \frac{1.1244}{(3/2)^{3/2}} = 0.612$$

This value is practically equal to that determined by Kirchhoff's free-stream theory $\pi / (\pi + 2) = 0.611$ (Kirchhoff, 1869), later extended by Michell (1890), corresponding to the contraction coefficient of an ideal jet issuing from a rectangular slot in a large tank, without energy loss and practically insignificant deflection under gravity effect.

CONCLUSIONS

Rectangular weirs with and without lateral contraction have been examined from a theoretical point of view. The energy equation was applied between two carefully chosen sections with the hypothesis that a critical state prevails at the location of the weir. The theoretical development led to a third degree equation whose variable is closely related to the discharge coefficient. The equation showed that the discharge coefficient is a function of the relative height of the weir as well as the contraction rate of the contracted device as the experiment predicted. The discharge coefficient of the suppressed weir was deduced by writing that the rate of contraction is equal to unity. The theoretical discharge coefficient was compared with the experimental results provided by the literature as well as with the well-established experimental relationships of some research workers. A deviation of 4% to 5% was observed between the theoretical and experimental discharge coefficients, which led to correct the theoretical equation. This deviation could be attributed to the assumptions made with regard to head loss and pressure distribution of the flow passing over the weir. Regarding the contracted weir, the values of the correction factor of the discharge coefficient have been tabulated for a wide range of the contraction rate. On the other hand, with regard to the suppressed weir, the theoretical discharge coefficient has been corrected by a function depending on the relative height of the weir.

The numerous calculations carried out have clearly shown that the corrected theoretical relationship of the discharge coefficient is in perfect conformity with the experimental data.

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