



STUDY OF A STILLING BASIN WITH A SWIRLING FLOW

ETUDE D'UN DISSIPATEUR D'ÉNERGIE AVEC RECIRCULATION DE L'ÉCOULEMENT

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ABSTRACT

The stability of a rectilinear axis dissipator can be evaluated on the basis of the study of the hydrodynamic forces, in terms of pressure and pressure fluctuation acting on the raft downstream.

This article analyzes the operation of existing designs of devices for dissipating excess energy from water discharges with regard to the hydraulic structures. The simplest and most widely used design for dissipating flow energy are straight-axis, prismatic or trapezoidal stilling basins, but the main drawback of the latter is high material consuming and the requirement for ensure a uniform distribution of unit flows in the inlet section, which makes them ineffective for circular bottom spillways.

A new model is proposed in this article with the aim of minimizing the agitation of the stilling basin flow in the form of a spiral by shortening the length of the basin, and simultaneously creating the possibility of a planned intensive spreading of flow in the

outlet section of the water basin, allowing it to be used when a multiple widening of the flow behind the spillway is required. This model not only promotes a more stable flow in the area near the dissipator, but also dissipates the energy of streams with any hydraulic characteristics.

Keywords: Enlarged dissipation basin, flow disturbance, energy dissipation, swirling flow, excess flows.

RESUME

La stabilité d'un dissipateur à axe rectiligne peut être évaluée sur la base de l'étude des efforts hydrodynamiques, en termes de pression et de fluctuation de pression, agissant sur le radier en aval.

Cet article analyse le fonctionnement des conceptions existantes de dispositifs de dissipation d'énergie excédentaire des rejets d'eau au niveau des structures hydrauliques. La conception la plus simple et la plus utilisée pour amortir l'énergie d'écoulement sont les bassins de tranquillisation à axe rectiligne, de forme prismatique ou trapézoïdale, mais le principal inconvénient de ces derniers est une consommation de matière importante et l'exigence d'assurer une répartition uniforme des débits unitaires dans la section d'entrée, ce qui les rend inefficaces pour les déversoirs à fond circulaire.

Un nouveau dispositif est proposé dans cet article dans le but de minimiser les agitations de l'écoulement du bassin de tranquillisation sous la forme d'une spirale en raccourcissant la longueur du bassin, et en créant simultanément la possibilité d'un étalement intensif planifié de l'écoulement dans la section de sortie du bassin d'eau, ce qui lui permet d'être utilisé lorsqu'un élargissement multiple du débit derrière le déversoir est nécessaire. Ce dispositif favorise non seulement un écoulement plus stable dans la zone proche du dissipateur, mais aussi dissipe l'énergie des cours d'eau avec n'importe quelles caractéristiques hydrauliques.

Mots clés : Bassin de dissipation élargi, perturbation d'écoulement, dissipation d'énergie, recirculation de l'écoulement, débits excédentaires.

INTRODUCTION

Energy dissipation structures are a good way to minimize the extreme forces caused by high velocity flows and restore the proper flow to the riverbed (Rajan and Shivashankara Rao, 1980; Tung and Mays 1982; Lopardo et al., 1985). However, the energy dissipation generates a large macro-turbulence field with which are associated severe fluctuations in velocity, water level and shear forces liable to generate erosion which can sometimes go as far as destruction neighboring structures (Lavallée et al., 2000; Armenio et al., 2001). Almost all hydraulic support structures, erected on natural streams, have weirs designed

for the slow release of excess flows. The discharge of treated water from the high-load reservoir inevitably leads to erosion of the channel downstream of the structure and its destruction.

To avoid this risk, it is planned to dissipate excess energy from the flow in all hydropower structures. In medium and low pressure hydrosystems, energy dissipation in a stilling basin is mainly used. In the simplest case, the latter is a prismatic structure with vertical walls, the bottom of which is equal to the bottom of the channel of the river thus ensuring the energy dissipation of the maximum water flow, which is determined by appropriate calculations (Rozanov 1985, Chertaousov 1962, Zuikov 2015, Agroskine 1964, Volchenkov 1992).

The drawback of prismatic stilling basins is the presence of high flow per unit width in the outlet section, equal to the value of flow per unit width at the inlet. Since the spillway is the most expensive structure in the hydroelectric complex, efforts are made to make it as small as possible, which is ensured by determining the maximum possible unit flows. Ultimately, this leads to the need to increase the length and depth of the stilling basin, the thickness of its walls and bottom, as well as to increase the capacity of the protective structure of the section of water canal adjacent to the stilling basin against erosion.

One of the ways to increase the efficiency of the use of stilling basin is to use enlarged sections (Chertaousov 1962, Zuikov 2015, Agroskine 1964). The most effective application of these basins is when the bottom pipes are used as spillways (Rasskazov 1996, Zuikov 2015, Volchenkov 1992, Emtsev 1967).

In this case, it is possible to increase the width of the outlet section of the stilling basin by 2 to 3 times, respectively, reducing the value of the unit flow at the outlet of the basin and reducing the risk of erosion by downstream.

One of the major disadvantages of stilling basins enlarged in plan is the limitation of the angle ϑ of the side walls. Exceeding the angle ϑ leads to the appearance of a disturbed flow in the basin and the limitation of its functioning as an energy dissipator.

Expanded stilling basins are particularly susceptible to this problem when used as an energy dissipating structure of the discharge lines to a plurality of tubes, since when the flow rates are lower than those calculated, it is impossible to ensure a distribution uniform unit flow rates in the inlet section.

When the flow propagates freely in the horizontal plane, the angle ϑ of the lateral widening according to Agroskine (1964), Emtsev (1967) and Slissky (1986) can be determined from the dependence:

$$\vartheta_{Lateral} = \arcsin \frac{1}{\sqrt{Fr_{CS}}} \quad (1)$$

Where Fr is the Froude number calculated in the contracted section at the inlet of the stilling basin by the following relation:

$$Fr_{CS}^2 = V_{CS}^2 / gH_{CS} \quad (2)$$

Where:

V_{CS} = velocity of flow in contracted section of stilling basin;

g = gravity acceleration;

H_{CS} = depth in the contracted section of stilling basin.

At the limit of the jet defined by expression (1), the flow depth is zero at the wall and the jet has the maximum water depth on the axis.

When calculating the parameters of the water basin, they are guided by the average parameters of the jet characteristics (the average depth and the average velocity over the cross section). Accordingly, when the threshold of the stilling basin is reached, such a jet will gush in the center with the possibility of ejection into the outlet conduit with the formation of a disturbed flow. (Chertaousov 1962, Zuikov 2015, Agroskine 1964, Volchenkov 1992).

To avoid this undesirable effect and to obtain approximately averaged parameters, as shown by the practice of model studies, the maximum angle with the flow axis of the side wall of the stilling pond should not exceed the value $\vartheta_{max} \leq 7^\circ$ according to Zuikov (2015) and Agroskine (1964) with free flow spreading and $\vartheta_{max} \leq 12^\circ$ with supported flow spreading (Volchenkov 1992, Novak 2003).

The present work analyzes the operation of existing designs of devices for energy dissipation excess energy from water discharges by proposing a new model in order to minimize the agitation of the flow of the stilling basin.

For example, one of the used design is that of Volga-Don Canal 2, which was designed in the 80's to compensate for water loss from the Don River to the lock when navigating along the Volgo-Don channel (Technical Report 1957). On the outline of the Volgo-Don 2 canal, it was planned to build 3 waterfalls with a height of about 20 meters each with a calculated channel flow $Q_{calculated} = 393 \text{ m}^3/\text{s}$ and a width of the spout $b = 18.0$ meters, the length of the hydraulic jump is $L = 46\text{m}$, and at a flow angle of $\vartheta_{max} = 14^\circ$ at the outlet of the extension, the flow width is $B = 29 \text{ m}$. The width of the water channel was 105 meters.

According to studies on physical models, the interaction of the inlet channel flow with that of the stilling basin leads to the formation of vortex zones. This generates an alternating downstream flow pattern (in sinusoidal form) between the two walls resulting in the formation of erosion zones of the bed, which is inadmissible and to be avoided.

With a planned free expansion of the flow in a stilling basin with an enlargement angle of $\vartheta_{max} = 14^\circ$, it would be necessary to create an enlarged basin of length L such that:

$$L = \frac{B - b}{2tg\vartheta_{\max}} = \frac{150 - 18}{2tg7^{\circ}} = 354m. \quad (3)$$

Which is unrealistic for execution.

This work follows from an earlier study published in 2021 (Gouryev et al 2021).

EXPERIMENTAL SETUP

A solution has been proposed according to the recommendations of Ghivotovski, Gouryev and Elenson (Ghivotovski 1984, Report Sc.1982, Report Djedra 2003) to avoid these problems of disturbance and formation of vortex zones. These technical solutions represent an enlarged stilling basin whose threshold is paired with the side walls by cylindrical surfaces which return the corresponding parts of the flow to its inlet section. At the entrance to the stilling basin, two ridges are made, adjacent to the side walls.

The proposed design of a stilling basin with helicoidally flow (with swirling) was developed and tested on a 1:40 scale physical model for the Nizhne-Kafir Niganskaya hydroelectric power station in Tajikistan, later on a scaled-down model. 1:40 of the Volgo-Don canal and in the version of the Djedra dam in Algeria on a scale model of 1:60. This model works as follows: In the case of a strongly sheared flow at the bottom of the basin, the flow of the spillway along the bottom between the side faces reaches the cutting edge, on impact on which it is divided into two parts, moving to the side walls.

Arriving on the curvilinear sections of the sharp edge, these halves of the flow return to the inlet section of the basin, fall on the inclined end faces up to their upper horizontal faces. Then, moving in the direction of the inlet section, collide and interact simultaneously with the flow entering the stilling basin between the corners of the vertical walls.

At the meeting point of the return streams, their collisions, mutual dissipation of energy and their interaction with the flow of the steed occur according to Faktorovich (1956). As a result of this interaction, a reverse flow above the bottom flow is formed. This flow forms a return flow above the transit jet, which forms a direct branch of the vortex at the second stage of the flow.

On entering the stilling basin's dissipation zone, the expected widening of this flow and its exit into the outlet channel is over its entire width. Indeed, with this design of the stilling basin, the hydraulic jump is a helical line wound in space which allows to physically reduce the length of the basin, to dissipate the energy more intensely due to the impact of the net on the basin threshold and the interaction of the return nets and during the return movement in the vortex of circulation.

According to Ghivotovski et al. (1984), in such a still basin structure, the widening angle can have different values depending on the design reasons.

Fig. 1 shows a photo of a still-basin model with recirculating flow at 1:40 scale, which was developed in the design at overflow of the Volgo-Don 2 channel (Report Volgo-Don 1982).

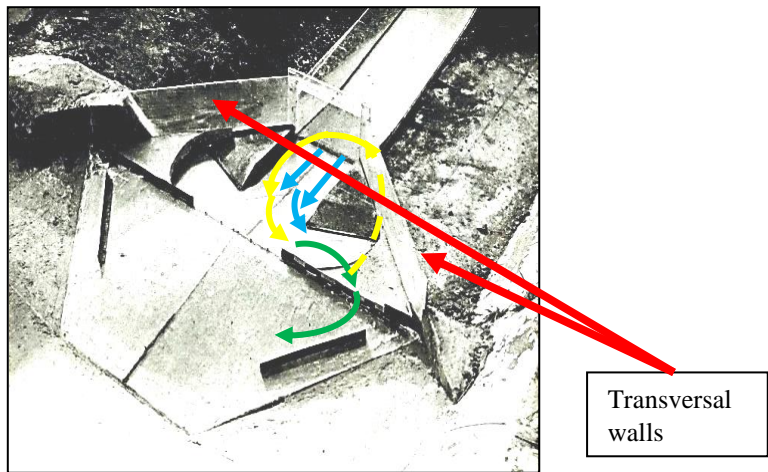


Figure 1: Photo of a model with a flow of the Volgo-Don canal 2. The direction of flow is shown by the arrows.

- ➡ current from the bottom of spillway;
- the climb of the higher jet to the surface of the flange;
- rotation of the flow on the 2nd stage towards the outlet channel
- flow spread to the second floor and exit to the outlet channel directed by installed transversal walls.

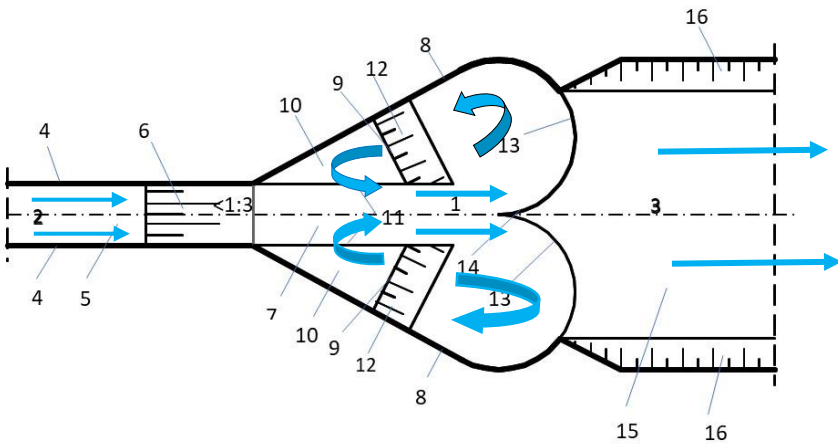
The same phenomenon appears on the right side.

Fig. 2 shows a photo of how this model works when the calculated flow rate is $Q = 393\text{m}^3/\text{s}$. As can be seen in the photo of figure 2, in the center of the stilling basin threshold, a vertical ejection of water is formed, which forms a hydraulic jump. To reduce this increase, it was necessary to install transverse walls, visible in the photo in Fig. 1. (Gouryev et al. 1986).

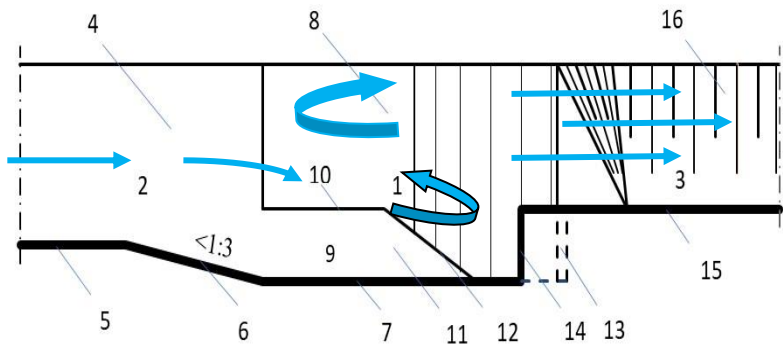


Figure 2: Photo of a model with a swirling flow from the Volga-Don 2 canal.

Flow rate $Q = 393\text{m}^3/\text{s}$.



a) Plan view with a swirling flow.



b) Longitudinal section along the axis of basin.

Figure 3: stilling basin with curvilinear threshold.

The indications of experimentation model are as follows:

1 - stilling basin; 2 - water supply pipe; 3 - outlet water pipe; 4 and 5 - wall and bottom of the supply conduit; 6 - bottom end section of the supply conduit; 7 - the bottom of the basin; 8 - side walls of the stilling basin; 9 -ledge; 10 - horizontal edge of the rim; 11 - inner edge of the flange;12 - inclined face of ledges; 13 - curvilinear section of the basin threshold; 14- sharp edge of the troughs; 15 and 16 - the bottom and side walls of the outlet conduit.

To eliminate flow agitation in the stilling basin with an uneven distribution of unit flows in the inlet section, the design of the basin with the flow circulation of the Volgo -Don 2, has been modified, whose main parameters are presented in (Report Sc. 1982).

The distinction consisted in the fact that the threshold of stilling basin 1 was produced in the plane in the form of two curved vertical surfaces 13 (figure 3) intersecting on the axis of the structure, located with a convexity towards the outlet conduit 3 and mating with the wall side 8 of the stilling basin, forming a sharp edge 14 on the axis of the basin modified, including the main parameters which are exposed in (Report Sc. 1982).

With this implementation of the threshold, the bottom flow is divided into two jets with their shock-free mating with its curvilinear surfaces and the reversal of these jets to the inlet section of the threshold. The advantage of this execution of the threshold of the stilling basin is with an uneven distribution of unit flows in the inlet section, the part of the flow which represents the major part of the flow, for example the left, forms a more powerful vortex.

Under the action of the concurrent flow of the vortex, all the lower flow is expelled from the side of the section with a lower flow, which leads into the second half of the basin, in this case, the one on the right, the vortex intensifies and the moment of flow stabilization

occurs, at which the flow past the threshold is automatically centered, and the transit flow entering the outlet conduit forms a symmetrical velocity profile.

Thus, a flow with negative feedback is formed in the stilling basin: more the jet creates an intense vortex; more the vortex pushes it back with a lower part of the flow.

In the Nizhne-Kafirniganskaya hydroelectric power station project (Tajikistan), a bottom spillway was planned, consisting of a pipe of cross section $b \times h = 4 \times 5$ m with intermediate ledges with a thickness of $t = 1.5$ (m). The energy dissipation has been provided in a stilling basin enlarged with a length of $L_{\text{basin}} = 90.0$ m and a depth of 19 m with a change in plane width from 15.0 to 50.0 meters.

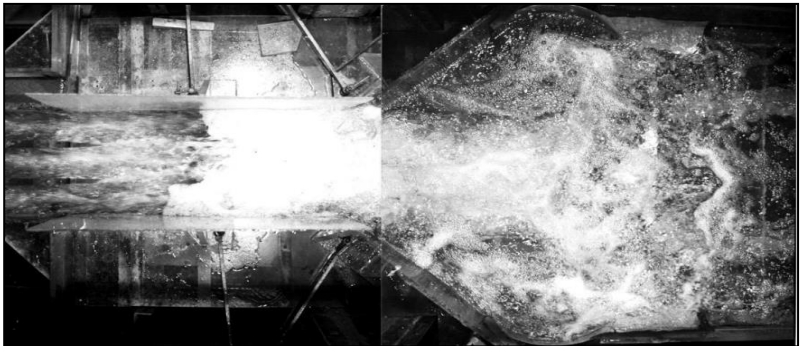
Tests on physical models of this basin have revealed its inability to operate with an uneven distribution of flow in the inlet section.

When operating under a flow rate of the spillway $Q = 322 \text{ m}^3/\text{s}$ and a calculated head of $H = 67\text{m}$, a powerful one-way vortex was formed, due to which the flow, without dissipation of energy reaches the threshold of the stilling basin and sprung in the form of a flowing range in the outlet water conduit. As can be seen in the photo of Fig. 4, the flow enters the discharge channel in the form of a unilateral range.

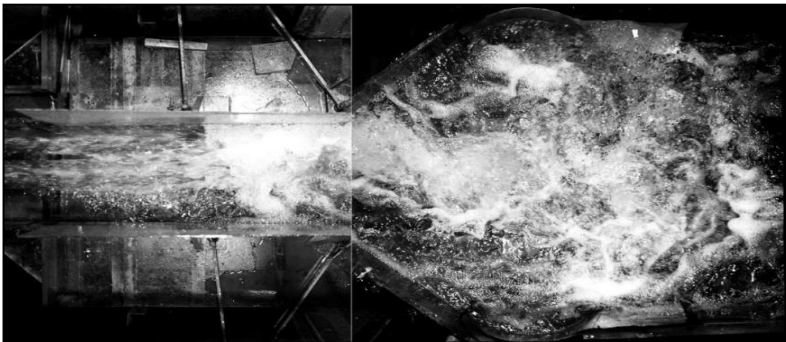


Figure 4: The operation of a stilling basin $Q = 322\text{m}^3/\text{s}$.

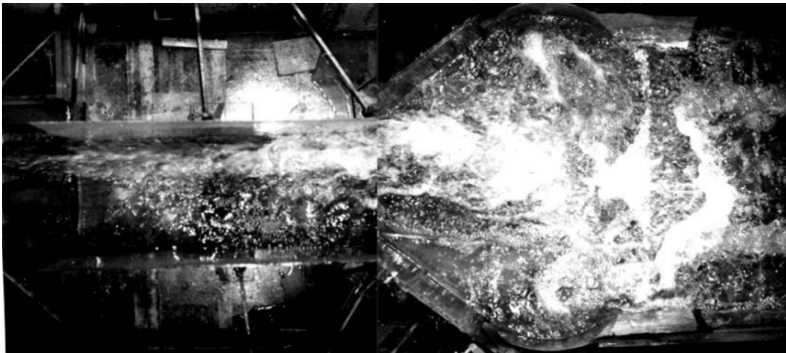
Fig. 5 shows a photo of the stilling basin operation of the Nizhne-Kafirniganskaya operational spillway, consisting of 3 pipes of cross section $b \times h = 4.0 \times 5.0$ m with a wall thickness of $t = 1, 5$ m, the entry width was $b = 18.0$ m and the exit width $B = 50.0$ m.



a) Operation of three tubes.



b) Operation of two tubes.



c) Operation of one tube $b \times h = 4.0 \times 5.0$ m, passing $322 \text{ m}^3/\text{s}$ each under pressure head, $H=67$ m.

Figure 5: Photo of the stilling basin operation with a swirling flow.

From Fig. 5 it can be seen that for all the operating modes of the spillway at the outlet of the basin with recirculation of the flow and a curvilinear threshold, a symmetrical velocity profile is observed in the inlet section of the conduit output, even in such a hard mode with only one pipe. This mode seems particularly interesting compared to the end pipe operation in a stilling basin with rectilinear axial enlargement illustrated in the photo of Fig. 4.

Figure 6 shows photos of the scale model of the rectilinear stilling basin and the model with an enlarged stilling basin with flow recirculation for the design of the Djedra dam (Algeria) of two different flows ($Q_p = 377 \text{ m}^3/\text{s}$; $Q_p = 638 \text{ m}^3/\text{s}$).

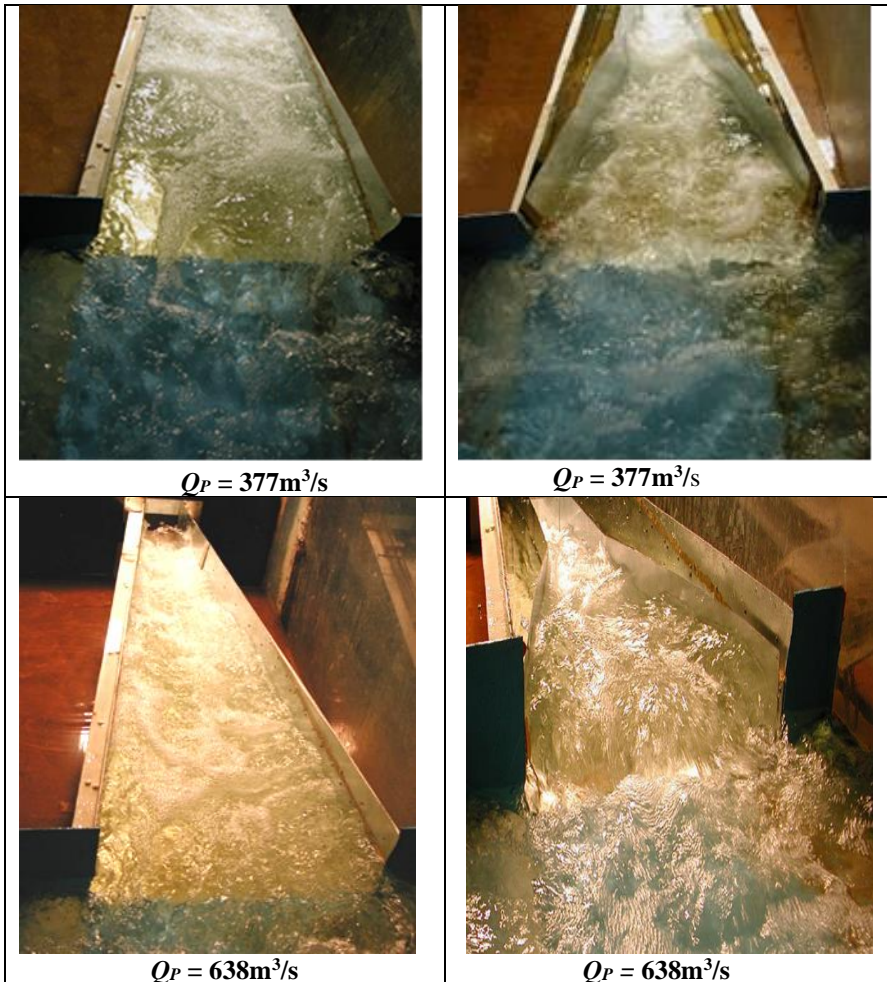
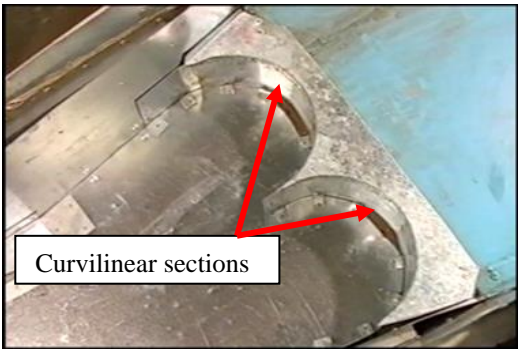


Figure 6: Scale models of the stilling basin of the Djedra dam (Algeria). (Scale 1: 60)

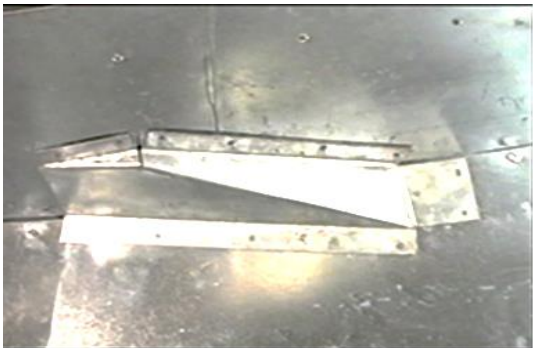
RESULTS AND DISCUSSION

In the study of the operation of shaft spillway of the Djedra dam (Algeria), as well as in the design version of its stilling basin, this design of the enlarged basin was considered (Djedra Report 2003). As part of the project, an experimental model was developed at the Research Laboratory of Hydroelectric Power Stations of the Moscow State University of Environmental Engineering - Russia (Djedra Report 2003). The stilling basin of 89.0 m length and width ranging from 6.0 m inlet section to 30.0m outlet section.

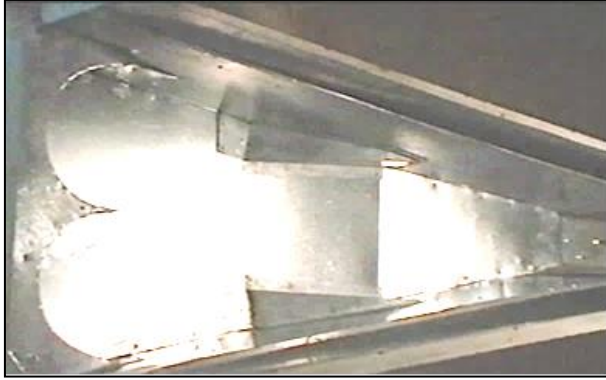
The stilling basin with swirling flow (Fig.7) had a length of 54.0 m, taking into account the length of the transition section. The flow calculated with probability $P = 0.1\%$ was $Q_{p = 0.1\%} = 377 \text{ m}^3/\text{s}$, taking into account the transformation of the flood hydrograph by the reservoir, and the transformed flow with probability $P = 0, 01\%$ was $Q_{p = 0.01\%} = 638 \text{ m}^3/\text{s}$. The unit flow rate (for one pipe) at the inlet of the stilling basin was $q = 638/6 = 106 \text{ m}^3/\text{s}$ compared to $q = 53.7 \text{ m}^3/\text{s}$ of the spillway of the Nizhne-Kafirniganskaya hydroelectric power station (Russia).



a) Curvilinear section



b) Edge for recirculation of the flow;



c) Channel with new model.

Figure 7: Photo of the scale model with recirculation of the flow.

Comparison of the velocity diagrams of Figure 8 shows that a stilling basin with swirling flow provides a more favorable hydraulic flow regime downstream. Nevertheless, in the final version of the Djedra dam project, an enlarged linear stilling basin was adopted, since in the practice of hydraulic engineering there are no structures built with swirling flow.

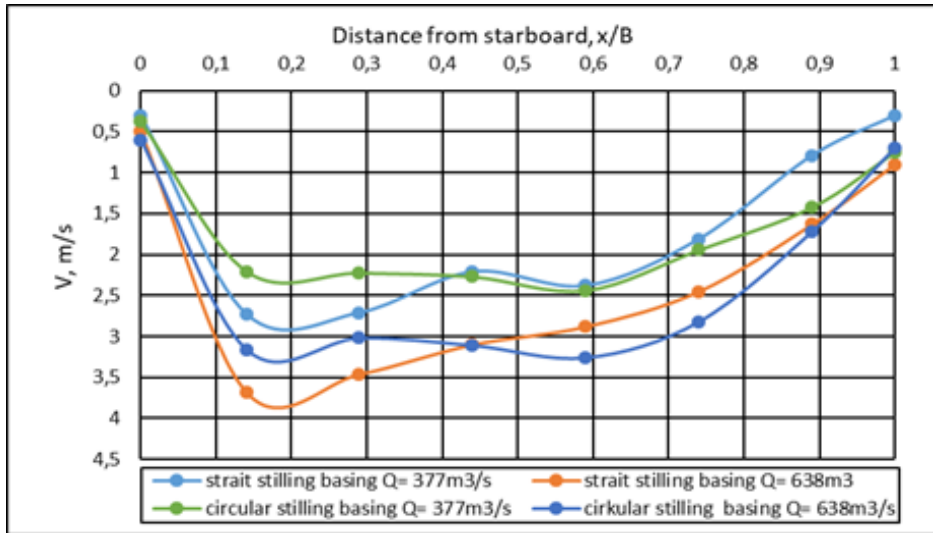


Figure 8: Comparison of flow velocity profiles.

- Calculated flow $Q_{\text{calculated}, p = 0.01\%} = 377 \text{ m}^3/\text{s}$;
- Transformed flow $Q_{\text{Transf.}, p = 0.01\%} = 638 \text{ m}^3/\text{s}$.

At present, no works has been found demonstrating the functioning of such a structure and the theoretical justification of its parameters. Therefore, the preliminary parameters can be taken according to (Report Sc 1982):

- Basin depth h_B , in accordance with the calculation of the prismatic stilling basins;
- The length of the basin $L_B = 0.5 L_{BP}$ length of the prismatic basin;
- The height of the inner edges of the $h_{edges} = 2h_{Cr}$ at the entrance to the stilling basin;
- The length of the inner edges of $L_{edges} = (2/3) \cdot L_B$ of the length of the basin;
- The slope of the inclined edges of the rim is from 1: 1 to 1: 1.5;
- The inclination of the slope of the end section of the bottom of the supply conduit $< 1: 3$;
- Radius of curvature of the curved sections of the threshold $R = (0.2 \dots 0.25) B$ of the width of the basin.

The efficient operation of a stilling basin with swirling flow is only possible if the bottom flow regime of the spillway is guaranteed. The coupling mode of the spillway flow and the flow in the basin can be determined according to the recommendations cited by Slissky (1970) and Kissilev (1972).

The distinction of the parameters of this type of basin must be clarified on the hydraulic model after the determination of the preliminary parameters.

CONCLUSIONS

The analysis of all the results allowed us to conclude that with all the advantages of prismatic stilling basins, the impossibility of their use in structures with unequal distribution of flows, limits the field of their application in hydraulic engineering. In addition, the great length of the rectilinear axial enlargement of the stilling basin and the need for them to support large dynamic loads lead to the need to use a large amount of materials during their construction. However, stilling basins with swirling flow can halve the length and material consumption of the basin and the smooth assembly of the flow with minimizing dynamic loads on its structural elements. Also, these tranquilizer basins are applicable to dissipate the energy of streams with any hydraulic characteristics.

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