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NEW THEORETICAL CONSIDERATIONS ON THE GRADUALLY VARIED FLOW IN A WIDE RECTANGULAR CHANNEL

NOUVELLES CONSIDERATIONS THEORIQUES SUR L'ECOULEMENT GRADUELLEMENT VARIE EN CANAL RECTANGULAIRE LARGE

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ABSTRACT

The computation of backwater curves is often encountered in various applications of hydraulic engineering, especially those related to the gradually varying flow in open channels. The knowledge and the mastery of the varied flow condition the correct sizing of the canal.

The objective of our study is to contribute to the establishment of an analytical protocol, as simple as possible, with the aim of proposing a generalized relation to the calculation of the gradually varied flow in a wide rectangular channel. The introduction of new dimensionless parameters leads to a differential equation different from those proposed in previous works.

The attention will focus mainly on the cases of horizontal and critical channel slopes given their interest on the mathematical side of the problem.

Keywords: Gradually varied flow, GVF, backwater curve, drawdown curve, slope, wide rectangular channel, control section, analytical solution.

RESUME

Le calcul des courbes de remous est souvent rencontré dans diverses applications de l'ingénierie hydraulique, en particulier celles liées à l'écoulement graduellement varié dans les canaux ouverts. La connaissance et la maîtrise des écoulements variés conditionnent le dimensionnement correct du canal.

L'objectif de notre étude est de contribuer à l'établissement d'un protocole analytique, le plus simple possible, dans le but de proposer une relation généralisée au calcul de l’écoulement graduellement varié dans un canal rectangulaire de grande largeur. L'introduction de nouveaux paramètres sans dimension conduit à une équation différentielle différente de celles proposées dans le passé.

L'attention sera portée principalement sur les cas de pentes horizontales et critiques du canal compte tenu de leur intérêt au niveau mathématique du problème.

Mots clés : Ecoulement graduellement varié, courbe de remous, courbe de rabattement, pente, canal rectangulaire large, section de contrôle.

INTRODUCTION

The term "Backwater Curve" is used herein as the longitudinal profile of the water surface in a non-uniform steady flow in an open prismatic channel (Chen, and Wang, 1969; Valentine, 1967). The flow depth varies gradually along the length of the channel. In fact, when the depth of water increases in the direction of flow then the surface profile is classified as backwater curve and when it decreases then it is known as drawdown curve. Non-uniform or varied flow occurs when the water depth as well as the other hydraulic parameters, namely the slope, the roughness, and the flow rate, vary from one section to another for a given channel (U.S. Army Corps of Engineers, 1959; U.S. Geological Survey, 1955; Henderson, 1966; Valentine, 1964). It is even found in a uniform channel with a rectilinear axis, of constant slope and section, and of homogeneous roughness, but only in the vicinity of its ends or when there is a singularity such as a weir, a waterfall, a sluice gate, ... etc. The non-uniform flow can be accelerated or decelerated depending on whether the flow velocity increases or decreases in the direction of the flow. Therefore, it can be classified into two categories: gradually varied flow and rapidly varied flow as a hydraulic jump for example (Chow, 1959). When a flow entering a channel is slow, the velocity and the resistance of the flow are low. The gravity forces are then predominant and the flow is then accelerated from upstream. Both velocity and resistance increase as one moves downstream, until the gravity forces are balanced. At this stage, uniform flow appears. Then, the gravity forces become more and more predominant because of the local hydraulic resistances which are downstream of the uniform flow. As a result, the uniform flow disappears and the non-uniform flow appears. The brief passage from a uniform flow to a varied flow which corresponds to a variation of depths and velocities is called a transition zone, the length of which depends on the flow rate as well as the characteristics of the channel such as roughness and slope. In the study of the gradually varied flow, some simplifying assumptions are necessary, assuming that the liquid current is rectilinear and parallel to the bottom of the channel. Therefore, the transverse velocities are neglected with respect to their longitudinal components parallel to the general direction of flow. It is considered in any section that the liquid stream is uniform with the same depth and the same flow rate, which amounts to saying that the Manning-Strickler or Chezy formulas are applicable in gradually varied flow regime, relationships which, strictly speaking, are only valid for the uniform flow regime. This assumption simplifies the calculation such as that of the friction slope S_f as well as the critical slope S_c. The errors associated with this assumption are considered quite small. The validity of the hypothesis is all the more justified as the varied motion approaches the uniform regime. It is also assumed that all the velocities are assumed to be equal to the average velocity in a given section of the channel and that the slope S_0 of the latter is quite low, meaning that the depth of flow is the same in both vertical and normal directions. Another simplifying assumption is that the coefficient of resistance to flow is independent of depth and remains constant along the entire reach of the channel. It is now accepted that this hypothesis is not realistic since it has been shown that the coefficient of resistance to flow such as that of Chezy or Manning depend in particular on the depth of the flow and many other factors (Achour, 2020; 2015).

Backwater curves, or the water surface profiles, can be classified according to the slope S_0 of the canal. There is, for a given flow rate, a channel slope S_c for which flow occurs at the critical depth. The corresponding backwater curves form the C-type group. Slopes of the canal less than this critical slope will be considered low. It is said to be a mild slope. This will form the M-type backwater curves group. The channel slopes greater than the critical slope are so-called steep slopes and will form the S-type backwater curves group. The horizontal slopes are associated with the type H group, while for the adverse slopes corresponds the H-type group.

The computation of the water surface profiles is based on the principle of energy, that is to say quite simply on the Bernoulli equation applied between two chosen flow sections. It is easily demonstrated that the final result is a differential equation called the gradually varied flow equation expressed as:

$$\frac{dh}{dL} = \frac{S_0 - S_f}{1 - \frac{Q^2 B}{g A^3}} \qquad (1)$$

where Q is the flow rate, B is the channel width, A is the water area, g is the acceleration due to gravity, h is the water depth at a given section, and dh/dL gives the variation of water depth along the channel in the flow direction. Note that dL, which is the distance between two given sections, is taken on the horizontal reference datum and not along the bottom channel.

The study of the relationship (1) allows the construction of the curves of the free surface of the flow. For this, different methods have been proposed in order to proceed with the calculations and the exact construction of the shapes of the free surface. Among the most widespread methods, one may distinguish the following three methods: method by successive approximations, method by direct integration, and method by graphical integration. The first method includes the method of sections where ΔL is fixed as well as a first depth h_1, the depth variation procedure where Δh is fixed, and finally the finite difference method. The direct integration method allows making the differential equation expressed by (1) integrable. By integrating between two sections A_1 and A_2 of respective depths h_1 and h_2, equation (1) becomes:

$$L_2 - L_1 = \int_{h_1}^{h_2} f(h)dh \tag{2}$$

This last equation is difficult to solve analytically because the second member is a complex function. It is quite easy to solve for a few simple cases such as the case of the large width rectangular channel which is of interest to our study. For this case, the preferred calculation methods are those of Bresse (Lencastre, Bakhmeteff (1932) and Chow (1955; 1959).

The graphical integration method puts the differential equation of the gradually varied flow into the following form, provided the flow rate and the channel profile are known:

$$dL = \frac{dh}{f(h)} \tag{3}$$

That is:

$$L_2 - L_1 = \int_{L_1}^{L_2} dL = \int_{L_1}^{L_2} \frac{dh}{f(h)} \tag{4}$$

One thus obtains a first-order differential equation, integrable by means of a constant of integration known thanks to the boundary conditions.

The calculation methods usually used for gradually varied flow, such as those previously indicated, do not take into account the effect of the viscosity of the flowing liquid. As a result, their application would be reserved exclusively for gradually varied flows in rough turbulent regime.

The present study proposes a new theoretical approach allowing the calculation as well as the plotting of the free surface of a one-dimensional flow. The differential equation governing the gradually varied flow is transformed into a function defined by dimensionless terms. The integration of this function leads to a mathematical formulation allowing the direct solution of all the problems of the one-dimensional gradually varied flow in a rough turbulent flow regime. Particular attention is paid to the particular cases of critical and horizontal slopes which are of remarkable mathematical interest.

TRANSFORMATION OF THE DIFFERENTIAL EQUATION FOR GRADUALLY VARIED FLOW

The Manning-Strickler equation gives the mean velocity of a uniform flow in the following form (Strickler, 1923):

$$V = \frac{Q}{A} = k\, R_h^{2/3} S_f^{1/2} \tag{5}$$

where Q is the flow rate, A is the water area, k is the Strickler coefficient, and R_h is the hydraulic radius. Recall that S_0 is the slope of the channel and that S_f is the slope of the hydraulic grade line or the linear hydraulic head loss. For a two-dimensional flow of depth h flowing on a bottom formed by a wide inclined plane of width B, one can write:

$q = Q/B$: The unit flow rate; $A = Bh$: The water area; $P = B$: The wetted perimeter; $R_h = h$: The hydraulic radius.

Taking into account the above parameters, Eq. (5) becomes:

$$V = \frac{Q}{Bh} = \frac{q}{h} = k\, h^{2/3} S_f^{1/2} \tag{6}$$

One may write finally what follows:

$$S_f = \frac{q^2}{k^2\, h^{10/3}} \tag{7}$$

On the other hand, one may write:

$$\frac{Q^2 B}{g A^3} = \frac{q^2}{g h^3} \tag{8}$$

Inserting both Eqs. (7) and (8) into Eq. (1) results in:

$$dL = \frac{1 - q^2/\left(gh^3\right)}{S_0 - \dfrac{q^2}{k^2\, h^{10/3}}}\, dh \tag{9}$$

Knowing that $q^2 = gh_c^3$, where h_c is the critical depth, Eq. (9) can be rewritten, after some arrangements, as:

$$dL = \frac{1 - h_c^3 / h^3}{S_0 - \dfrac{g}{k^2 (h / h_c)^{10/3} h_c^{1/3}}} dh \quad (10)$$

Let us adopt the following dimensionless parameters:

$$h^* = h / h_c ; \; L^* = L / h_c \quad (11)$$

Eq. (10) then becomes:

$$dL^* = \frac{1 - h^{*-3}}{S_0 - \dfrac{g}{k^2 \, h^{*10/3} h_c^{1/3}}} dh^* \quad (12)$$

By multiplying the numerator and the denominator of Eq. (12) by ($k^2 \, h^{*10/3} h_c^{1/3}$), it becomes:

$$dL^* = \frac{\left(1 - h^{*-3}\right) k^2 \, h^{*10/3} h_c^{1/3}}{S_0 k^2 \, h^{*10/3} h_c^{1/3} - g} dh^* \quad (13)$$

For a wide rectangular channel, the critical slope is expressed as:

$$S_c = \frac{g}{k^2 h_c^{1/3}} \quad (14)$$

Let δ be $\delta = 1 / S_c$, which amounts to writing that:

$$\delta = \frac{k^2 h_c^{1/3}}{g} \quad (15)$$

By virtue of Eq. (15) and some arrangements, Eq. (13) becomes:

$$dL^* = \delta \frac{h^{*10/3} - h^{*1/3}}{S_0 \delta \, h^{*10/3} - 1} dh^* \quad (16)$$

Introducing the following dimensionless number:

$$\beta = S_0 \delta = S_0 / S_c \quad (17)$$

Eq. (16) is then rewritten as:

$$dL^* = \delta \frac{h^{*10/3} - h^{*1/3}}{\beta h^{*10/3} - 1} dh^* \tag{18}$$

Integrating Eq. (18) between two relative depths h_1^* and h_2^*, yields:

$$\overset{h_2^*}{\underset{h_1^*}{L^*}} = \delta \int_{h_1^*}^{h_2^*} \frac{h^{*10/3} - h^{*1/3}}{\beta h^{*10/3} - 1} dh^* \tag{19}$$

Consider the following function:

$$F(h^*, \beta) = \int_{h_1^*}^{h_2^*} \frac{h^{*10/3} - h^{*1/3}}{\beta h^{*10/3} - 1} dh^* \tag{20}$$

One may then write what follows:

$$\overset{h_2^*}{\underset{h_1^*}{L^*}} = \delta \left[F(h_2^*, \beta) - F(h_1^*, \beta) \right] \tag{21}$$

That is:

$$\overset{h_2}{\underset{h_1}{L}} = h_c \delta \left[F(h_2^*, \beta) - F(h_1^*, \beta) \right] \tag{22}$$

Thus, it is the function $F(h^*, \beta)$ expressed by Eq. (20) which makes it possible to solve the problems of the gradually varied one-dimensional flow.

THE DIMENSIONLESS PARAMETER β

By definition and for a uniform flow for which $S_0 = S_f$, one may write:

$$h = h_n \tag{23}$$

where h_n is the normal depth.

Dividing the two members of Eq. (23) by h_c results in:

$$h^{*} = h_n^{*} \tag{24}$$

In uniform flow regime, Eq. (7) is written as:

$$S_0 = \frac{q^2}{k^2\, h_n^{10/3}} \tag{25}$$

Eq. (25) can be rewritten as:

$$S_0 = \frac{g\, h_c^3}{k^2 \left(\dfrac{h_n}{h_c}\right)^{10/3} h_c^{10/3}}$$

That is:

$$S_0 = \frac{g}{k^2\, h_n^{*10/3} h_c^{1/3}} \tag{26}$$

Combining Eqs. (15), (17), and (26) yields:

$$\beta = S_0\, \delta = h_n^{*\,-3/10} \tag{27}$$

It is thus shown that the dimensionless parameter β is closely related to the normal relative depth. This implies that β is independent of the depth h and that it is constant for a given case. Eq. (27) has never been established before.

Without having demonstrated, Bakhmeteff (1932) suggests that β is a constant because the S_0 / S_f ratio varies little with depth, according to the author. This is not the case with Chow (1959) who considers that β is not constant, which is an unreliable assertion when referring to Eq. (27). Considering β as being a variable further complicates the solution of the problem as presented by Chow (1959).

The interest and advantage of Eq. (27) lies in the fact that it allows the classification of slopes. This is done as follows:

i) $\beta > 1$, $h_n^{*} < 1$, i.e. $h_n < h_c$

This means that the normal flow regime is supercritical and the slope is of steep type (S).

ii) $\beta = 1$, $h_n^{*} = 1$, i.e. $h_n = h_c$

This means that the normal flow regime is critical, which implies that the slope is of the critical type (C).

iii) $0 < \beta < 1$, $1 < h_n^{*} < \infty$, $h_c < h_n < \infty$

This means that the normal flow regime is subcritical, which implies that the slope is of the Mild type (M).

iv) $\beta = 0,\ h_n^* = \infty,\ \delta \neq 0$, so $S_0 = 0$

The normal flow regime does not exist and the slope is horizontal.

v) $\beta < 0,\ h_n^* < 0$, so $h_n < 0$ and $S_0 < 0$

The normal flow regime does not exist and the slope is adverse (A).

SPECIAL CASES

Horizontal slope

This case corresponds to $S_0 = 0$ and Eq. (16) becomes:

$$dL^* = \delta\left(h^{*1/3} - h^{*10/3}\right)dh^* \tag{28}$$

Integrating this equation between two relative depths h_1^* and h_2^* results in:

$$\int_{h_1^*}^{h_2^*} dL^* = \delta \int_{h_1^*}^{h_2^*} \left(h^{*1/3} - h^{*10/3}\right)dh^* \tag{29}$$

which gives:

$$\overset{h_2}{\underset{h_1}{L}} = h_c\,\delta\left(\frac{3}{4}h^{*4/3} - \frac{3}{13}h^{*13/3} + C_0\right)_{h_1}^{h_2} \tag{30}$$

Eq. (30) can be rewritten as:

$$\overset{h_2}{\underset{h_1}{L}} = h_c\,\delta\left[F(h_2^*,0) - F(h_1^*,0)\right] \tag{31}$$

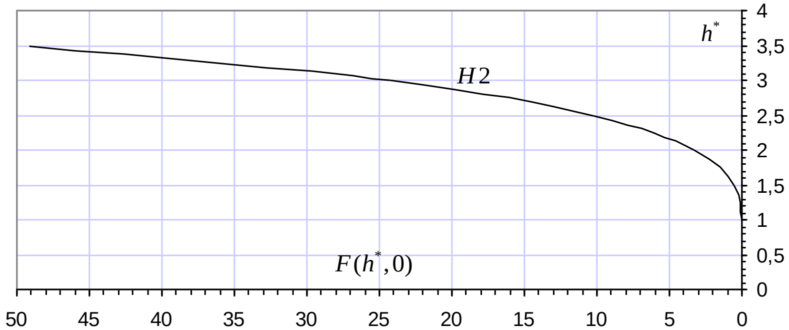
where:

$$F\left(h^*,0\right) = \frac{3}{4}h^{*4/3} - \frac{3}{13}h^{*13/3} + C_0 \tag{32}$$

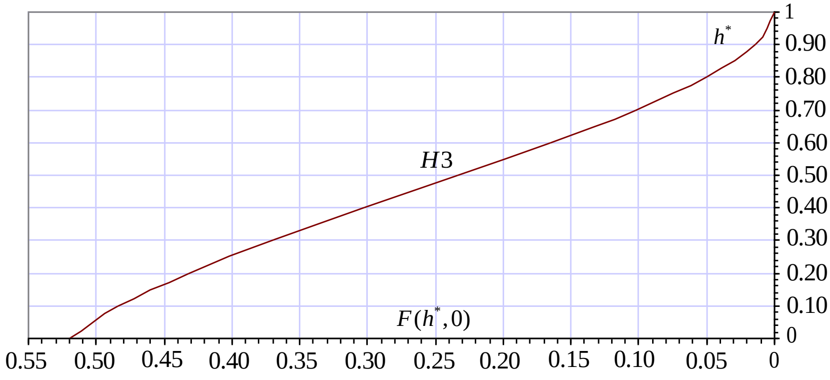
C_0 is an integration constant which can be determined by the initial conditions. According to Eq. (20), for $h^* = 1$ and $\beta = 0$ one may write $F\,(1, 0) = 0$. Inserting this result into Eq. (32) yields $C_0 = - 27/52$. Therefore, Eq. (32) can be written as:

$$F\left(h^*,0\right) = \frac{3}{4}h^{*\,4/3} - \frac{3}{13}h^{*13/3} - \frac{27}{52} \qquad (33)$$

Eq. (33) is used to plot the H2-type backwater curve for h^* values greater than 1 ($h^* > 1$), as well as the H3-type backwater curve for h^* values less than 1 ($h^* < 1$). Theses curves are shown in Fig.1. When calculating the function $F\left(h^*,0\right)$ using Eq. (33), the values can be negative. One must then consider the absolute value.



a) $h^* > 1$



b) $h^* < 1$

Figure 1: Graphical representation of H2 and H3-type drawdown and backwater curves respectively according to Eq. (33)

Critical slope

In this case, on may write:

$$S_0 = S_c\,, \beta = S_0 / S_c = 1,\; h_n^* = 1, \text{ i.e. } h_n = h_c$$
$$\delta = 1/S_c = 1/S$$

The first case to study is for which:

$$h > h_n = h_c \text{ ; i.e. } h^* > 1$$

Eq. (18) becomes:

$$dL^* = \frac{1}{S}\frac{h^{*10/3} - h^{*1/3}}{h^{*10/3} - 1}dh^* \tag{34}$$

Since $h^* > 1$, the numerator and the denominator of Eq. (34) are positive, i.e. $dL/dh > 0$. If $h^* \to \infty$, then $dL/dh \to 1/S$ which allows concluding that the curve approaches asymptotically to the horizontal.

For $h = h_c$, i.e. $h^* = 1$, Eq. (34) leads to the following indeterminacy:

$$\frac{dL^*}{dh^*} = \frac{1}{S}\frac{0}{0}$$

Using the L'hopital's rule, the limit of dL / dh when h^* approaches 1 is:

$$\lim_{h^* \to 1}\frac{dL^*}{dh^*} = \frac{1}{S}\lim_{h^* \to 1}\left(\frac{h^{*10/3} - h^{*1/3}}{h^{*10/3} - 1}\right) = \frac{1}{S}\left(\frac{10/3 - 1/3}{10/3}\right) = \frac{0.9}{S} \tag{35}$$

In this case, it is a C1-type backwater curve (Fig. 2).

The second case to study is for which:

$$h < h_n = h_c \text{ ; i.e. } h^* < 1$$

Since $h^* < 1$, the numerator and the denominator of Eq.(34) are both negative, i.e. $dL/dh > 0$. The water depth h increases downstream. Fro $h = 0$, i.e. $h^* = 0$, Eq. (34) becomes:

$$\frac{dL^*}{dh^*} = 0 \tag{36}$$

In this singular point of the flow, the depth becomes zero and the tangent of the curve is perpendicular to the bottom of the channel. This is the C3-type backwater curve (Fig.2).

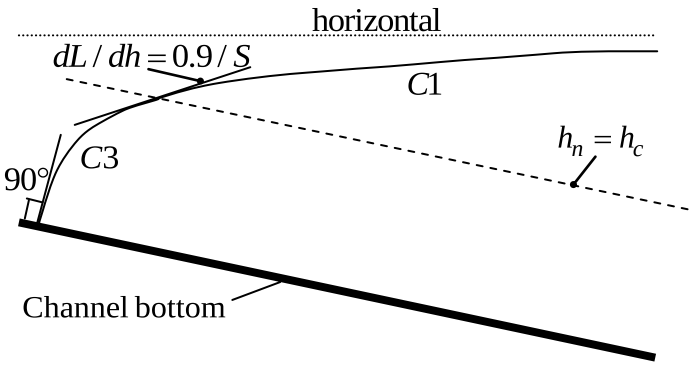


Figure 2: C-type backwater curves

The function $F(h^*, 1)$ corresponding to critical flow has been the subject of an in-depth regression study for the values $h^* > 1$ and $h^* < 1$.

For the values of h^* such as $h^* \le 1$, the function $F(h^*, 1)$ can be reasonably represented by the following equation, obtained with a coefficient of determination $R^2 = 0.9999$:

$$F\left(h^*,1\right) = 0.73 h^{*\,1.325} \tag{37}$$

The maximum deviation caused by Eq. (37) is 1.72 % only. Eq. (37) allows a simplified and rapid calculation of the C3-type backwater curve, in particular the calculation of the length L separating two given depths h_1 and h_2.

On the other hand, for the values of h^* such as $h^* \ge 1$ and more precisely in the wide range $1 \le h^* \le 2$, the function $F(h^*, 1)$ can be governed with fairly great accuracy by the following equation, obtained with a coefficient of determination $R^2 = 0.9999$:

$$F\left(h^*,1\right) = 0.9389 h^* - 0.2265 \tag{38}$$

The maximum deviation caused by Eq. (38) is 0.73 % only, which allows making a fast and precise calculation of the C1-type backwater curve.

USE OF THE H-TYPE BACKWATER CURVES TO DETERMINE THE ABSOLUTE ROUGHNESS OF A WIDE RECTANGULAR CHANNEL

The experimental measurement of the coordinates of the H3-type backwater curve allows estimating the value of the absolute roughness ε. In this section, the utility of the H3-type backwater curve in the estimation of ε will be highlighted. The backwater curve is caused by the installation of a sluice gate in a horizontal rectangular channel of large width. Supercritical flow is generated by passing a unit flow rate q controlled upstream or downstream, under a sluice gate (Fig. 3).

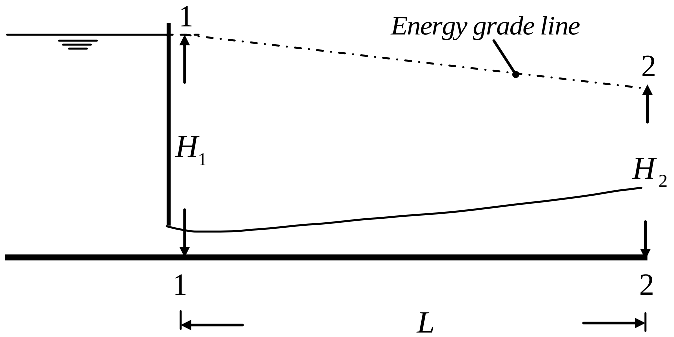


Figure 3: H3-type backwater curve downstream a sluice gate

The unit flow rate q being known, one may calculate the critical depth h_c according to the following well known relationship:

$$h_c = \left(\frac{q^2}{g} \right)^{1/3} \tag{39}$$

In carefully chosen sections 1-1 and 2-2, the total heads H_1 and H_2 are measured using a Pitot tube. Dividing H_1 and H_2 by the critical depth h_c results in:

$$H_1^* = H_1 / h_c$$
$$H_2^* = H_2 / h_c$$

The relative total head $H^* = H / h_c$ can be expressed as:

$$H^* = H / h_c = \frac{h}{h_c} + \frac{q^2}{2gh_c h^2} \tag{40}$$

The ratio h/h_c corresponds by definition to the dimensionless parameter h^*. Taking into account Eq. (39), Eq. (40) becomes:

$$H^* = h^* + \frac{1}{2h^{*2}} \tag{41}$$

This is a third degree equation in h^*. Since $H_1^* = H_1/h_c$ and $H_2^* = H_2/h_c$ are known, then Eq. (41) analytically gives the values of h_1^* and h_2^* both chosen less than 1 since the backwater curve is of H3-type. With the calculated values of h_1^* and h_2^*, the corresponding values of $F\left(h^*, 0\right)$ are worked out using Eq. (33). Then, thanks to Eq. (31), one may calculate the value of the parameter δ as:

$$\delta = \frac{\overset{h_2}{\underset{h_1}{L}}}{h_c\left[F(h_2^*, 0) - F(h_1^*, 0)\right]} \tag{31a}$$

The distance L separating the depths h_1 and h_2 in the respective sections 1-1 and 2-2 (Fig.3) is measured experimentally.

Once the parameter δ has been calculated, one may deduce the value of the Strickler coefficient k according to Eq. (15) as:

$$k = \sqrt{\frac{g\delta}{h_c^{1/3}}} \tag{15a}$$

Finally, using the relationship proposed by Hager (1987), one can calculate the required value of the absolute roughness ε as:

$$\varepsilon = \left(\frac{8.2\sqrt{g}}{k}\right)^6 \tag{42}$$

EXAMPLE 1

Let consider a horizontal channel of width B = 7m, conveying a flow rate:

$Q = 28$ m^3/s

The Manning roughness coefficient was previously determined as being:

n =0.025 m$^{-1/3}$/s

From the critical flow depth corresponding to a control section, calculate and draw the the water surface profile over a length of L = 6 m following the control section.

Solution

Start from the control section, and then go up the liquid stream from downstream to upstream. The calculations will lead to the plot of the H2-type backwater curve.

According to Eq. (39), the critical flow depth is as:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(Q/B)^2}{g}\right]^{1/3} = \left[\frac{(28/7)^2}{9.81}\right]^{1/3} = 1.17710984\text{m}$$

It is therefore considered that the first depth of the flow corresponds to:

$$h_1 = h_c = 1.17710984\text{m}$$

That is:

$$h_1^* = h_1 / h_c = 1$$

For the purposes of the calculation, a depth step $\Delta h = 0.01m$ is considered. This means that the second depth to consider is:

$$h_2 = h_1 + \Delta h = 1.17710984 + 0.01 = 1.18710984\ m$$

More generally, one may write: $h_{i+1} = h_i + \Delta h$, i = 1,2,...

That is:

$$h_2^* = h_2/h_c = 1.18710984/1.17710984 = 1.00849538$$

The next step consists in calculating the dimensionless parameter δ according to Eq. (15), knowing that Strickler coefficient k is such as $k = 1/n$:

$$\delta = \frac{k^2 h_c^{1/3}}{g} = \frac{h_c^{1/3}}{n^2 g} = \frac{1.17710984^{1/3}}{0.025^2 \times 9.81} = 172.209315$$

Based on Eq. (33), calculate then the quantity

$$I = \left[F(h_2^*, 0) - F(h_1^*, 0)\right]$$

According to Eq. (33):

$$F\left(h^*, 0\right) = \frac{3}{4} h^{*\,4/3} - \frac{3}{13} h^{*13/3} - \frac{27}{52}$$

That is:

$$F\left(h_1^*,0\right) = F\left(1,0\right) = \frac{3}{4} - \frac{3}{13} - \frac{27}{52} = 0$$

$$\mathrm{F}(\mathrm{h}_2^*,0) = \frac{3}{4} \times 1.00849538^{4/3} - \frac{3}{13} \times 1.00849538^{13/3} - \frac{27}{52} = 0.00010908$$

That is:

$$I = \left[F(h_2^*,0) - F(h_1^*,0)\right] = 0.00010908$$

According to Eq. (31), the length separating the depths h_1 and h_2 is as:

$$\underset{h_1}{\overset{h_2}{L}} = \underset{1-2}{\Delta L} = h_c \delta \left[F(h_2^*,0) - F(h_1^*,0)\right] = h_c \times \delta \times I$$

Whence:

$$\underset{1-2}{\Delta L} = 1.17710984 \times 172.209315 \times 0.00010908 = 0.0221109\, m$$

That is:

$$\underset{h_1}{\overset{h_2}{L}} = 0 + \underset{1-2}{\Delta L} = \underset{1-2}{\Delta L} = 0.0221109\, m$$

Continue the calculation with the procedure described above writing that:

$$\underset{h_1}{\overset{h_3}{L}} = \underset{h_1}{\overset{h_2}{L}} + \underset{2-3}{\Delta L}$$

More generally, one may write:

$$\underset{h_1}{\overset{h_{i+1}}{L}} = \underset{h_1}{\overset{h_i}{L}} + \underset{h_i \to h_{i+1}}{\Delta L} \text{ , } i = 1, 2, \ldots$$

Note that for $i = 1$, $\underset{h_1}{\overset{h_1}{L}} = 0$.

Based on this calculation procedure, table 1 was dressed.

Table 1: H2-type backwater curve calculations according to the proposed method

$Q = 28 m^3 / s$, **B = 7 m,** $n = 0.025 m^{-1/3} s$, ***L*** **= 6 m,** $h_c = 1.17710984 m$, $\Delta h = 0.01 m$							
h_1 (m)	h_2 (m)	h_1^*	h_2^*	δ	I	ΔL (m)	L (m)
1.1771098	1.18710984	1	1.1871098	172.20931	0.0001090	0.022110	0.022110
1.18710984	1,19710984	1.00849538	1.0169907	172.20931	0.0003305	0.006700	0.089111
1,19710984	1.20710984	1.01699076	1.0254861	172.20931	0.0005569	0.112901	0.202012
1.20710984	1.21710984	1.02548615	1.0339815	172.20931	0.0007884	0.159822	0.361834
1.21710984	1.22710984	1.03398153	1.0424769	172.20931	0.0010249	0.207774	0.569609
1.22710984	1.23710984	1.04247692	1.0509423	172.20931	0.0012666	0.256769	0.826378
1.23710984	1.24710984	1.0509723	1.0594677	172.20931	0.0015135	0.306816	1.133195
1.24710984	1.25710984	1.05946768	1.0679630	172.20931	0.0017657	0.357928	1.491123
1.25710984	1.26710984	1.06796307	1,0764584	172.20931	0.0020231	0.410114	1.901237
1.26710984	1.27710984	1.07645845	1.0849538	172.20931	0.0022859	0.463385	2.364623
1.27710984	1.28710984	1.08495384	1.0934492	172.20931	0.0025541	0.517753	2.882376
1.28710984	1.29710984	1.09344922	1.1019446	172.20931	0.0028278	0.573228	3.455604
1.29710984	1.30710984	1.1019446	1.1104439	172.20931	0.0031070	0.629821	4.085425
1.30710984	1.31710984	1.1104439	1.1189353	172.20931	0.0033917	0.687544	4.772969
1.31710984	1.32710984	1.1189353	1.1274307	172.20931	0.0036821	0,746406	5.519376
1.32710984	1.33710984	1.1274307	1.1359261	172.20931	0.0039782	0.806421	6.325797

The curve representing $h = f(L)$ is plotted in Fig. 4 according to the values of Table 1. Over a length of 6.326 m, the line of the water surface evolves from the critical depth $h_c = 1.17710984 m$, in the control section, to the depth $h = 1.337 m$. The evolution takes place according to the horizontal H2-type backwater curve.

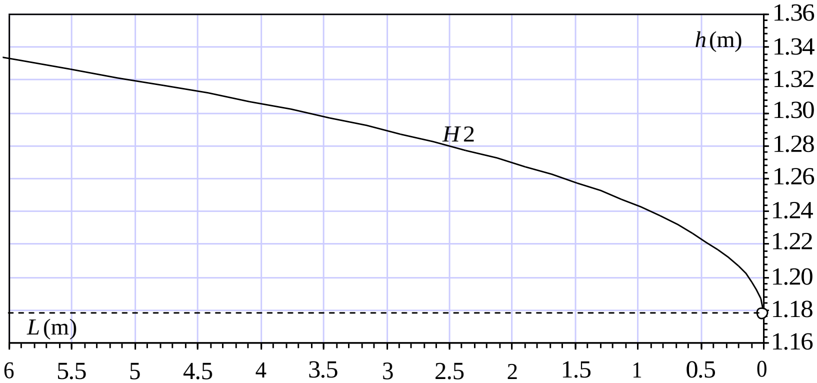


Figure 4: H2-type backwater curve according to the data of example 1.
(o) Control section $h_c = 1.17710984 m$

Let's check the calculations by directly using Eq. (19):

$$\int_{h_1^*}^{h_2^*} dL^* = \delta \int_{h_1^*}^{h_2^*} \left(h^{*1/3} - h^{*10/3} \right) dh^*$$

That is:

$$L = h_c \delta \int_{h_1^*}^{h_2^*} \left(h^{*1/3} - h^{*10/3} \right) dh^*$$

With h_1^*=1 and h_2^*= 1.1359261, the value of the integral calculated by appropriate software is such that:

$$\int_{h_1^*}^{h_2^*} \left(h^{*1/3} - h^{*10/3} \right) dh^* = -0.0312062$$

Whence:

$$L = h_c \delta \int_{h_1^*}^{h_2^*} \left(h^{*1/3} - h^{*10/3} \right) dh^* = 1.17710984 \times 172.20931 \times 0.0312062 = 6.32578613\, m$$

This value is identical to that calculated by the previous procedure and reported in table 1, i.e. L = 6.325797 m.

EXAMPLE 2

This example is considered in order to show how to estimate the absolute roughness ε of a wide rectangular channel from the knowledge of the H3-type backwater curve.

A two-dimensional flow of a unit flow q = 0.099 m^2/s passes through a supercritical flow regime on a horizontal plane.

In a first section 1-1, the total head is H_1 = 0.80 m, while in a second section 2-2 the total head is H_2 = 0.40 m. Sections 1-1 and 2-2 are separated by a length L = 4 m.

Determine the absolute roughness of the channel bottom.

Solution

According to Eq. (39), the critical flow depth is as:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{0.99^2}{9.81}\right)^{1/3} = 0.09996941m \approx 0.10m$$

Let's calculate the relative total heads in sections 1-1 and 2-2, respectively such as:

$$H_1^* = \frac{H_1}{h_c} = \frac{0.80}{0.10} = 8$$

$$H_2^* = \frac{H_2}{h_c} = \frac{0.40}{0.10} = 4$$

Using Eq. (41), one may obtain:

$$h_1^* = 0.254067$$

and

$$h_2^* = 0.371196$$

The relative depths h_1^* and h_2^* are less than 1, so it is an H3-type backwater curve.

According to Eq. (33):

$$F\left(h_1^*, 0\right) = \frac{3}{4} h_1^{*\,4/3} - \frac{3}{13} h_1^{*13/3} - \frac{27}{52}$$

That is:

$$F(h_1^*, 0) = \frac{3}{4} \times 0.254067^{4/3} - \frac{3}{13} \times 0.254067^{13/3} - \frac{27}{52} = -0.3991532$$

On the other hand:

$$F\left(h_2^*, 0\right) = \frac{3}{4} h_2^{*\,4/3} - \frac{3}{13} h_2^{*\,13/3} - \frac{27}{52}$$

Whence:

$$F(h_2^*, 0) = \frac{3}{4} \times 0.371196^{4/3} - \frac{3}{13} \times 0.371196^{13/3} - \frac{27}{52} = -0.32230159$$

According to Eq. (31) and considering the absolute value of the functions *F*, the parameter δ is as:

$$\delta = \frac{\underset{h_1}{\overset{h_2}{L}}}{h_c\left[F(h_2^*,0)-F(h_1^*,0)\right]} = \frac{4}{0.10\times\left[0.3991532-0.32230159\right]}$$

That is:

$\delta = 520.483566$

With regard to Eq. (15), the Strickler coefficient is such that:

$$k = \sqrt{\frac{g\delta}{h_c^{1/3}}} = \sqrt{\frac{9.81\times 520.483566}{0.10^{1/3}}} = 104.882899\, m^{1/3}/s$$

According to Eq. (42), the absolute roughness is:

$$\varepsilon = \left(\frac{8.2\sqrt{g}}{k}\right)^6 = \left(\frac{8.2\times\sqrt{9.81}}{104.882899}\right)^6 = 0.00021561 m \approx 0.216\, mm$$

EXAMPLE 3

This example concerns the calculation of the C-type backwater curve by the proposed method. The calculation procedure will be identical to that described in example 1. The example of the C1-type backwater curve will be taken and whose first depth will be defined as h_1 = 1.35 m. The following depths will be calculated considering a depth step $\Delta h = 0.01 m$. So, for depths, one may write:

$$h_{i+1} = h_i + \Delta h \text{ , } i = 1,2,...$$

On the other hand, for lengths, on may write:

$$\underset{h_1}{\overset{h_{i+1}}{L}} = \underset{h_1}{\overset{h_i}{L}} + \underset{h_i\to h_{i+1}}{\Delta L} \text{ , } i = 1, 2,\ldots$$

Consider a wide rectangular channel with slope S_0 = 0.001147. The Manning's n resistance coefficient was estimated to be n =1/90 $m^{-1/3}/s$, implying that the Strickler coefficient is k = 90 $m^{1/3}/s$. The unit flow rate through the channel is q = 4 m^2/s.

Calculate the length L separating the depths h_1 = 1.35 m and h_2 = 1.36 m.

Perform the calculation using the differential equation which governs the gradually varied flow as well as the approximate relationships proposed in the theoretical part of the study.

Solution

According to Eq. (39), the critical flow depth is as:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{4^2}{9.81}\right)^{1/3} = 1.17710984\,m$$

The next step consists in calculating the dimensionless parameter δ according to Eq. (15):

$$\delta = \frac{k^2 h_c^{1/3}}{g} = \frac{90^2 \times 1.17710984^{1/3}}{9.81} = 871.809657$$

According to Eq. (27), one may write:

$$\beta = S_0 \delta = h_n^{*\,-3/10}$$

That is:

$$\beta = 0.001147 \times 871.809657 = 0.99996568 \approx 1$$

It is therefore concluded that the slope of the channel is critical, meaning that:

$$S_0 = S_c = S\ ,\ \beta = S_0 / S_c = 1,\ h_n^* = 1,\ \text{i.e. } h_n = h_c$$

In this section, attention is paid to the following case:

$$h > h_n = h_c\ ;\ \text{i.e. } h^* > 1$$

Therefore, equation (34) applies:

$$dL^* = \frac{1}{S} \frac{h^{*10/3} - h^{*1/3}}{h^{*10/3} - 1} dh^*$$

That is:

$$L = \frac{h_c}{S} \int_{h_1^*}^{h_2^*} \frac{h^{*10/3} - h^{*1/3}}{h^{*10/3} - 1} dh^*$$

where, according to Eq. (20):

$$F(h^*, 1) = \int_{h_1^*}^{h_2^*} \frac{h^{*10/3} - h^{*1/3}}{h^{*10/3} - 1} dh^*$$

Whence, one may write:

$$L = \frac{h_c}{S} I$$

where:

$$I = \int_{h_1^*}^{h_2^*} \frac{h^{*10/3} - h^{*1/3}}{h^{*10/3} - 1} dh^* = \left[F(h_2^*, 0) - F(h_1^*, 0) \right]$$

The first depth of the flow corresponds to:

$$h_1 = 1.35\, m$$

That is:

$$h_1^* = h_1 / h_c = 1.35 / 1.17710984 = 1.14687692$$

Since a depth step $\Delta h = 0.01 m$ is considered, this means that the second depth to accounting for is:

$$h_2 = h_1 + \Delta h = 1.35 + 0.01 = 1.36\, m$$

That is:

$$h_2^* = h_2 / h_c = 1.36 / 1.177109841 = 1.15537221$$

Using appropriate software package, the integral I is such that:

$$I = 0.00781364$$

The exact length L separating the two defined depths is then:

$$L_{h1}^{h2}(\text{exact}) = \frac{h_c}{S} I = \frac{1.17710984}{0.001147} \times 0.00781364 \approx 8.0187555\, m \approx 8.02\, m$$

Recall the approximate Eq. (38):

$$F\left(h^*, 1\right) = 0.9389\, h^* - 0.2265$$

That is:

$$F\left(1.14687692; 1\right) = 0.9389 \times 1.14687692 - 0.2265 = 0.85030274$$

The length L_1 separating the depth h = 0 ($h^* = 0$) and the depth h = 1.35 m ($h^* = 1.14687692$) is:

$$L_0^{1.35} = \frac{h_c}{S} I = \frac{1.17710984}{0.001147} \times 0.85030274 \approx 872.623998\, m$$

On the other hand:

$$F\left(1.15537221;1\right) = 0.9389 \times 1.15537221 – 0.2265 = 0.85827897$$

Whence, the length L_2 separating the depth h = 0 m ($h^* = 0$) and the depth h = 1.36 m ($h^* = 1.15537221$) is:

$$L_0^{1.36} = \frac{h_c}{S} I = \frac{1.17710984}{0.001147} \times 0.85827897 \approx 880.80961 m$$

So, the approximate length L separating the two defined depths h_1 = 1.35 m and h_2 = 1.36 m is:

L (approximate) = $L_0^{1.36}$ - $L_0^{1.35}$ = 880.80961 – 872.623998 = 8.18561137 m ≈ 8.2 m

That is:

$$\frac{\Delta L}{L} = 100 \times \left(\frac{8.18561137 - 8.0187555}{8.0187555} \right) = 2.065\,\%$$

One thus commits a relative error of about 2% when using the approximate Eq. (38).

It should be noted that, for the considered C1-type backwater curve, the depth increases by only 1 cm over a distance of approximately L = 8 m.

EXAMPLE 4

This example addresses the problem of the gradually varied flow passing under a sluice gate, where it is required to determine the depth h_2 of the flow at a fixed distance L and for a given initial depth h_1. The slope of the channel is horizontal.

The discharge is Q = 22.4 m^3/s

The channel width is B = 10 m

The initial depth is h_1 = 0.32 m

The absolute roughness is ε = 5 mm

Determine the depth h_2 at the distance L = 60 m from h_1.

Solution

The unit flow rate is:

$$Q = Q/B = 22.4/10 = 2.24 \text{ m}^2/\text{s}$$

According to Eq. (39), the critical flow depth is as:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{2.24^2}{9.81}\right)^{1/3} = 0.79972808\,m \approx 0.80\,m$$

Since the slope is horizontal, the dimensionless parameter β is as:

$$\beta = S_0 / S_c = 0$$

On the other hand, the relative depth h_1^* is:

$$h_1^* = h_1 / h_c = 0.32 / 0.80 = 0.40$$

Thus, since $h_1^* < 1$, the flow is characterized by a H3-type backwater curve.

According to Eq. (33):

$$F\left(h_1^*, 0\right) = \frac{3}{4} h_1^{*\,4/3} - \frac{3}{13} h_1^{*\,13/3} - \frac{27}{52}$$

That is:

$$F\left(h_1^*, 0\right) = \frac{3}{4} \times 0.4^{4/3} - \frac{3}{13} \times 0.4^{13/3} - \frac{27}{52} = -0.3025417$$

Knowing the absolute roughness ε, Hager's relationship (42) allows computing the Strickler's k roughness coefficient as:

$$k = \frac{8.2\sqrt{g}}{\varepsilon^{1/6}} = \frac{8.2 \times \sqrt{9.81}}{0.005^{1/6}} \approx 62.11\,m^{1/3} / s$$

The next step consists in calculating the dimensionless parameter δ according to Eq. (15):

$$\delta = \frac{k^2 h_c^{1/3}}{g} = \frac{62.11^2 \times 0.8^{1/3}}{9.81} \approx 365.05$$

According to Eq. (31), one may write:

$$F(h_2^*,0)=\frac{L_{h_1}^{h_2}}{h_c\delta}+F(h_1^*,0)$$

That is:

$$F(h_2^*,0)=\frac{60}{0.80\times 365.05}-0.3025417=-0.09709039$$

Applying Eq. (33) to h_2^* results in:

$$F\left(h_2^*;0\right)=\frac{3}{4}h_2^{*\,4/3}-\frac{3}{13}h_2^{*\,13/3}-\frac{27}{52}=-0.09709039$$

Since the backwater curve is of H3-type, the relative depth h_2^* must be less than 1 like its h_1 counterpart. Therefore, calculations show that the required solution of the previous equation is:

$$h_2^*=0.70896439036\approx 0.709$$

The sought depth h_2 is then:

$$h_2=h_2^*h_c=0.70896439036\times 0.80=0.56717151\,m\approx 0.567\,m$$

CONCLUSIONS

The study focussed on the backwater curves generated by a gradually varied flow occurring in a wide rectangular channel. The differential equation governing such a flow has been transformed by the introduction of dimensionless parameters, such as δ, β, and h^* which represents the ratio of the depth h to the critical depth h_c [Eq. (18)]. The dimensionless β parameter has been shown to be closely related to the normal relative depth [Eq. (27)] with which it plays an important role in the backwater curves classification.

In order to show how to apply the new equation governing the flow, the cases of the horizontal and critical slopes have been considered. Theoretical development applied to these slopes has shown real mathematical interest.

Four numerical examples have been proposed to enlighten the reader on the practical application of the advocated method.

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APPENDIX

This appendix is intended to show that the proposed method can be extended to the case of the non-wide rectangular channel. A theoretical demonstration will be given for this purpose and a new differential equation governing the gradually varied flow in a rectangular channel will be drawn. A numerical example will be exposed in order to show how to use the equation in the case of a horizontal channel slope.

For a rectangular channel of width b and a flow of depth h, one can write:

The unit flow rate: $q = Q/b$

The water area: $A = bh$

The wetted perimeter: $P = b + 2h$

The hydraulic radius: $R_h = A/P = bh/(b + 2h)$

Taking into account these considerations, Eq. (5) can be written after some modifications as:

$$V = \frac{Q}{A} = k\, R_h^{2/3} S_f^{1/2} \tag{5}$$

$$V = \frac{Q}{bh} = \frac{q}{h} = k h^{2/3} \left(\frac{b/h}{b/h + 2} \right)^{2/3} S_f^{1/2} \tag{43}$$

Let's the aspect ratio be $\eta = b/h$. Thus, Eq. (43) is rewritten as:

$$V = \frac{Q}{bh} = \frac{q}{h} = k \left(\frac{\eta}{\eta + 2} \right)^{2/3} h^{2/3} S_f^{1/2} \tag{44}$$

From Eq. (44), one may deduce that:

$$S_f = \frac{q^2}{k^2 \left(\frac{\eta}{\eta + 2} \right)^{4/3} h^{10/3}} \tag{45}$$

Let us put σ as:

$$\sigma = k \left(\frac{\eta}{\eta + 2} \right)^{2/3} \tag{46}$$

Inserting Eq. (46) into Eq. (45) results in:

$$S_f = \frac{q^2}{\sigma^2 h^{10/3}} \tag{47}$$

It can be easily demonstrated that the critical slope is expressed as:

$$S_c = \frac{g}{\sigma^2 h_c^{1/3}} \tag{48}$$

Recall the following relationship:

$$dL = \frac{1 - q^2 / \left(gh^3\right)}{S_0 - S_f} dh \tag{49}$$

Inserting Eq. (47) into (49) results in:

$$dL = \frac{1 - q^2 / \left(gh^3\right)}{S_0 - \dfrac{q^2}{\sigma^2 h^{10/3}}} dh \tag{50}$$

The quantity $q^2 / (\sigma^2 h^{10/3})$ can be written as:

$$q^2 / (\sigma^2 h^{10/3}) = \frac{g}{\sigma^2 (h / h_c)^{10/3} h_c^{1/3}} \tag{51}$$

As well as the quantity $q^2 / (gh^3)$ is written as:

$$q^2 / (gh^3) = (h / h_c)^{-3} \tag{52}$$

Taking into account Eqs. (51) and (52), Eq. (50) becomes:

$$dL = \frac{1 - \left(h / h_c\right)^{-3}}{S_0 - \dfrac{g}{\sigma^2 (h / h_c)^{10/3} h_c^{1/3}}} dh \tag{53}$$

Let's recall the following dimensionless parameters:

$$h^* = h / h_c \text{ and } L^* = L / h_c$$

Thus, Eq. (53) can be written as:

$$dL^* = \frac{1 - h^{*-3}}{S_0 - \dfrac{g}{\sigma^2 h^{*10/3} h_c^{1/3}}} dh^* \tag{54}$$

Let's ψ be:

$$\psi = \frac{\sigma^2 h_c^{1/3}}{g} \tag{55}$$

Comparing Eqs. (48) and (55) results in:

$$\psi = \frac{1}{S_c} \tag{56}$$

After some arrangements, Eq. (54) reduces to:

$$dL^* = \psi \frac{h^{*10/3} - h^{*1/3}}{\psi S_0 h^{*10/3} - 1} dh^* \tag{57}$$

That is:

$$L_{h_1}^{h_2} = h_c \psi \int_{h_1^*}^{h_2^*} \frac{h^{*10/3} - h^{*1/3}}{\psi S_0 h^{*10/3} - 1} dh^* \tag{58}$$

Or:

$$L_{h_1}^{h_2} = h_c \psi \int_{h_1^*}^{h_2^*} \frac{h^{*10/3} - h^{*1/3}}{\zeta h^{*10/3} - 1} dh^* \tag{59}$$

where:

$$\zeta = \psi S_0 = S_0 / S_c \tag{60}$$

Eq. (57) is of the same form as Eq. (18) which was determined for the case of the wide rectangular channel. δ is similar to ψ and $\beta = S_0 \delta = S_0 / S_c$ is identical to $\zeta = \psi S_0 = S_0 / S_c$.

Eq. (57) is the definitive form of the differential equation which governs the gradually varied flow in a rectangular channel.

EXAMPLE 5

Take the data from example 4 and solve the problem by considering the channel as being rectangular and no longer as being wide as assumed in example 4. Eq. (55) is then applicable.

Let us calculate the σ parameter defined by Eq. (46):

$$\sigma = k\left(\frac{\eta}{\eta+2}\right)^{2/3} = 62.11\left(\frac{10/0.32}{10/0.32+2}\right)^{2/3} = 59.5937128\, m^{1/3}/s$$

Thus, the ψ parameter defined by Eq. (55) is such that:

$$\psi = \frac{\sigma^2 h_c^{1/3}}{g} = \frac{59.5937128^2 \times 0.80^{1/3}}{9.81} = 336.069069$$

On the other hand, the relative depth h_1^* is:

$$h_1^* = h_1 / h_c = 0.32/0.80 = 0.40$$

Thus, since $h_1^* < 1$, the flow is characterized by a H3-type backwater curve.

According to Eq. (33):

$$F\left(h_1^*, 0\right) = \frac{3}{4} h_1^{*4/3} - \frac{3}{13} h_1^{*13/3} - \frac{27}{52}$$

That is:

$$F\left(h_1^*, 0\right) = \frac{3}{4} \times 0.4^{4/3} - \frac{3}{13} \times 0.4^{13/3} - \frac{27}{52} = -0.3025417$$

According to Eq. (59) for $S_0 = 0$ ($\zeta = 0$), one may write:

$$L_{h_1}^{h_2} = h_c \psi \int_{h_1^*}^{h_2^*} \left(h^{*1/3} - h^{*10/3}\right) dh^*$$

which can be written in the following form:

$$L_{h_1}^{h_2} = h_c \psi \left[F(h_2^*, 0) - F(h_1^*, 0) \right]$$

Whence:

$$F(h_2^*, 0) = \frac{L_{h_1}^{h_2}}{h_c \psi} + F(h_1^*, 0)$$

That is:

$$F(h_2^*, 0) = \frac{60}{0.80 \times 336.069069} - 0.3025417 = -0.0793729$$

Applying Eq. (33) to h_2^* results in:

$$F\left(h_2^*; 0\right) = \frac{3}{4} h_2^{*\,4/3} - \frac{3}{13} h_2^{*\,13/3} - \frac{27}{52} = -0.0793729$$

Since the backwater curve is of H3-type, the relative depth h_2^* must be less than 1 like its h_1 counterpart. Therefore, calculations show that the required solution of the previous equation is:

$$h_2^* = 0.74083644771 \approx 0.741$$

The sought depth h_2 is then:

$$h_2 = h_2^* h_c = 0.74083644771 \times 0.80 = 0.59266916\, m \approx 0.592\, m$$

In example 4, considering the rectangular channel to be wide, the calculated depth h_2 was:

$$h_2 = 0.567\, m$$

So, there is a difference of about 2.55 cm only, corresponding to a deviation of 4.5% approximately.

Note that h_2^* can also be determined by reading Fig. 1b corresponding to $h^* < 1$ or to the H3-type backwater curve.