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TECHNICAL NOTE

DIMENSIONLESS EQUATION OF THE RADIUS OF CURVATURE OF THE DOWNSTREAM TOE OF A SPILLWAY

EQUATION ADIMENSIONNELLE DU RAYON DE COURBURE DU PIED DES BARRAGES DEVERSOIRS

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ABSTRACT

The exactness in determining the radius of curvature of the downstream toe of a spillway ensures a smooth hydraulic transit from the upstream to the downstream of the structure. The literature mentions that this radius takes half the height of the dam if the latter is less than 10 m. Beyond this value, the values of the connection radius are numerically tabulated as a function of the height of dam and the head of water over the crest. The purpose of this technical note is to transform the numeric values into multiple dimensional curves and then present them as a single curve with a unique dimensionless mathematical formulation that is easy to apply. The relation thus proposed avoids to the users any kind of interpolation and intermediate dubious calculation.

Keywords: Spillway, discharge head, radius of curvature, dimensionless relation

RESUME

La précision dans la détermination du rayon de courbure du pied aval d'un déversoir assure un transit hydraulique fluide de l'amont vers l'aval de l'ouvrage. La littérature mentionne que ce rayon prend la moitié de la hauteur du barrage si celui-ci est inférieur à 10 m. Au-delà de cette valeur, les valeurs du rayon de raccordement sont chiffrées numériquement en fonction de la hauteur du barrage et de la hauteur d'eau sur la crête. Le but de cette note technique est de transformer les valeurs numériques en courbes multidimensionnelles, puis de les présenter sous la forme d'une courbe unique avec une formulation mathématique sans dimension unique et facile à appliquer. La relation ainsi proposée évite aux utilisateurs toute sorte d'interpolation et de calcul intermédiaire incertain.

Mots-clés : Déversoir, charge, rayon de courbure, relation sans dimension.

INTRODUCTION

The spillway dam is a structure often encountered in hydraulic installations. A spillway is a structure built to divert or evacuate the water retained behind a fixed valve or dam, the height of which would exceed a certain limit (for example the crest of the structure). One of its fundamental functions is to release surplus waters from the reservoir in order to prevent overtopping and possible failure of the dam.

The main function of dams and weirs is to divert the flow of a river in a water transport system towards a hydropower plant. They can also produce an additional drop and provide storage capacity. The weir is, hydraulically speaking, the ideal solution giving the greatest discharge coefficients. Its curved shape corresponds to that which a water jet would have on a thin weir for the corresponding design discharge. For higher or lower flow rates, areas of overpressure or depression appear on the downstream side. Depressions can cause cavitation and damage to the structure. Fortunately, work suggests that such a phenomenon will not occur before the load on the crest of the weir is greater than three times the design load (Sinniger and Hager, 1989; Chow, 1959).

The two main types of spillways are controlled spillway and uncontrolled spillway. The first one has mechanical structures or gates to regulate the flow rate. This design allows nearly the full height of the dam to be used for water storage year-round, and flood waters can be released as required by opening one or more gates. The second one, in contrast, does not have gates. When the water rises above the crest of the spillway, it begins to be released from the reservoir. The discharge is controlled only by the depth of water above the reservoir's spillway.

As water passes over a spillway and down the chute, potential energy converts into increasing kinetic energy. Failure to dissipate the water's energy can lead to scouring and erosion at the dam's toe (base). This can cause spillway damage and undermine the dam's stability. Energy can be dissipated in several ways such as steps, flip bucket, ski jump, or stilling basin (Chanson, 2015; Hager, 1992). It is this last work which interests our study. In fact, the downstream toe of the spillway is connected to the stilling basin by a curved shape, with a radius of curvature R, in order to damp the supercritical flow occurring in this part of the spillway.

There is empirical data in the literature (Naoumenko, 1975; Ibrahim and Alkhader, 1995) giving the value of R as a function of the height of the weir and the flow head above the crest. These data are tabulated and the present study proposes to put them in the form of an easy-to-use dimensionless equation.

PROBLEMATIC

In this technical note, we aim to transform the empirical data recorded in table 1 into graphs of several dimensional formulations, giving the variation of the radius of curvature R of the downstream toe of spillways as a function of the corresponding heights of dams P for each value of the head of water over the crest H, and then look for the possibility of transforming them into a single dimensionless curve (formulation). The dimensional curves and the final dimensionless equation give the values of the connection radius of the spillways of the overflow dams (Figure 1) with the stilling basins witch dissipating hydraulic energy due to the flood (discharge) releases above the spillways.

Table 1: Connection radius as a function of the height of dam *P* and the head of water over the crest *H* (Naoumenko, 1975; Ibrahim and Alkhader, 1995)

***H* (m)**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
***P* (m)**				*R* (m)					
10	3	4.2	5.4	6.5	7.5	8.5	9.6	10.6	11.6
20	4	6	7.8	8.9	10	11	12.2	13.3	14.3
30	4.5	7.5	9.7	11	12.4	13.5	14.7	15.8	16.8
40	4.7	8.4	11	13	14.5	15.8	17	18	19
50	4.8	8.8	12.2	14.5	16.5	18	19.2	20.3	21.3
60	4.9	8.9	13	15.5	18	20	21.2	22.2	23.2

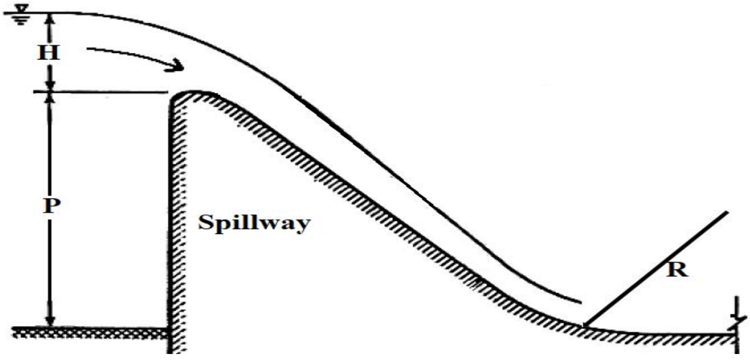

Figure 1: Sketch of the radius of curvature of the downstream toe of a spillway

STATISTICAL DESCRIPTION OF THE DATASET

The data from Table 1 can be statistically considered in two ways, one considers the values of the connection radius as a function of the heads of water H discharged above the dam; this point of view is represented by figure 2. The second considers the same values of so-called radius as a function of the heights of the dams P; this point of view is represented by figure 3. Figures 2 and 3 show the violin graphs of radius values against of heads of water H and heights of dams P, respectively. Figure 2 indicates that the means and medians of the values of the radius approach more and more as the values of H increase, thus reflecting a tendency towards normality for the large values of the head water. Contrariwise in figure 3, this finding is reversed for the values of the same radius with respect to the values of the heights of the dams.

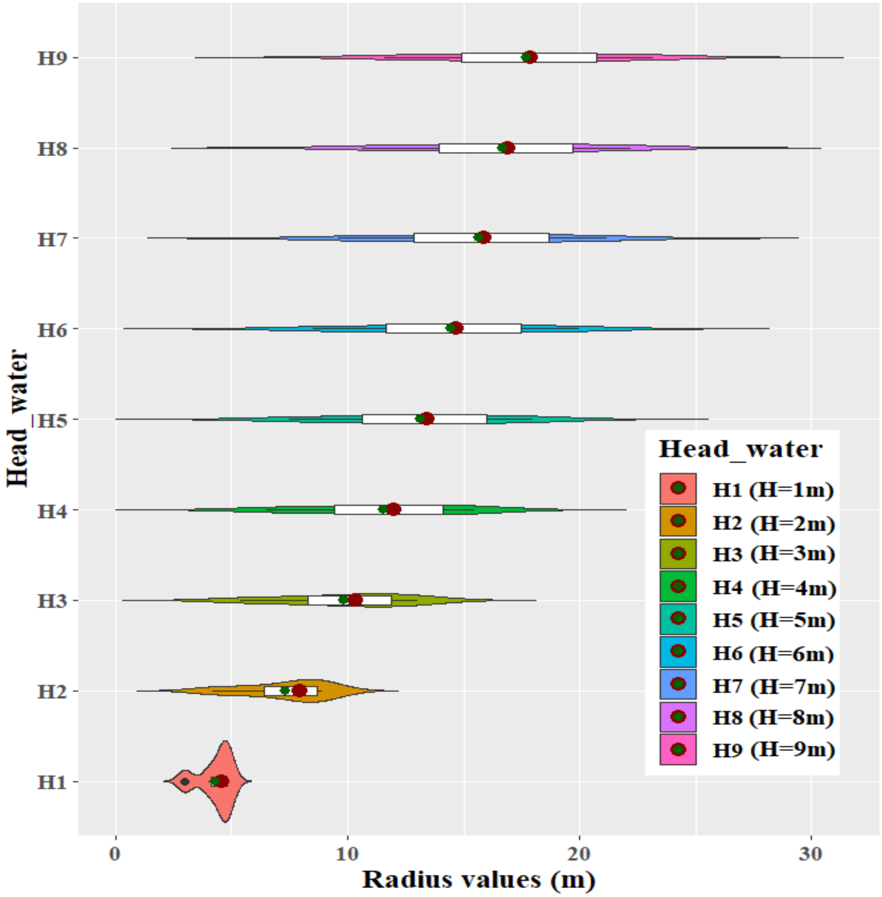

Figure 2: Violin diagrams of different values of heads of water over the dams against of radius values

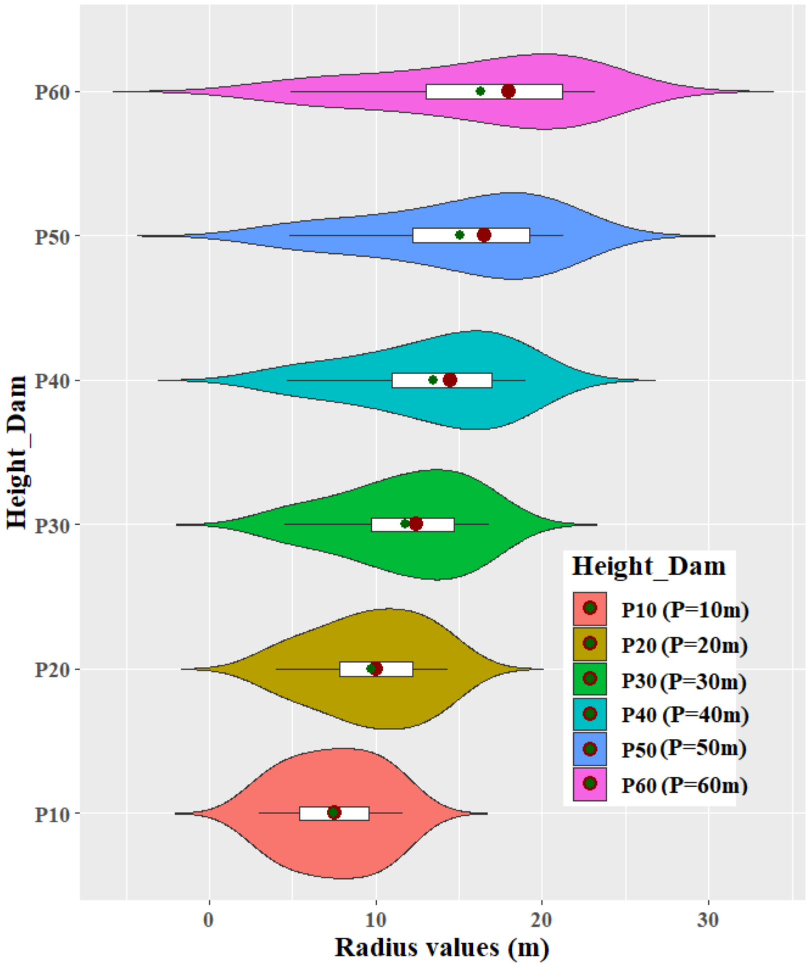

Figure 3: Violin diagrams of different heights of dams against of radius values

TRANSFORMATION OF TABULATED DATA INTO DIMENSIONAL CURVES

The graphical representation of the connection radius R as a function of the height of the dam P gives nine curves corresponding to the number of values of the head of water over the crest H of the dam. These curves are illustrated in figure 4. For $P < 10$ m, a value of $R = P/2$ is recommended by Naoumenko (1975) and indicated in (Ibrahim and Alkhader, 1995).

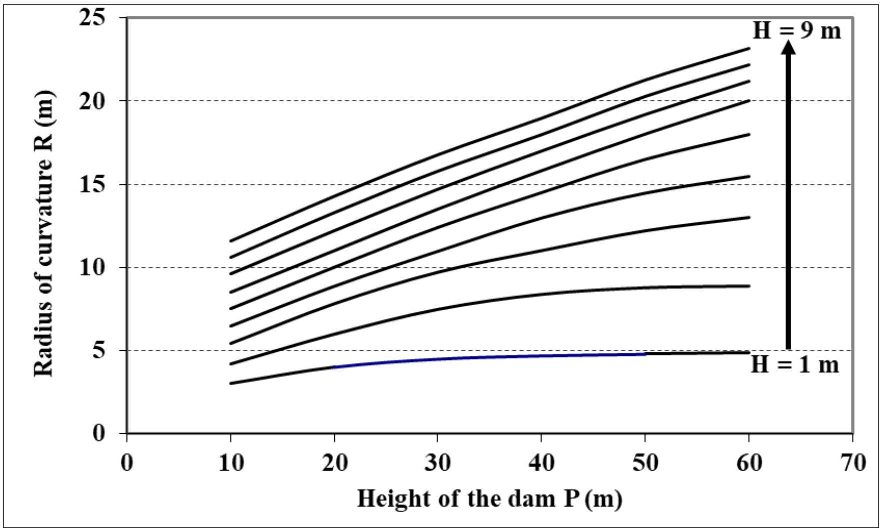

Figure 4: The graphical representation of the connection radius as a function of the height of the dam corresponding to the head of water over the crest

From Figure 4 it can be seen that each of the nine curves, with a number of values of *H*, has a tendency which can be given, with statistical satisfaction, by a mathematical formulation. The nine mathematical formulations are given in Table 2 where we note that the trends are of parabolic type at almost perfect performances (R^2 very close to 1). The relationships are ideal ($R^2 = 1$) for the *H* equal to 3 m and 4 m and the weakest relation concerns the value of equal to 9 m ($R^2 = 0.9792$).

Table 2: Mathematical formulations of radius *R* as a function of the height of the dam p corresponding to the head of water over the crest H

Value of H (m)	Formulation	Coefficient of determination R^2
1	$R = -0.0009P^2 + 0.2920P + 8.78$	0.9999
2	$R = -0.\ 0009P^2 + 0.2920P + 778$	0.9999
3	$R = -00007P^2 + 0.2848P + 6.82$	1
4	$R = -0.0007P^2 + 0.2771P + 5.77$	1
5	$R = -0.0012P^2 + 0.2942P + 4.64$	0.9998
6	$R = -0.0017P^2 + 0.2998P + 3.62$	0.9995
7	$R = -0.0019P^2 + 0.2850P + 2.80$	0.9992
8	$R = -0.0023P^2 + 0.2550P + 1.87$	0.9994
9	$R = -0.0011P^2 + 0.1108P + 2.09$	0.9792

DIMENSIONLESS RELATIONSHIP

In this section the concern is focused on the possibility of existence of a single curve with dimensionless relation which encloses all the possibilities and scenarios and gives the value of the connection radius R whatever the values of P and H.

To this end, the goal is reached by means of an adequate change of variable similar to those carried out in (Houichi, 2007; Houichi and Achour, 2007). The mathematical trick is to consider dimensionless variables according to Table 3. These new variables will have H/P in abscissa and R/P as ordinate.

The unique relation is universal and it is of the power type with a coefficient of determination $R^2 = 0.9923$ (figure 5), that is to say that this unique formulation explains 99.23% of the variability of R/P according to H/P values. The relationship is written:

$$\frac{R}{P} = 1.2\left(\frac{H}{P}\right)^{0.6} \tag{1}$$

The relation proposed is simple and easy to remember and encloses all the cases of Table 1 for the radius of connection of the toe of the spillway and the stilling basin.

Table 3: Dimensionless transformation of the variables *H* and *R*

H (m)	P (m)	H/P (-)	R/P (-)	H (m)	P (m)	H/P (-)	R/P (-)	H (m)	P (m)	H/P (-)	R/P (-)
1	10	0.100	0.300	4	10	0.400	0.650	7	10	0.700	0.960
	20	0.050	0.200		20	0.200	0.445		20	0.350	0.610
	30	0.033	0.150		30	0.133	0.367		30	0.233	0.490
	40	0.025	0.118		40	0.100	0.325		40	0.175	0.425
	50	0.020	0.096		50	0.080	0.290		50	0.140	0.384
	60	0.017	0.082		60	0.067	0.258		60	0.117	0.353
2	10	0.200	0,420	5	10	0.500	0.750	8	10	0.800	1.060
	20	0.100	0.300		20	0.250	0.500		20	0.400	0.665
	30	0.067	0.250		30	0.167	0.413		30	0.267	0.527
	40	0.050	0.210		40	0.125	0.363		40	0.200	0.450
	50	0.040	0.176		50	0.100	0.330		50	0.160	0.406
	60	0.033	0.148		60	0.083	0.300		60	0.133	0.370
3	10	0.300	0.540	6	10	0.600	0.850	9	10	0.900	1.160
	20	0.150	0.390		20	0.300	0.550		20	0.450	0.715
	30	0.100	0.323		30	0.200	0.450		30	0.300	0.560
	40	0.075	0.275		40	0.150	0.395		40	0.225	0.475
	50	0.060	0.244		50	0.120	0.360		50	0.180	0.426
	60	0.050	0.217		60	0.100	0.333		60	0.150	0.387

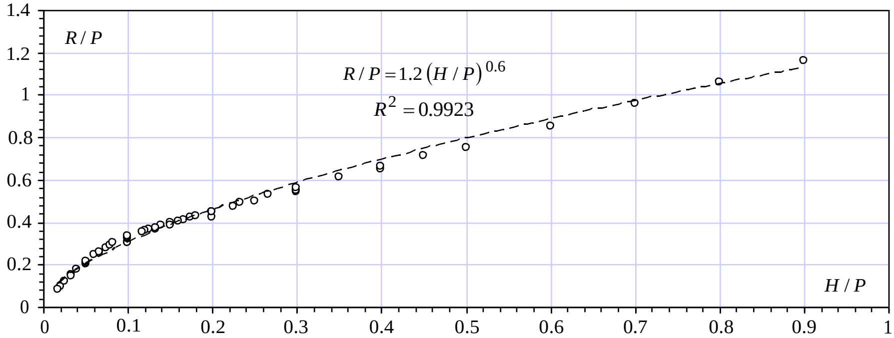

Figure 5: Graphical representation of *R/P* as a function of *H/P* and dimensionless relationship

RESIDUALS ANALYSIS FROM THE APPLICATION OF THE PROPOSED RELATIONSHIP

The residuals resulting from the application of the proposed relation and which are in fact the difference between the actual values of R/P and the approximated values from the so-called proposed relation, supposed to obey the normality estimated by the famous test of Shapiro-Wilk and would present a quantile-quatile cloud as regular as possible.

Test of Shapiro-Wilk

The Shapiro-Wilk statistic is equal to W = 0,96165, having a p-value = 0.08165 > 0,05, where the residual follows normality according to the Shapiro-Wilk test.

Graphical visualization of the normality of the residuals

The graphic visualization (figure 6) of the histogram of the residuals with respect to the theoretical density (curve in blue) and the Gaussian bell (curve in dotted red) as well as the approximation in value of the median and the mean of the residues (0,0016 and -0,0011, respectively) indicate the aspect of normality of the residuals already confirmed by the Shapiro-Wilk test. The quantile-quatile cloud of figure 7 shows a regularity of distribution of the points which further confirms the aspect of desired normality.

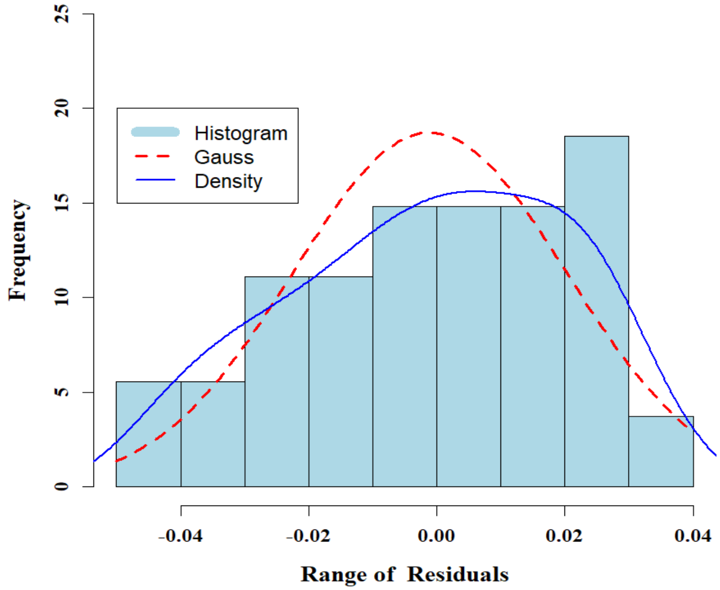

Figure 6: Graphical comparison of the histogram of the residuals of the proposed relation and the normality through the Gaussian bell and the theoretical density

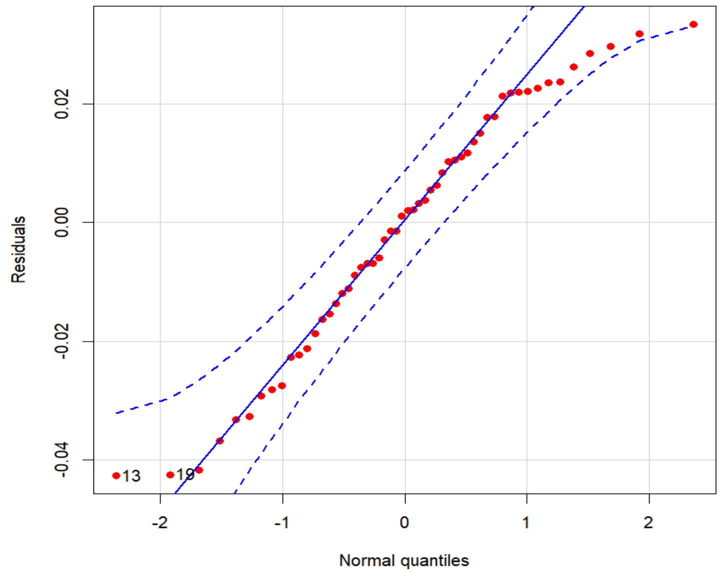

Figure 7: Quantile-Quantile graph of the residuals of the proposed relation

CONCLUSIONS

In this technical note the tabulated empirical data were transformed into graphs of several dimensional curves of parabolic variation (polynomial of order two) and then these were then transformed into a single dimensionless curve of variation power. The dimensional curves and the dimensionless final curve give the value of the connection radius of the toe of the spillways with hydraulic energy dissipation basins. The proposed relationship was ultimately justified by the analysis of the residuals by testing the aspect of normality statistically and graphically.

NUMERICAL EXAMPLE

Consider the following data: H = 2m, P = 40m. Compute the radius of curvature R and compare it with actual one.

According to Eq. (1), R is:

$R = 40 \text{ x } 1.2 \text{ x } (2/40)^{0.6} = 7.955$m.

For the given data, the actual value is R = 8.4m derived from table 3 corresponding to R/P = 0.21.

Thus, the deviation is:

$$100 \times \left| \frac{7.955 - 8.4}{8.4} \right| = 5.3\%$$

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