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TECHNICAL NOTE

ANALYTICAL RELATIONSHIP BETWEEN THE STRICKLER ROUGHNESS COEFFICIENT AND THE ABSOLUTE ROUGHNESS IN ROUGH TURBULENT FLOW REGIME

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ABSTRACT

Manning-type relations do not contain the term viscosity, which limits their use to the field of rough turbulent flow. Strickler roughness coefficient *k* or *n* of Manning are estimated with great difficulty, requiring a proven practical experience. In order to simplify the problem and allow a rapid calculation of *k*, the main objective of the present study is to establish an explicit analytical relation which links *k* and the absolute roughness ε. To do this, it was useful to combine the rational Darcy-Weisbach relationship and the empirical Manning-Strickler formula. After some development, the function $\Phi(k;\varepsilon)=\zeta(\varepsilon^*)$, where $\varepsilon^*=\varepsilon/D_h$ is the relative roughness, was clearly defined. It was possible to calculate the mean value of the function ζ in the practical range of ε^* such that Φ is equal to a constant. This constant is different from that proposed by the literature.

Keywords: Darcy-Weisbach, Friction factor, Manning-Strickler, Relative roughness, Rough turbulent flow, Roughness coefficient.

INTRODUCTION

Although it was developed for pipe-flow, the enthusiasm to use the Colebrook formula (1939) to estimate the friction factor *f* in open-channels is not recent. As an example, in 1958 and 1959, Ackers used the Colebrook formula in open channel flow.

In 1963, a special task group of the American Society of Civil Engineers recommended the use of the Colebrook equation. This group felt that, with the exception of the rough turbulent regime, Colebrook's formula is more reliable than Manning's relationship for which the resistance coefficient *n* is constant.

Another example is that of Frederiksen and De Vries (1965) who compared a large number of friction equations and ultimately decided to use Colebrook's formula to calculate the friction losses in wide channels. More recently, in 1985, on the recommendation of the Committee on Channel Stabilization of the Corps of Engineers, US Army, the Los Angeles District uses the Colebrook formula to calculate the friction losses in the Arizona diversion channel.

In his conclusion, Falvey (1987) makes it clear that most of the major engineering organizations in the world are now using the Colebrook equation to estimate the frictional resistance of both open and closed conduit flows. He said that "The universal use of the Colebrook-White equation is recommended".

In Europe, the preference to use Colebrook's formula for the computation of open channel flow resistance is growing (Hager, 1985).

Note finally that Sinniger and Hager (1989) assert that the Colebrook’s formula remains applicable to any shape of channels and conduits.

On the other hand, several research workers have developed friction factor formulae in the form of the Colebrook equation but whose constants differ (Henderson, 1966; Progress Report of the Task Force, 1963). However, compared to the values used for closed conduit flow, the range of variation of these constants is not at all very large. In fact, several researchers use the same values as those used for closed conduit flow, i.e. they favour the use of the original Colebrook formula as mentioned above (Progress Report of the Task Force, 1963; Ackers, 1958; 1959).

For the calculation of the flow resistance coefficient, Colebrook's formula has the particularity of being applied for ranges of hydraulic parameters far beyond those reached by the tests.

In hydraulic engineering, virtually all flows of interest such as rivers and man-made open channels are considered rough and the occurring flow is in complete turbulent state. The other two regimes of turbulent flow, that is to say smooth and transition are often encountered during laboratory tests but the second regime can take place in some practical cases of flow in pipes (Chow, 1959: Henderson, 1966).

For the establishment of the hydraulically rough flow, the following condition must be satisfied (Hager, 1989):

$$\varepsilon \geq 30\nu\left[Q(gS_0)^2\right]^{-0.2} \tag{1}$$

where ε is the absolute roughness (m), ν is the kinematic viscosity (m^2/s), Q is the volume flow rate (m^3/s), g is the acceleration due to gravity (m/s^2), and S_0 is the channel bed slope.

If the flow is in a complete turbulent flow state, where effects of viscosity are neglected, the computation can be performed by the use of the well known Manning-Strickler formula. Owing to its simplicity of form and to the satisfactory results it lends to practical applications, the Manning-Strickler formula has become the most widely used of all uniform flow formulae for open-channel flow computation. This formula relates the mean velocity V of the flow to the hydraulic radius R_h as follows:

$$V = k\, R_h^{2/3} \sqrt{S_0} \tag{2}$$

where k ($m^{1/3}/s$) is the Strickler resistance coefficient which is also a coefficient of roughness.

According to Christensen (1984), Eq. (2) is restricted to:

$$0.004 \le \varepsilon / R_h \le 0.04 \tag{3}$$

Moreover, the range of applicability of Eq. (2) is also limited by both slope and observed data (Chow, 1959; Falvey, 1987).

The coefficient k is equal to the inverse of Manning's coefficient n, i.e. $k = 1/n$ and varies from 20 for rough stone and rough surface to 80 $m^{1/3}/s$ for smooth concrete and cast iron. With the exception of the Darcy-Weisbach friction factor, most coefficients of resistance are all estimated empirically and are applied only to rough turbulent flow. There is no exact method to estimate the coefficient of resistance k or n so much so that different people can get different values. It takes a lot of experience to be able to estimate somewhat the average value of these coefficients.

For mainly man-made channels of trapezoidal, circular shape or rectangular one Achour and Amara (2014, 2020a, 2020b) derive analytical relationships for estimating the Manning’s coefficient.

For all these reasons, the idea of proposing a relationship which makes it possible to estimate the value of k from the known value of the absolute roughness constitutes the basis of the present technical note. The idea was proposed first by Hager in 1987. Nonetheless, the authors believe that Hager's theoretical approach is not rigorous and somewhat approximate, suggesting correcting Hager's relationship based on a more elaborate and congruous procedure.

Given that the measurement of the absolute roughness ε is not very precise, the relation must be such that k does not undergo large variations under the effect of the relative error made on the measurement of ε. The k-ε relationship that this study intends to establish will be very useful for man-made open channels.

FRICTION FACTOR

According to Colebrook (1939), the friction factor *f*, known also as the Darcy-Weisbach friction factor, is expressed as:

$$f^{-1/2} = -2\log\left(\frac{\varepsilon^*}{3.7} + \frac{2.51}{R\sqrt{f}}\right) \tag{4}$$

where $\varepsilon^* = \varepsilon / D_h$ is the relative roughness, D_h is the hydraulic diameter equal to four times the hydraulic radius R_h, and R is the Reynolds number.

The Darcy-Weisbach relationship, which may also be applied to uniform flow in open channels, relates the channel slope S_0 as:

$$S_0 = \frac{f}{D_h}\frac{V^2}{2g} \tag{5}$$

For higher values of R, the flow may be in a complete turbulent flow state, i.e. rough turbulent flow. It is characterised by the friction factor f also governed by Eq. (4) for $R \to \infty$, yielding:

$$f^{-1/2} = -2\log\left(\frac{\varepsilon^*}{3.7}\right) \tag{6}$$

Combining Eqs. (2) and (5) results in:

$$f^{-1/2} = \frac{k}{4^{2/3}\sqrt{2}\sqrt{g}} D_h^{1/6} \tag{7}$$

Eq. (7) shows clearly that Eq. (6) can be formulated as the following power law**:**

$$f^{-1/2} = \beta D_h^{\vartheta} \tag{8}$$

where $\vartheta = 1/6$ is assumed as a constant and $\beta = 0.280615\, k / \sqrt{g}$.

To verify the previous average value of ϑ, use was made of the theoretical computation values of equation (6) for different values of both hydraulic diameters and roughnesses. Considering the following practical ranges 0.10 m ≤ D_h ≤ 10 m and 0.1 mm ≤ ε ≤ 50 mm, corresponding to a variation of the relative roughness ε^* within the wide practical range 0.0001 to 0.05, the exponent ϑ was found to vary slightly from 0.092 to 0.22 so that ϑ could be effectively assumed as a constant, independent on both D_h and ε. Nevertheless, 1/6 ≈ 0.167 seems to be a slightly overestimated value of the exponent ϑ, knowing that 0.156 would be the most appropriate value. In the above mentioned ranges of D_h and ε, the parameter β has been found strongly dependent on D_h and ε values, since its corresponding range was from 3.60 to 9.10. The obtained coefficient of determination R^2, greater than 0.99, proves the goodness of the fit of the derived relationships (7) or (8). Taking into account the range of variation thus obtained for β, it is concluded that k varies between 40.3m$^{1/3}$/s and 102m$^{1/3}$/s theoretically. It is important to note that these values have been observed in practice (George et al., 1989).

K-ε RELATIONSHIP

Eliminating $f^{-1/2}$ between Eqs. (6) and (7) yields:

$$\frac{k}{\sqrt{g}} = \frac{7.127}{D_h^{1/6}} \log\left(\frac{3.7}{\varepsilon^*}\right) \tag{9}$$

To make equation (9) dimensionless, multiply the two sides by $\varepsilon^{1/6}$. Thus, one may obtain the following result:

$$\frac{k\,\varepsilon^{1/6}}{\sqrt{g}} = 7.127\ \varepsilon^{*\,1/6} \log\left(\frac{3.7}{\varepsilon^*}\right) \tag{10}$$

Let us denote by Φ the following function:

$$\Phi = \frac{k\,\varepsilon^{1/6}}{\sqrt{g}} \tag{11}$$

It is therefore clear, from both Eqs. (10) and (11), that on the one hand the function Φ depends exclusively on the relative roughness ε^* and is not, on the other hand, a constant as stated by Hager (1987) who derived that $\Phi = 8.2$ = constant.

The function Φ is represented graphically in Fig. 1 as a function of the relative roughness ε^*, according to Eq. (10).

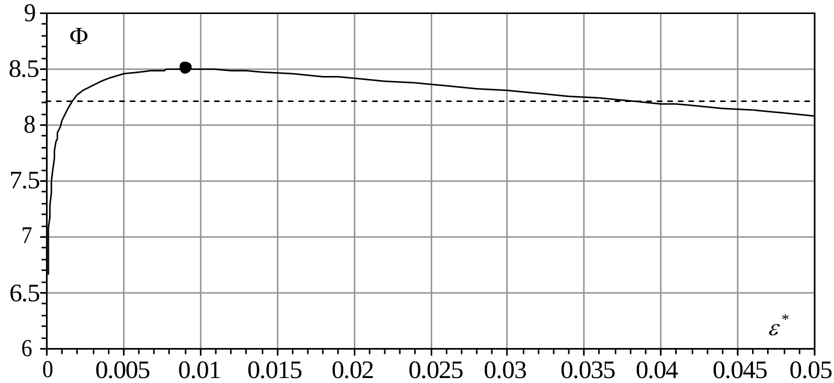

Figure 1. Variation of Φ versus ε^* according to Eq. (10). (•): Φ_{max} . (- - -): Φ = 8.2 as Hager's result (1987).

Note how Φ increases sharply within a rather narrow range for lower values of the relative roughness ε^* and increases slightly beyond the horizontal dashed line, tending finally to the maximum value Φ_{max} . Beyond Φ_{max} , the curve decreases slightly and its variation may be approximate linearly to ε^* . Setting the first derivative of Eq. (10) with respect to relative roughness ε^* equal to zero, i.e.:

$$\frac{d\Phi}{d\varepsilon^*} = \frac{0.515869\ \ln\left(3.7/\varepsilon^*\right) - 3.09522}{\varepsilon^{*\,5/6}} = 0 \tag{12}$$

yields:

$$\varepsilon^* = 0.00917128$$

Inserting this result into Eq. (10) results in:

$$\Phi_{max} = 7.127 \times 0.00917128^{\ 1/6} \times \log\left(3.7/0.00917128\right) = 8.49665777 \approx 8.497$$

Inserting this result into Eq. (10), the following $k_{max} - \varepsilon$ relationship is obtained:

$$\frac{k_{max}\varepsilon^{1/6}}{8.497\sqrt{g}} = 1 \tag{13}$$

In order to compare our result with that of Hager (1987), let us calculate the mean value of the function Φ by integrating Eq. (10) in the practical range [0.0001; 0.05] of the relative roughness ε^* mentioned previously. The mean value Φ_m is as:

$$\Phi_m = \frac{1}{(0.05-0.0001)} \int_{0.05}^{0.0001} 7.127\, \varepsilon^{*1/6} \log\left(3.7/\varepsilon^{*}\right) d\varepsilon^{*} \tag{14}$$

That is:

$$\Phi_m = \frac{3.09522\, \varepsilon^{*7/6}}{(0.05-0.0001)} \left[0.857143 \ln\left(\varepsilon^{*-1} \right) + 1.85612 \right]_{0.0001}^{0.05} \tag{15}$$

After calculations, the final result is:

$$\Phi_m = 8.31470769 \approx 8.315$$

Inserting this result into Eq. (11) and rearranging results in the following k-ε relationship:

$$\frac{k\, \varepsilon^{1/6}}{8.315\sqrt{g}} = 1 \tag{16}$$

This is the relationship that this study sought. It explicitly relates the Strickler roughness coefficient k to the absolute roughness ε. Thanks to the power 1/6, even if one assumes that the measurement of the absolute roughness ε is affected by a relative error of 20%, k only undergoes a variation of 20/6 ≈ 3.34%.

Note finally that calculation shows that the deviation between Eq. (16) and that of Hager is about 1.4%.

CONCLUSIONS

The main objective of the study is to determine the relationship between the Strickler roughness coefficient k and the absolute roughness ε in the rough turbulent flow domain, then compare it to the Hager relation established in 1987. To do so, it was necessary to combine the Darcy-Weisbach and Manning-Strickler formulae. After performing some manipulations, it was possible to define the dimensionless function Φ which contains both k and ε [Eq. (11)]. It has been shown that this function is not a constant, but it depends exclusively on the relative roughness $\varepsilon^* = \varepsilon / D_h$ [Eq. (10)]. The graphical representation showed, in particular, that Φ reached a maximum for $\varepsilon^* = 0.00917128$ corresponding to $\Phi_{max} \approx 8.497$. Finally, the mean value of the function Φ was calculated as $\Phi_m \approx 8.315$ thus deviating by 1.4% from the value proposed by Hager.

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