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THEORETICAL AND EXPERIMENTAL ANALYSIS OF SEQUENCE DEPTH RATIO AND ENERGY LOSS IN AN ABRUPTLY ENLARGED TRAPEZOIDAL CHANNEL

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ABSTRACT

Among the hydraulic parameters describing the classical hydraulic jump are the inflow Froude number, the sequence depth ratio, the jump surface profile, and the energy loss.

Literature analysis showed that these parameters varied according to the shape of the studied section and the upstream flow conditions, such as the flow discharge and the initial height at the toe of the jump.

The present study examines, theoretically and experimentally, the effect of enlargement on the sequence depth ratio, the relative energy loss, and the jump surface profile. The study was based on an experimental model in which an inserted abruptly enlarged trapezoidal channel was directed towards a rectangular channel.

Comparison of experimental and theoretical results showed that the physical and economic interest of flow through the considered section is conditioned by the frictional force effect due to the enlargement. These findings can be useful in the design of energy stilling basins and in hydraulic jump performance.

Keywords: Hydraulic jump; Stilling basin; Trapezoidal channel; Abrupt expanding; Rectangular channel; Froude number.

INTRODUCTION

Hydraulic jumps in gradual or abrupt enlargements have been extensively discussed over the past two decades. Hydraulic jumps in gradually enlarged stilling basins, also called circular or radial hydraulic jumps, have been studied in particular by Rubatta (1963 and 1964), for divergent and converging channels, and by Koloseus (1969), who considered a circle segment as a control volume by formulating an equation for the sequence depth ratio. This explains the linear variation in the longitudinal surface profile. The range of observations is relatively wide, but this can be attributed to measurement difficulties. The authors showed that the relative jump length is smaller than in a prismatic channel.

Arbhabhirama (1971) extended the previous study by assuming an elliptical quarter longitudinal surface profile. The relationship for the sequence depth ratio is presented by analogy with the equation of Bélanger (1828), and the concordance with the observations is favorable for a range of divergence angles examined (10° to 26°). The comparison between their approach and that of Koloseus (1969) indicates a better fit of data by assuming a nonlinear surface profile and that the jump position can be estimated with an empirical relationship.

Concerning energy stilling basins of sharply enlarged rectangular cross-section, numerous studies have been carried out and the resulting hydraulic jump has been qualified as a spatial hydraulic jump (Unny, 1963), (Sharma, 1965) and (Macha, 1963).

Applying the momentum theorem, Hager (1985) gave the sequence depth ratio Y as a function of the inflow Froude number Fr_1 and a widening ratio $\beta = B/b$. Taking into account the frictional force F_x caused by the enlargement and the fact that the liquid entrained in the enlargement area has been pumped.

Using a symmetric (bilateral) model and the corresponding unilateral model (Herbrand, 1973), they highlighted the inconsistency of the depth-width parameter and experimentally demonstrated the insignificance of the parameters in question. The data from Herbrand (1973) are particularly interesting because they apply to the case where the jump toe is located at the level of a transition section.

In contrast to this type of jump, Rajaratnam (1968) examined the formation of the downstream jump of the transition section. Novel theoretical approaches are proposed by Khattaoui and Achour (2012), applying the equation of momentum, to the determination of the sequence depth ratio and the hydraulic jump efficiency in a compound rectangular channel. Inspired by the work of Achour (2000), relating to the jump hydraulic in a sharply enlarged circular gallery. The authors illustrated the influence of each geometric and hydraulic variable; the theoretical resulting equations are represented on graphs. These showed the variation in the sequence depth ratio as well as the yield as a function of the inflow Froude number, the width ratio of the minor bed and the major bed, as well as their depth ratio.

For the hydraulic jump in trapezoidal channels, Kateb et al. (2013) analyzed the effect of a positive step on this type of jump for the upstream relative height of the trapezoidal channel of $0.03 \leq M \leq 0.09$. The authors found that the forced A-type jump by positive step reduced the sequence depth ratio compared to the classical jump. The same result is obtained with regard to the variation of relative positive step height. Ebrahimiyan (2020) presented a comparative study on the effects of suspended sediments on the sequence depth ratio, the jump length and the energy loss in two types of channels: the rectangular and trapezoidal channels.

A new model is proposed by Brakeni et al. (2021). The authors aim to minimize the agitation of the stilling basin flow in the form of a spiral by shortening the length of the basin. This device not only favors a more stable flow in the dissipation zone, but also dissipates the energy at the level of the watercourses, whatever the hydraulic characteristics.

For a trapezoidal channel, Benmalek et al. (2022) proposed empirical relationships between the sill relative height as well as the sequence depth ratio with the inflow Froude number and the jump compactness. Also, the effect of the compactness ratio on the jump surface profile is being studied.

This study aims to theoretically and experimentally find the effect of the abruptly enlarged trapezoidal channel being directed towards a rectangular channel on the jump sequence depth ratio and the relative energy loss. Thus, a form of the jump surface profile has been proposed. The purpose of this analysis is to verify the economic interest of the considered section in the design of the stilling basin.

THEORETICAL ANALYSIS

Fig. 1 shows a hydraulic jump controlled by a thin sill in a trapezoidal channel abruptly widened to a rectangular channel. The controlled jump formation is conditioned by setting a sill on the flow downstream.

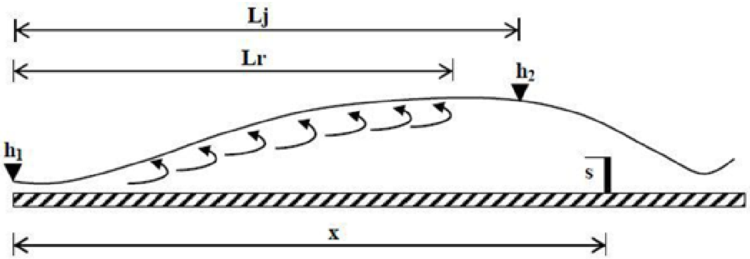

Figure 1: A controlled jump caused by a thin sill.

The literature study showed that the hydraulic jump is governed by the momentum equation applied between its initial and final sections. In this case, it is a question of applying the equation of momentum for a controlled hydraulic jump by a thin sill moving in an abruptly enlarged trapezoidal channel towards a rectangular channel as shown in Fig. 2.

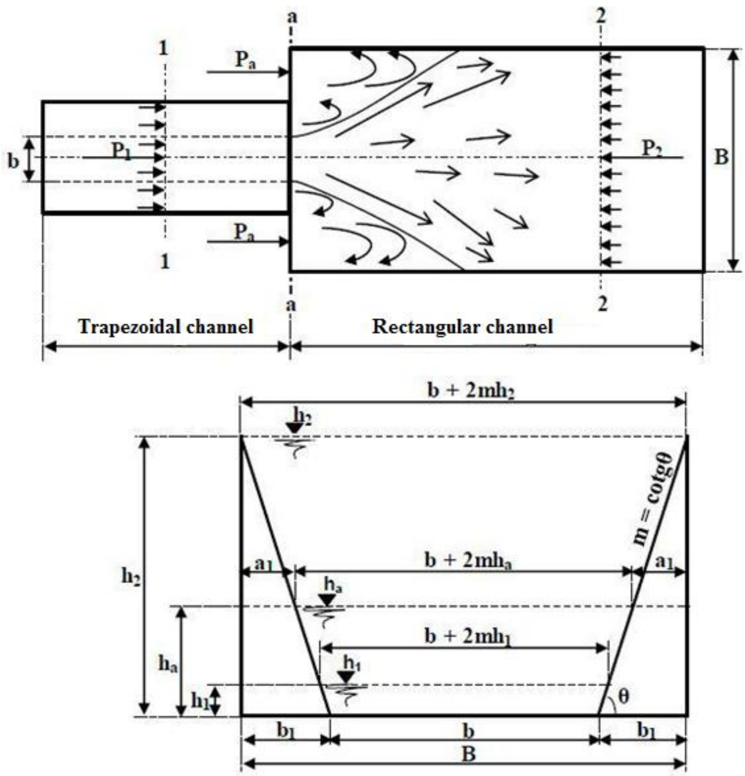

Figure 2: Forces acting on controlled hydraulic jump by sill in an abruptly enlarged trapezoidal channel towards a rectangular channel

The momentum equation will be applied, taking into consideration the following simplifying assumptions: The pressure distribution in sections 1-1 and 2-2 is hydrostatic. Friction on the walls and bottom of the channel along the short distance between 1-1 and 2-2 is negligible compared to the pressure drop due to the turbulence created by the jump. The velocities of the different liquid nets in each of sections 1-1 and 2-2 are parallel to the average velocity V and are considered uniform. Air resistance is negligible. The application of Newton's second law gives:

$$\zeta_2 \rho Q V_2 - \zeta_1 \rho Q V_1 = P_1 + P_a - P_2 \quad (1)$$

Where from:

P_1, P_2 and P_a are the external forces. These forces are:

The hydrostatic pressure force P_1 applied to section 1-1 (trapezoidal section) [Kg.m.s^{-2}]

The hydrostatic pressure force P_2 applied to section 2-2 (rectangular section) [Kg.m.s^{-2}]

The relative pressure force P_a on the side walls with a length of (b-b) applied to the section (a-a) (abruptly enlargement) [Kg.m.s^{-2}].

ζ_1 and ζ_2 are the momentum correction factors that are considered equal to the unit since the velocity distribution is assumed to be uniform.

ρ is the density of the moving liquid [Kg.m^{-3}]

Q is the volume flow. [m^3.s^{-1}]

V is the average velocity of the flow. [m.s^{-1}]

Taking into account all these forces, Eq. (1) is written.

$$P_1 + P_a + \rho Q V_1 = P_2 + \rho Q V_2 \tag{2}$$

Note that the application point of each of these forces coincides with the gravity center of the section under consideration. From Fig. 2, these forces can be expressed by applying the laws of hydrostatics.

$$\bar{h}_1 A_1 + \bar{h}_a A_a + \frac{Q^2}{gA_1} = \bar{h}_2 A_2 + \frac{Q^2}{gA_2} \tag{3}$$

Where from: A_1 is a [m^2] cross-section of the trapezoidal canal at (1-1) given by:

$$A_1 = bh_1 + mh_1^2 \tag{4}$$

h_1 is a jump upstream from the water depth [m].

m is a cotangent of the slope angle in the trapezoidal channel [-].

A_a is a cross section [m^2] at the level of enlargement (a-a) given by:

$$A_a = (B - b)h_a - mh_a^2 \tag{5}$$

h_a is the jump water depth at an enlargement level [m].

A_2 is the cross section [m^2] of a rectangular channel at level 2-2 given by:

$$A_2 = Bh_2 \tag{6}$$

Where: h_2 is a jump downstream from the water depth [m].

The centers of gravity for h_1, h_a, and h_2, [m] for each section are given by Eqs. (7), (8) and (9): For the trapezoidal section, Eq. (4) writes:

$$\bar{h}_1 = \frac{h_1^2}{6} \frac{\left(3b + 2mh_1\right)}{A_1} \tag{7}$$

At an enlargement level, the gravity center is given by the following Eq. (8).

$$\frac{\bar{h}_a^2}{6} \frac{\left[3\left(B - b\right) - 4mh_a\right]}{A_a} \tag{8}$$

For a rectangular section at level 2-2, the gravity center is given by Eq. (9).

$$\bar{h}_2 = \frac{1}{2} h_2 \tag{9}$$

Inserting Eqs. (4), (5), (6), (7), (8), and (9) into Eq. (3) results in:

$$\begin{aligned} &\frac{h_1^2}{6}\left(3b + 2mh_1\right) + \frac{h_a^2}{6}\left(3B - 3b - 4mh_a\right) + \frac{\left(bh_1 + mh_1^2\right)^2}{\left(b + 2mh_1\right)} Fr_1^2 = \\ &\frac{1}{2} Bh_2^2 + \frac{\left(bh_1 + mh_1^2\right)^3}{\left(b + 2mh_1\right) Bh_2} Fr_1^2 \end{aligned} \tag{10}$$

b is the base width of the trapezoidal channel [m].

B is the width of the rectangular channel [m].

Fr_1 is the inflow Froude number defined by the universal relation for the trapezoidal channel by Eq. (11).

$$Fr_1^2 = \frac{Q^2\left(b + 2mh_1\right)}{gA_1^3} \tag{11}$$

g is the gravitational acceleration [$m.s^{-2}$].

When dividing the equality in Eq. (10) by b,

$$\frac{h_1^2}{6}\left(3+2\frac{mh_1}{b}\right)+\frac{h_a^2}{6}\left(3\frac{B}{b}-3-4\frac{mh_a}{b}\right)+\frac{h_1^2\left(1+\frac{mh_1}{b}\right)}{\left(1+2\frac{mh_1}{b}\right)}Fr_1^2=$$
$$\frac{1}{2}\frac{B}{b}h_2^2+\frac{h_1^3\left(1+\frac{mh_1}{b}\right)^3}{\frac{B}{b}h_2\left(1+2\frac{mh_1}{b}\right)} \tag{12}$$

When Eq. (12) is divided by h_1^2,

$$\left(\frac{1}{2}+\frac{1}{3}\frac{mh_1}{b}\right)+\frac{h_a^2}{h_1^2}\left(\frac{1}{2}\frac{B}{b}-\frac{1}{2}-\frac{2}{3}\frac{mh_a}{b}\right)+\frac{\left(1+\frac{mh_1}{b}\right)}{\left(1+2\frac{mh_1}{b}\right)}Fr_1^2=$$
$$\frac{1}{2}\frac{B}{b}\frac{h_2^2}{h_1^2}+\frac{h_1\left(1+\frac{mh_1}{b}\right)^3}{\frac{B}{b}h_2\left(1+2\frac{mh_1}{b}\right)}Fr_1^2 \tag{13}$$

When setting, $Y=h_2/h_1$ (the jump sequence depth ratio at levels 1-1 and 2-2) and $Y_a=h_a/h_1$ (the jump sequence depth ratio at level a-a)

$\beta=B/b$ (expansion ratio)

$M=(mh_1)/b$ (the upstream relative height of the trapezoidal channel)

$MY_a=(mh_a/b)$, $m=\text{cotg}\ (\theta)$

Eq. (13) will be written.

$$\left(\frac{1}{2}+\frac{1}{3}M\right)+Y_a^2\left(\frac{1}{2}\beta-\frac{1}{2}-\frac{2}{3}MY_a\right)+\frac{\left(1+M\right)^2}{\left(1+2M\right)}Fr_1^2=$$
$$\frac{1}{2}\beta Y^2+\frac{1}{Y}\frac{\left(1+M\right)^3}{\beta\left(1+2M\right)}Fr_1^2 \tag{14}$$

Finally, we obtain Eq. (14) giving the variation of the inflow Froude number as a function of the upstream relative height *M*, the sequence depths ratio *Y*, Y_a, and the expansion ratio.

$$Fr_1^2 = \frac{\frac{(1+2M)}{2(1+M)^2}\left[\beta Y^2 - \left(1+\frac{2}{3}M\right) - Y_a^2\left(\beta - 1 - \frac{4}{3}MY_a\right)\right]}{\left[1 - \frac{(1+M)}{\beta Y}\right]} \tag{15}$$

The relative pressure force on the length of the side walls (*B*-*b*) is given by $P_a=Y_a^2[\beta-1-(4/3)MY_a]$. It can be demonstrated experimentally that for the upstream and downstream flow conditions, the flow pattern is constant when the liquid is pumped into the separation zone (A-A) according to the case shown in Fig. 2.Therefore, the minimum flow conditions are P_a=0 and Y_a=0 (because no negative pressure can appear). For *M* = 0 (the case of the rectangular channel), Eq. (15) takes the form of Eq. (16).

$$Fr_1^2 = \frac{\beta Y\left(\beta Y^2 - 1\right)}{2\left(\beta Y - 1\right)} \tag{16}$$

This is the expression of Hager (1985) relating to the evaluation of hydraulic jumps in an abruptly enlarged rectangular channel. In the case where the pressure force P_a is taken into account and for *M* = 0 (the case of the rectangular channel), as follows:

$$Y^3 - Y\left(\beta + Y_a^2 - \beta Y_a^2 + 2\beta Fr_1^2\right) + 2\beta^2 Fr_1^2 = 0 \tag{17}$$

This is the expression of Herbrand (1973) relating to the spatial hydraulic jump. The implicit form of Eq. (15) can only be easily solved if some simplifying assumptions about the non-uniform distribution of pressure at the separation zone (*B*-*b*) are available. The relative pressure force on the side walls (*B*-*b*) is neglected by the pumping (Hager, 1985). Taking these assumptions into account, the resulting Eq. (15) takes the form of Eq. (18):

$$Fr_1^2 = \frac{\frac{(1+2M)}{2(1+M)^2}\left[\beta Y^2 - \left(1+\frac{2}{3}M\right)\right]}{\left[1 - \frac{(1+M)}{\beta Y}\right]} \tag{18}$$

EXPERIMENTAL ANALYSIS

Description of the experimental model

The experimental model (Fig. 3) consists of a supply basin connected to a measuring channel of symmetrical trapezoidal straight section of slope inclination angle 73° by a circular PVC pipe of 115 mm in diameter. The assembly operates in a closed circuit in which two pumps are inserted in parallel, feeding a convergent load leading into the measuring channel. The measuring channel of trapezoidal cross-section and length of 5 m is connected, in its downstream part, to a second channel of rectangular straight section, in which is inserted a rectangular weir allowing direct measurement of flow (Fig. 4). The experimental study aims to analyze the controlled hydraulic jump by a thin sill evolving in an abruptly enlarged trapezoidal channel towards a rectangular channel (Fig. 5). The experiments were conducted under an initial flow height of h_1=3 cm. A wide range of inflow Froude numbers was obtained, corresponding to $4.33<Fr_1<8.41<$ and an upstream relative height M=0.05. The formation of a controlled jump is conditional on the creation of a sill in the direction of the downstream flow. For this purpose, sills of different heights were used. A representative sample of experimental measurement points for each of the parameters involved in the phenomenon also made it possible for this type of jump to achieve significant results. These parameters are: the flow discharge Q, the initial height h_1, the final height h_2, the sill height s, and the relative sill position x at the toe of the jump. These make it possible to compose the following dimensionless products: the inflow Froude number Fr_1, the sequence depth ratio $Y=h_2/h_1$ and the relative length of the surface profile $X=x/Lr$.

Figure 3: Measurement Channel

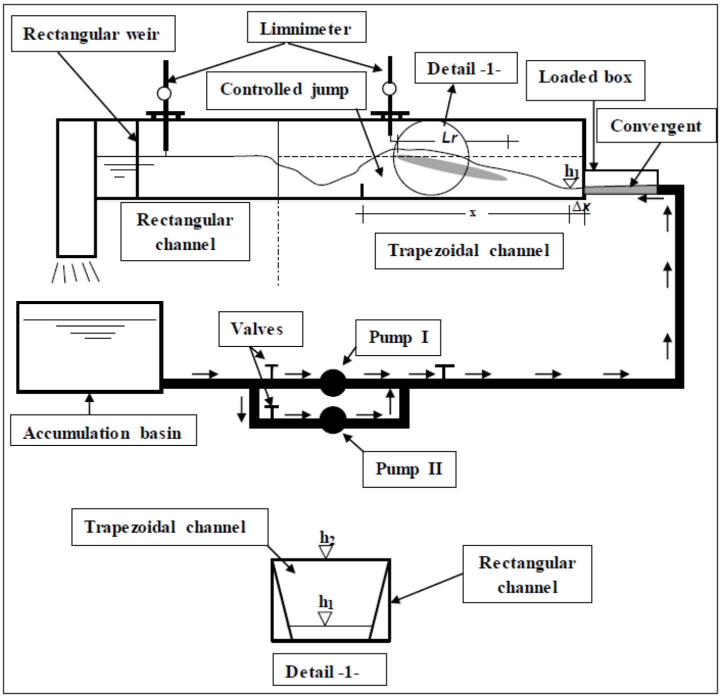

Figure 4: Description of the Experimental Model

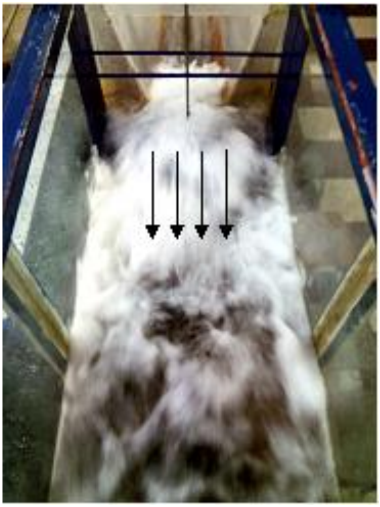

Figure 5: Photos of the hydraulic jump in an abruptly enlarged trapezoidal section towards a rectangular section

EXPERIMENTAL RESULTS

Sequence depth ratio

Fig. 6 shows the variation in the jump sequence depth ratio $Y=h_2/h_1$ as a function of the inflow Froude number Fr_1 in an abruptly enlarged trapezoidal channel towards a rectangular channel. The adjustment of the experimental data by the least squares method showed that the variation $Y=f(Fr_1)$ is linear of type $Y=aFr_1+b\approx 0.76Fr_1+0.41$ for Froude number values in the range $4.33\leq Fr_1\leq 8.41$, an expansion ratio $\beta=3$ and an upstream relative height $M=0.05$.

The experimental results show that for a given Froude number value, the jump sequence depth ratio suffered a significant shift from the results obtained by applying Eq. (15). This shift is due to the initial simplifying assumptions and neglect of the frictional forces of the liquid against the canal wall. On the other hand, the values of Y plotted according to Eq. (17) are very important, and this is due to the neglect of the relative pressure force P_a on the side walls (B-b). In the case where the force P_a is taken into account, the values of the sequence depth ratio Y decrease. As a result, it is more advantageous for economic reasons to eliminate the effect of the Pa force through a system of continuous pumping of stagnant water at the level of enlargement. This solution causes a decrease in Y, which gives a minimum configuration regarding the height of the stilling basin.

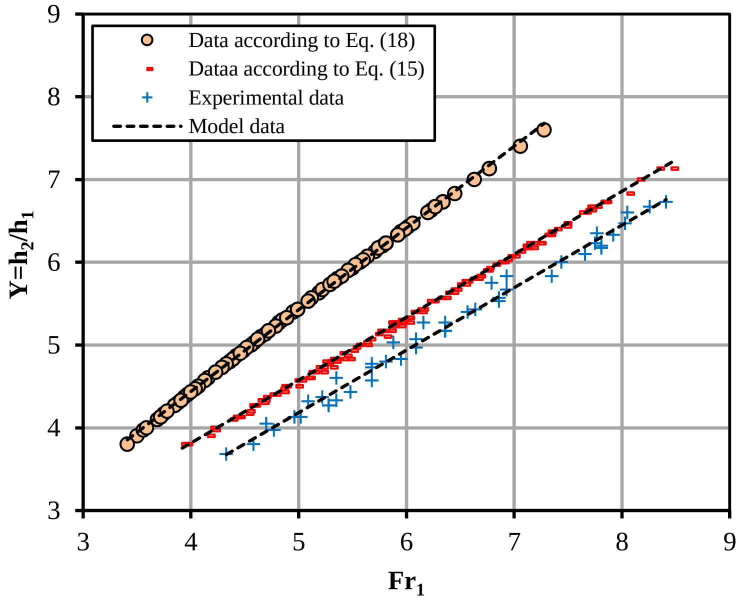

Figure 6: Experimental variation of $Y=f(Fr_1)$ in an abruptly enlarged trapezoidal channel for $\beta=3$

Fig. 6 shows a significant lag between the line of Eq. (15) and the experimental data. This shift is attributed to the initial simplifying assumptions when applying the momentum theorem. This makes it possible to correct Eq. (15) by experimental data. Fig. 7 shows the experimental variation of the Fr_{1exp} inflow Froude number according to the theoretical values Fr_{1th}. This figure denotes a shift of the scatter plot from the first bisector that increases as the number of inflow Froude increases. To overcome this discrepancy, we propose an adjustment of the theoretical relationship by the least squares method, based on the experimental results. The adjustment between the two parameters resulted in a line in Fig. 8 which passes through the slope origin at 1.07.

$$Fr_{1exp} = 1.07 Fr_{1th} \tag{19}$$

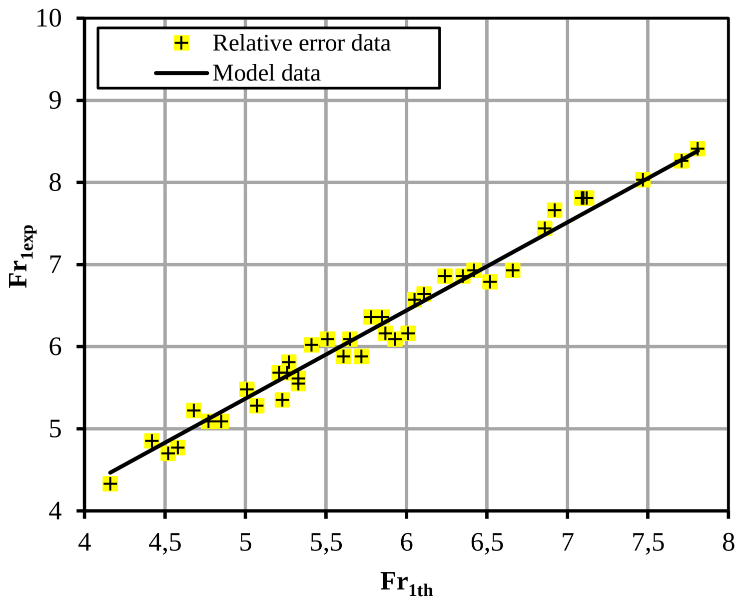

Figure 7: Graph of Fr_{1exp} and Fr_{1th}

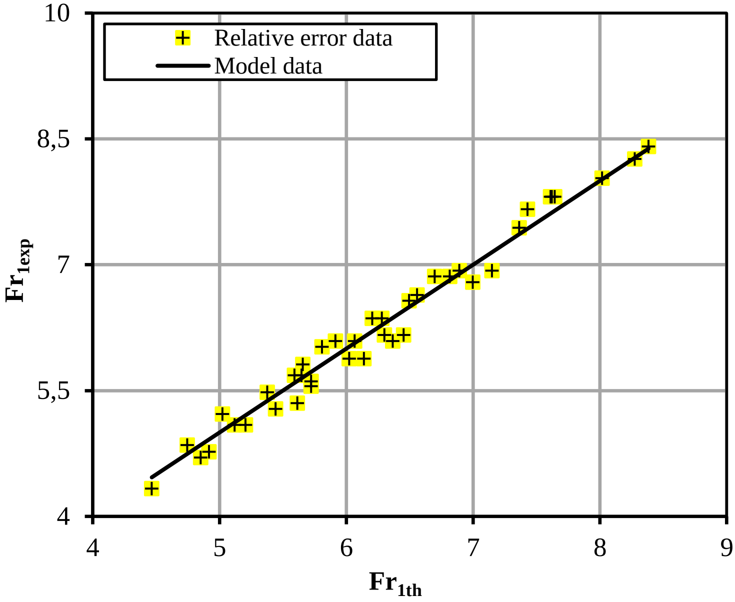

Figure 8: Correction of Fr_{1th} by experimental data

The purpose of this adjustment is to correct Eq. (15), by the slope coefficient of 1.07.

$$Fr_1^{\,2} = 0.57\,\frac{\dfrac{(1+2M)}{(1+M)^2}\left[\beta Y^2 - \left(1+\dfrac{2}{3}M\right) - Y_a^2\left(\beta - 1 - \dfrac{4}{3}MY_a\right)\right]}{\left[1 - \dfrac{(1+M)}{\beta Y}\right]} \tag{20}$$

Fig. 7 shows that the correction of Eq. (15) minimized the discrepancy between the line representing experimental and theoretical data of Y.

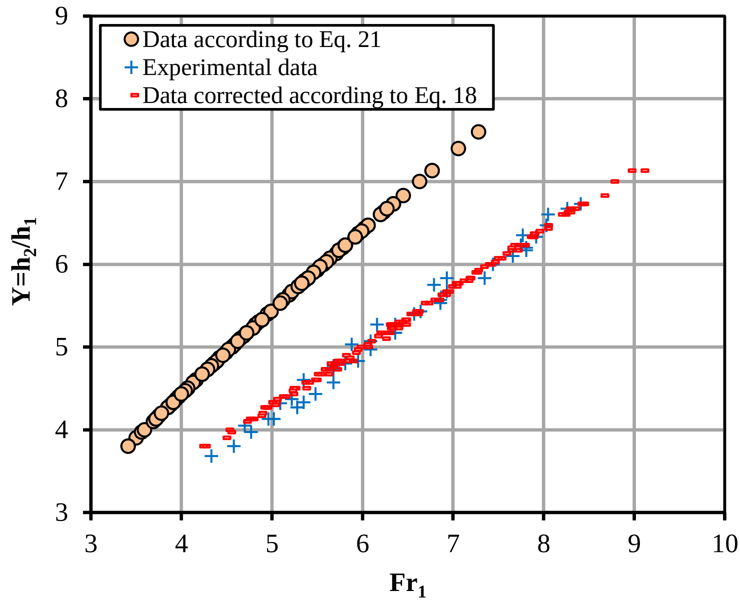

Figure 9: Correction of Eq. (15)

The jump surface profile

The jump surface profile $y=f(x)$ was represented for a value of h_1 = 3 cm, such as: $y=(h_x-h_1)/(h_2-h_1)$, $X=x/Lr$ (Fig. 10).

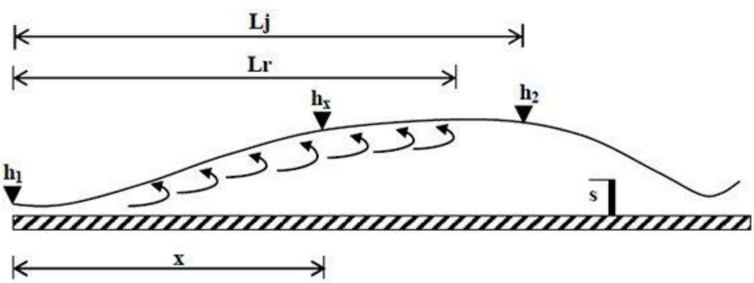

Figure 10: Characteristics of the jump surface profile

Fig. 11 represents the jump profile surface. The analysis of the experimental data corresponding to the classical jump shows that the variation of $y=f(X)$ is of the hyperbolic tangent form where $y=a\mathrm{Tanh}(bX)$, given by Eq. (21):

$$y = 1.09\mathrm{Tanh}(1.90X) \quad (21)$$

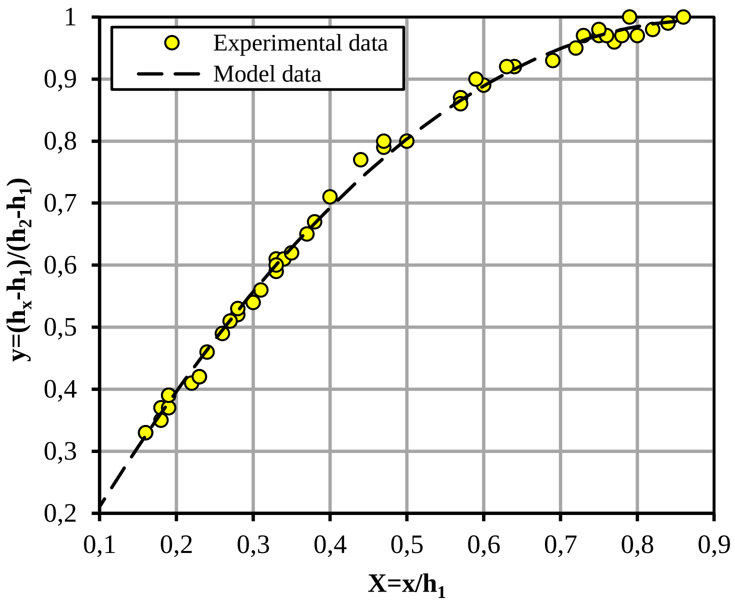

Figure 11: The jump surface profile

Eq. (21) is close to the equation of Bakhmetev and Matzky (1936) relating to the hydraulic jump in a rectangular channel y = Tanh (1.50X) where a = 1 and b = 1.50. Increasing the coefficient b generates an increase in the water level and a reduction in the length of jump, which offers an economic advantage for the length of the stilling basin.

Relative energy loss

The jump energy loss ΔH is by definition the difference between the initial and final charges, $\Delta H=H_1-H_2$. It is determined analytically using Bernoulli's theorem. Assuming that the channel is horizontal, Eq. (22) is obtained.

$$\Delta H = \left(h_1 + \frac{Q^2}{2gA_1^2}\right) - \left(h_2 + \frac{Q^2}{2gA_2^2}\right) \tag{22}$$

The relative energy loss is defined by Eq. (23) where $\eta=\Delta H/H_1$.

$$\eta = 1 - \frac{\left(h_2 + \frac{Q^2}{2gA_2^2}\right)}{\left(h_1 + \frac{Q^2}{2gA_1^2}\right)} \tag{23}$$

By introducing Eqs. (7, 9 and 14) into Eq. (23) with $Y=h_2/h_1$, $M=mh_1/b$ and $\beta=B/b$ we obtain Eq. (24).

$$\eta = 1 - \frac{Y + \dfrac{(1+M)^3 Fr_1^2}{2Y^2\beta^2(1+2M)}}{1 + \dfrac{(1+M)Fr_1^2}{2(1+2M)}} \tag{24}$$

The graphical representation of Eq. (24) is given by Fig. 12. The latter shows that the relative energy loss for a hydraulic jump in an abruptly enlarged trapezoidal channel towards a rectangular channel can reach 84% for an expansion ratio β=3 and for a variation of Froude number $3>Fr_1<8$. This yield is significant in the choice of the most advantageous form of the energy stilling basin.

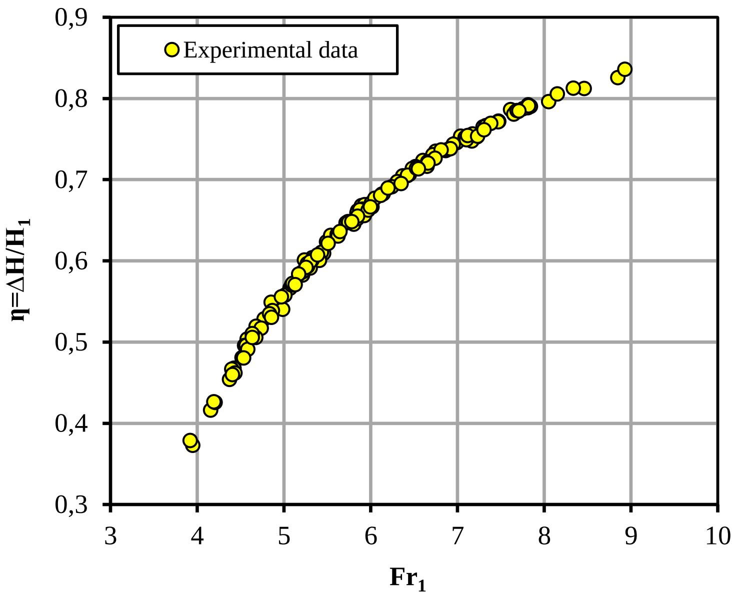

Figure 12: Relative energy loss, Δ*H*/*H*$_1$, in a horizontal trapezoidal channel abruptly enlarged towards a rectangular channel for different experimental Froude numbers and a *β*=3.

CONCLUSION

Through this study, some characteristics of hydraulic jump in a trapezoidal channel abruptly enlarged towards a rectangular channel were examined. The study analyzed the effect of enlargement on the jump sequence depth ratio as a function of the inflow Froude number and surface profile.

The experimental results obtained showed that the relative error is important in relation to the theoretical relationship and that the adjustment lines diverge as the values of the Froude number increase. This discrepancy is due to the initial simplifying assumptions. These results were used to correct the theoretical relationship with the experimental data and made it possible to determine a correction coefficient for the theoretical relationship.

With regard to the elimination of the frictional force due to enlargement, the pumping of stagnant water into the widening area is recommended to minimize the jump sequence depth ratio and decrease the height of the energy stilling basin. The experimental results of the jump surface profile $y=f(X)$ show that this function is of the hyperbolic tangent type.

The relative energy loss for a hydraulic jump in a trapezoidal channel abruptly enlarged towards a rectangular channel can reach 84% for an expansion ratio $\beta=3$ and a variation of Froude number $3>Fr_1<8$. This yield is significant in the choice of the most advantageous form of the energy stilling basin.

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