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CONTROL OF THE HYDRAULIC JUMP BY A THIN-CRESTED SILL IN A RECTANGULAR CHANNEL NEW EXPERIMENTAL CONSIDERATIONS

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ABSTRACT

The control of the hydraulic jump by a thin-crested sill was not studied from the theoretical point of view; experimental research only was able to investigate it quite appropriately, even though the tests undertaken in 1950 were quite restrictive. Although these tests are old, the conclusions of this research are still in effect today. The control of the hydraulic jump by a thin-crested sill is governed by three dimensionless parameters namely the incident Froude number F_1, the relative position X/h_2 of the sill, and the relative sill height $S = s/h_1$. The 1950 tests involved only three values of the relative position of the threshold, that is X/h_2 = 3; 5 and 10. This resulted in a diagram of three curves showing the variation in the relative sill height S as a function of the incident Froude number F_1. If the point defined by the coordinates (S; F_1) is located between two curves of the diagram, then it is necessary to carry out an interpolation that is quite binding.

By designing and using an adequate hydraulic installation, the current study brings something new from an experimental point of view by involving a very wide range of the relative position X/h_2 of the sill since 15 values of this parameter are considered. This quantitative supplement helped to generalize the problem for any flow condition. Even better, it allowed deriving an experimental equation enabling the explicit calculation of the position X of the sill, thus avoiding any interpolation operation. What is sought in general in the problem of controlling the hydraulic jump is to find the right value of the sill position X so that the hydraulic jump is formed completely on the stilling basin such that its length L_j is approximately equal to X. The present study responds perfectly and appropriately to this issue.

Keywords: Control of hydraulic jump, thin-crested sill, sill height, incident Froude number, stilling basin.

INTRODUCTION

In the field of hydraulic structures, particularly dams, it is very often a question of restoring the flow of water to a stream. At the toe of the spillway equipped with the dam, a stilling basin is constructed, designed so that the dissipation of energy takes place inside, where a hydraulic jump is formed for this purpose. To stabilize the position in the basin, a linear sill can be installed. The latter is the particular device promoting the formation of the jump, in addition to making it possible to make the dissipator more compact, to improve the flow stability under variations in flow parameters and to increase the dissipation capacity (Bretz, 1987).

A sill near the toe of the jump forms an effective energy dissipator. According to both experimental and theoretical considerations, it was shown that the thickness of the sill has no discernible effect on the flow pattern or the dissipation mechanism (Hager, 1992). As a result, a sill of suitable structural resistance and height can be considered. Unlike basins with steps, a sill only causes a local disturbance of the basin's bottom.

The first study of the forced hydraulic jump by a continuous sill in a fundamental way is due to Forster and Skrinde (1950). They studied the thin-walled sill to control the jump and fix its position in the channel to avoid erosion of the riverbed beyond the stilling basin. They were limited to the case where the flow is free downstream of the weir, i.e., the water level downstream has no influence on the formation of the hydraulic jump. The latter is practically finished to the right of the sill, which then functions as a weir. The authors have developed an intrinsic equation to determine the conjugate heights as a function of the upstream Froude number F_1 and the relative height of the sill $S=s/h_1$, including coefficients deduced from the theory of weirs. However, this formulation is inconvenient to use in practice. Weaver (1950), in discussing the work of Forster and Skrinde, conducted similar tests and measured the force on the sill as a function of its position in the jump. The result is that the force is maximal when the sill is placed at the toe of the jump and then decreases to a minimum at the end of the roll. The study also shows that a sill placed at the end of the jump causes a reduction in the downstream water depth required to form a classic jump. Shukry (1957) presented the effectiveness of basins with sills in the case of submerged hydraulic jumps. The author takes as a performance criterion the distribution of velocities downstream of the sill. He thus proposes placing the sill in the middle of the stilling basin. However, Bretz (1987) asserts that Shukry's study lacks rigor and that the work is qualitative only. Rand (1965) conducted systematic observations on sill flow, including a flow classification. The ratio of sequent depth as a function of sill height and hydraulic jump type, as well as length characteristics and energy loss, were studied. Later, Rajaratnam and Murahari (1971) found that the pressure on the rear sill face was uniformly distributed. Additionally, the mean flow features were observed and the forward flow upstream from the sill could be described by the turbulent wall jet. On the other hand, Bretz (1987), Junrui and Yongxiang (1988), and Sumi (1988) continued the studies of Stanley (1934), Rand (1965), and Ohtsu (1981). The effect of discharge variation on a sill-controlled stilling basin was analysed wherein the sill forms a secondary basin. Notably, Bretz (1987) conducted a very consistent experimental study on the effect of transverse sills. The synthesis of his work is given by Hager and Bretz (1988) and Hager and Li (1991).

In the present paper, a new experimental investigation of the control of hydraulic jumps by a thin-crested sill is developed. For that, a very wide range of the relative position of the sill involving 15 values of this parameter is investigated allowing the generalization of the problem for any flow condition. The main objective of the present study is to answer the question of the right value of the sill position X so that the hydraulic jump is formed completely on the stilling basin such that its length L_j is approximately equal to X.

AVAILABLE EXPERIMENTAL INVESTIGATIONS

This part of the study aims to review the work of Forster and Skrinde (1950) carried out on the hydraulic jump controlled by a thin-crested sill. The resulting conclusions are still valid today.

The hydraulic jump can be controlled by a thin-crested sill as shown in Fig. 1, or a broad-crested sill, continuous or discontinuous, as well as by a positive or negative step (references). All these obstacles serve to ensure the formation of the jump on the stilling basin by increasing the downstream water level, controlling its position during changes in parameters such as the flow rate, and finally contributing to better compactness of the stilling basin.

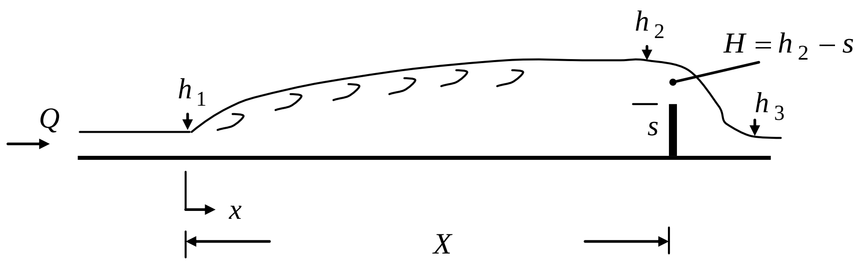

Figure 1: Hydraulic jump controlled by a thin-crested sill according to Forster and Skrinde (1950)

Dimensional analysis shows that the relationship between the Froude number F_1 of the incident flow, the sill height s, the initial depth h_1 of the hydraulic jump, the final depth h_2 of the hydraulic jump, the position X of the sill counted from the beginning of the hydraulic jump, and the depth h_3 downstream of the sill can be written in the following form:

$$\frac{s}{h_1} = f\left(F_1, X/h_2, h_3/h_1\right) \tag{1}$$

The position X of the sill (Fig. 1) cannot be analytically determined and the function f must then be defined experimentally.

In the tests carried out by Forster and Skrinde (1950), the relative position of the sill was defined by the ratio X/h_2 as indicated in Eq. (1). For each of the performed tests, the ratio X/h_2 was kept constant while ensuring the complete formation of the hydraulic jump. The length of the stilling basin was at the limit equal to position X of the sill counted from the beginning of the hydraulic jump (Fig. 1). The sill of height s was unsubmerged and worked as a weir. Thus, the depth h_3 of the flow downstream of the sill has no influence on the flow rate. According to Forster and Skrinde, this condition is satisfied when the following inequality holds:

$$h_3 < h_2 - 0.75\, s \tag{2}$$

Thus, the functional relationship expressed by Eq. (1) reduces to:

$$\frac{s}{h_1} = f\left(F_1, X/h_2\right) \tag{3}$$

To experimentally define the function f, Forster and Skrinde considered three values of the ratio X/h_2 which were 3, 5, and 10, kept constant for each test. The experimental measurements obtained by the authors have been translated graphically according to Fig.2.

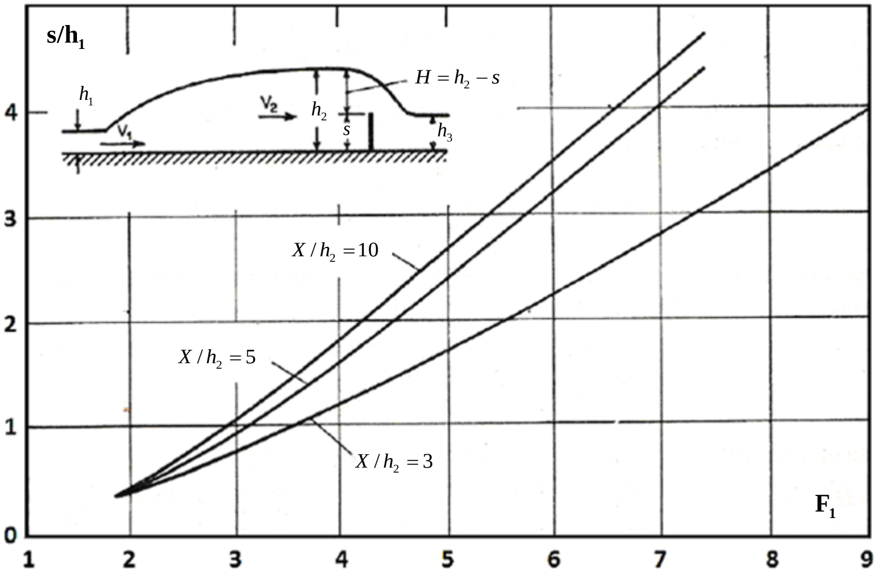

Figure 2: Experimental variation of the relative thin-crested sill height as a function of the incident Froude number for three values of the relative position of the sill according to Forster and Skrinde (1950).

Fig. 2 shows that any point on the diagram is represented by the pair of coordinates (F_1; s/h_1). When the values of these coordinates are defined and known, Fig. 2 then allows the determination of the parameter *X* which indicates the location of the sill. The distance X thus determinate will be the seat of a complete hydraulic jump. If a point in the diagram of Fig. 2 is located between two curves, the relative position X/h_2 of the sill must be determined by interpolation. If the point is above one of the curves in Fig. 2, the sill s is too high and the hydraulic jump can move upstream under the influence of the sill. If the point is below one of the curves in Fig. 2, the sill s is too low and the hydraulic jump can move downstream, giving way to a supercritical flow on the stilling basin whose security would then be threatened under the erosive effect of the opposing forces.

CURRENT EXPERIMENTAL APPROACH

One of the aims of this part of the study is to complement the restricted tests of Forster and Skrinde (1950). It is expected to consider several relative positions X/h_2 of the thin-crested sill and to derive above all an explicit experimental equation which would translate the functional relationship expressed by Eq. (3). The design of the stilling basin, i.e., the determination of the required distance *X* (Fig. 1), could then be ensured for all configurations of the flow. This will allow the designer to perform a direct calculation instead of performing interpolations as indicated in the previous section.

For this, our tests will involve 15 values of the relative position X/h_2 of the thin-crested sill such as $3 \leq X/h_2 \leq 10$ with a step of 0.5.

The hydraulic installation designed for the purpose of experimentally studying the control of the jump by a thin-crested sill in a rectangular channel of width b has been specially adapted. In particular, the incident flow is generated by a pressurized box-convergent set directly fed by the pump (Fig. 3).

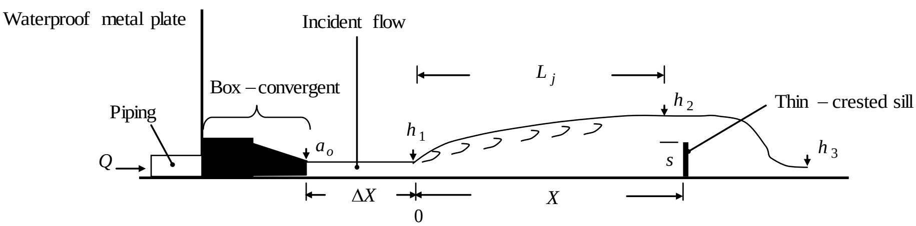

Figure 3: Schematic representation of the hydraulic jump controlled by a thin-crested sill adopted during the authors' tests; incident flow generated by a box-convergent set.

Two advantages characterize this system. The first resides in the fact that it generates a high velocity supercritical flow at the exit of the convergent, while the second resides in the fact that it is horizontally movable on the channel bed. This operation thus allows adjusting the position X of the sill at will to the values required by the experiment.

The box as well as the convergent are equipped with steering guides that properly direct the flow toward the entrance of the channel.

The distance ΔX (Fig. 3) was kept as short as possible, not exceeding 5 cm so that the initial depth h_1 of the hydraulic jump was rightly assimilated to the exit opening a_o of the convergent, i.e., $h_1 \approx a_o$. This allowed avoiding the delicate measurement of the depth h_1 due to the highly supercritical nature of the flow on the short distance ΔX. Consequently, for each pair of values (a_o; Q), where Q is the flow rate, the Froude number F_1 of the incident flow has been computed according to the following relationship:

$$F_1^2 = \frac{Q^2}{g\,b^2\,a_o^3} \tag{4}$$

where g is the acceleration due to gravity. The convergent part has been designed long enough in such a way as to obtain several openings a_o. These were obtained by a simple transverse cut of the convergent at the desired opening a_o using an appropriate metal plate cutting machine. The flow rates Q tested were measured using a diaphragm flow meter with a deviation of 0.5 l/s, previously calibrated in the wide range of flow rates of 1.18 l/s $\leq Q \leq$ 35 l/s using a 90° V-notched weir.

The only flow depth that needed to be measured was the final depth of jump h_2. This was accurately measured using a double precision Vernier gauge with an absolute deviation of only 0.02 mm.

The tests were carried out at increasing flow rates. For a fixed opening a_o, the increase in flow rate leads to an increase in the incident Froude number F_1 in accordance with Eq. (4) and the distance ΔX as well. Despite the increase in the incident Froude number, the flow slice of length ΔX can be reduced by raising the sill. The simultaneous increase in the incident Froude number and that of the sill height implies an increasingly large relative length L_j/X and can reach unity as a limit value; L_j denotes the length of the hydraulic jump as shown in Fig. 3. The greater the length X is, the higher the values of the incident Froude number and of the sill height s must be to obtain a relative length L_j/X equal to unity.

If one considers an unsubmerged thin-crested sill (Fig. 3) of width b equal to that of the channel, the depth h_3 of the flow downstream of the sill has no influence on the behavior of the hydraulic jump. The parameters involved in the phenomenon are then Q, h_1, b, s, X, L_j, and g. Therefore, the following functional relationship can be written, bearing in mind that $h_1 \approx a_o$:

$$\zeta\left(F_1; s/h_1; X/h_1; L_j/X\right) = 0 \tag{5}$$

For obvious economic reasons, the most interesting configuration for the considered hydraulic jump is that which corresponds to the smallest possible distance ΔX and to the ratio L_j/X equal to unity. Furthermore, since $X/h_1 = (X/h_2)Y$, where Y is the sequent depth ratio governed by Belanger's equation (1838), the functional relationship (5) reduces to:

$$\varphi\left(F_1; s/h_1; X/h_2\right) = 0 \tag{6}$$

It is the function φ that the present experimental investigation seeks to define.

The experiment was carried out in a rectangular channel 30 centimeters wide, 10 meters long and 45 cm deep fed in a closed circuit by a 35 l/s flow pump. The walls of the channel are made of transparent glass to visualize the flow while its bottom is metallic. The incident flow is generated by a pressurized convergent 1 m long and of the same width as the test channel. Its initial and final openings are 15 cm and 2 cm respectively and it is connected to a pressurized box of the same width. The box-converging set, made of sheet metal, is directly supplied by the pump through a flexible pipe. The box and the convergent are fitted inside with carefully arranged guides to best ensure the stability, uniformity, and correct orientation of the incident flow. A watertight vertical plate is placed above the box and over its entire width to avoid any water overflow when the hydraulic jump is submerged and the measurement channel is flooded (Fig. 3).

The height s of the sill is chosen so that the hydraulic jump forms at a distance $\Delta X = 5$ cm from the convergent generating the incident flow.

The measurements concerned the following interesting range of the incident Froude number, namely $3 \leq F_1 \leq 9$ which corresponds to the range of the relative sill height $S = s/h_1$ such that $1 \leq S \leq 6$.

Because the variation step chosen for the ratio X/h_2 is small (0.5), the resulting experimental curves $S(F_1)$ are very close together and will not all be represented.

RESULTS AND DISCUSSION

The analysis of the experimental measurements collected during the tests showed above all that the relative sill height $S = s/h_1$ was related to the incident Froude number F_1 by a power-law relationship such as:

$$S = C_o \left(F_1 - 1 \right)^{\beta} \quad (7)$$

For the particular case of the critical flow corresponding to $F_1 = 1$, Eq. (7) allows us to write that $S = 0$.

The main result to remember is that the exponent β takes a practically constant value equal to approximately 5/4 for the considered range values of the relative positions of the sill, i.e., $3 \leq X/h_2 \leq 10$. Only the parameter C_o varies and its variation can be related to that of the relative position X/h_2 of the sill. The experimental results, reported in Tables 2 to 8 in the appendix, allow us to define the function $C_o = F(X/h_2)$.

For all the experimental values of the incident Froude numbers and that of S, the corresponding value of C_o was calculated according to Eq. (7) for $\beta = 5/4$, namely,

$$C_o = \frac{S}{\left(F_1 - 1 \right)^{5/4}} \quad (8)$$

Table 1 groups together the experimental values of C_o which correspond to the various relative distances X/h_2 chosen for the sill.

Table 1: Experimental values of C_o for some positions X/h_2 of the thin-crested sill

X/h_2	3	3.5	4	4.5	5	5.5	6	6.5	7
C_o	0.300	0.330	0.360	0.390	0.421	0.425	0.429	0.433	0.444
X/h_2	7.5	8	8.5	9	8.5	10			
C_o	0.447	0.450	0.453	0.456	0.459	0.463			

Fig. 4 shows the variation in C_o = f (X/h_2) which results in two sections of straight lines $L1$ in the range $3 \leq X/h_2 \leq 5$ and $L2$ over the range $5 \leq X/h_2 \leq 10$.

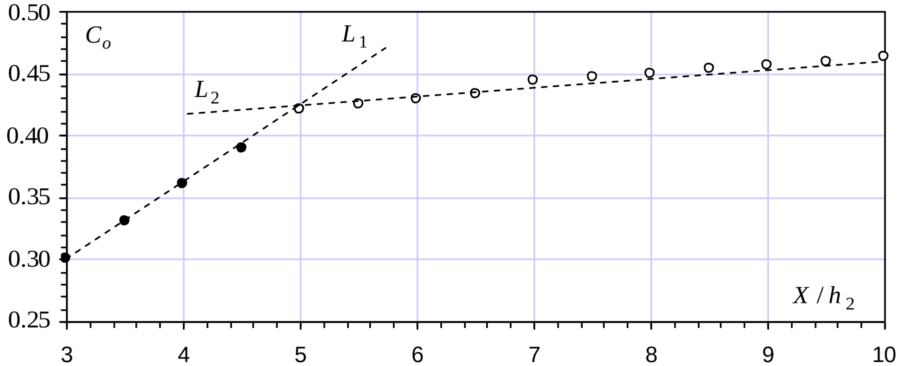

Figure 4: Experimental variation of the parameter C_o as a function of the relative position X/h_2 of the thin-crested sill, according to table 1

The use of the linear least-squares fitting method involving experimental data gave the following trend line relationships:

Line L_1: $C_o = 0.0604\,(X/h_2) + 0.1186$ in the range $3 \leq X/h_2 \leq 5$, $R^2 = 1$ (9)

Line L_2: $C_o = 0.0086\,(X/h_2) + 0.3791$ in the range $5 \leq X/h_2 \leq 10$, $R^2 = 0.9729$ (10)

Both Eqs. (9) and (10), along with Eq. (8) allows deducing the required position X of the thin-crested sill which should ensure the complete formation of the hydraulic jump on the stilling basin, for the known values of S and F_1.

From the known values of S and F_1, the parameter C_o is calculated according to Eq. (8). With the calculated value of C_o, the position X sought of the thin-crested sill is computed using either Eq. (9) if $3 \leq C_o \leq 0.421$, or Eq. (10) if $0.421 \leq C_o \leq 0.463$, in accordance with table 1.

The experimental values of the relative sill height S as a function of the incident Froude number F_1 are represented graphically in Fig. 5 for several values of the position X/h_2 of the sill. Not all 15 experimental values of X/h_2 are represented because beyond $X/h_2 = 7$, the curves overlap. It thus appears that for the relatively wide range $5 \leq X/h_2 \leq 7$, the variation that S undergoes for the same Froude number F_1 is not significant. This seems to indicate that the dimensioning calculation of the stilling basin, in particular with regard to its length X, could be carried out for the average value $X/h_2 = 6$. This remains valid when $C_o \geq 0.429$ in accordance with Table 1. Given that the length L_j of the hydraulic jump corresponds approximately to the length X of the stilling basin, it is then justified to write that $L_j/h_2 \approx 6$. This result coincides with that obtained by Peterka (1983) for the case of the classic hydraulic jump, thus indicating that the setup of the thin-crested sill for the control of the hydraulic jump has practically no effect on the compactness of the stilling basin when C_o reaches or exceeds 0.429.

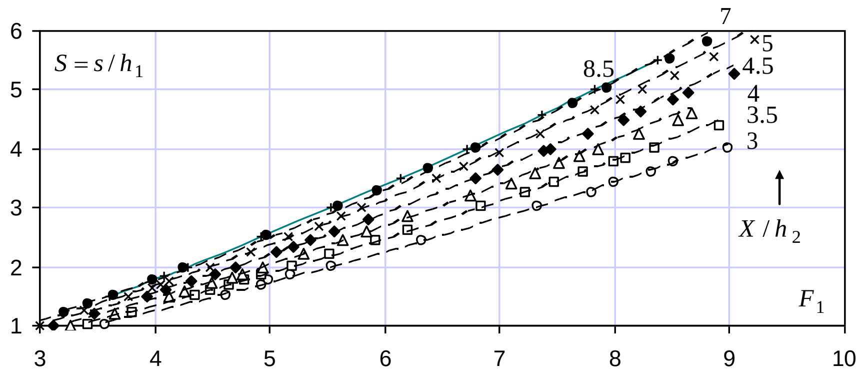

Figure 5: Experimental variation of the relative sill height *S* as a function of the incident Froude number *F*₁ for a thin-crested sill, according to tables in appendix

CONCLUSIONS

The present study had the merit of bringing something new from the experimental point of view during investigations on the control of hydraulic jumps by a thin-crested sill in a rectangular channel. The functional relationship $\varphi\left(F_1; s/h_1; X/h_2\right) = 0$ has been successfully defined [Eq. (8) along with Eqs. (9) and (10)] and the problem has been generalized to any flow condition by considering 15 experimental values of the sill position X/h_2, thus substantially completing the Forster and Skrinde tests of 1950.

The experiment was carried out under variable flow rate and initial depth h_1 of the hydraulic jump that was rightly assimilated to the exit opening a_o of the convergent generating the incident flow. For each series of measurements, the relative position X/h_2

of the sill was kept constant and was varied in the wide range $3 \leq X/h_2 \leq 10$ with a step of 0.5. The incident Froude number F_1 has been varied in the range $3 \leq F_1 \leq 9$, which corresponds to the range of the relative sill height $S = s/h_1$ such that $1 \leq S \leq 6$.

For each considered value of X/h_2, the experimental measurement analysis showed that the variation in the relative sill height S, as a function of the incident Froude number F_1, was correctly represented by the power-law formula expressed by Eq. (7). For the whole considered X/h_2 range values, the study revealed that the exponent β of Eq. (7) remained practically constant around the value 5/4. On the other hand, parameter C_o of Eq. (7) varied and its variation was related exclusively to the relative position X/h_2 of the sill.

The graphic representation of the variation of $C_o(X/h_2)$ clearly indicated that the curve obtained could reasonably be assimilated into two segments of straight lines L_1 and L_2 [Fig. (4)] corresponding to the ranges $3 \leq X/h_2 \leq 5$ and $5 \leq X/h_2 \leq 10$ respectively. The use of the linear least-squares fitting method involving experimental data gave the trend line relationships in the previous ranges [Eqs. (9) and (10)].

Moreover, the study showed that in the relatively wide range $5 \leq X/h_2 \leq 7$, the relative sill height S of the thin-crested sill undergoes only a very slight variation as a function of the incident Froude number F_1. This result led to the conclusion that, from a practical point of view, the dimensioning calculation of the stilling basin could be carried out by considering the arithmetic mean value $X/h_2 = 6$. This result corresponds to that of the relative length $L_j/h_2 = 6$ of the classical hydraulic jump, indicating that the setup of a thin-crested sill for the control of the hydraulic jump has no effect on the compactness of the stilling basin.

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APPENDIX

Table 2: Experimental values of the relative sill height S and the incident Froude number F_1 for $X/h_2 = 3$; average value of C_o: 0.300

$X/h_2 = 3$		
S	F_1	C_o Eq. (8)
1	3.583	0.30538274
1.5	4.625	0.29988604
1.667	4.941	0.30021166
1.75	5	0.30935922
1.833	5.191	0.30567885
2	5.541	0.30171063
2.416	6.333	0.29811405
3	7.333	0.29861347
3.25	7.816	0.29510148
3.416	8	0.30001662
3.583	8.325	0.297329
3.75	8.525	0.30088334
4	9	0.29730178

Table 3: Experimental values of the relative sill height S and the incident Froude number F_1 for X/h_2 = 3.5; average value of C_o: 0.330

X/h_2 = 3.5		
S	F_1	C_o Eq. (8)
1	3.427	0.33011312
1.2	3.808	0.3301299
1.5	4.36	0.32973661
1.585	4.495	0.33168077
1.67	4.658	0.33011241
1.75	4.798	0.3300611
1.835	4.935	0.33109693
2	5.21	0.33164799
2.18	5.533	0.32959024
2.426	5.935	0.32982369
2.6	6.21	0.3303134
3	6.85	0.32974364
3.25	7.232	0.33006468
3.41	7.48	0.32982656
3.585	7.742	0.32999186
3.75	8	0.3293508
3.82	8.1	0.32960245
4	8.359	0.33001713
4.385	8.922	0.32993322

Table 4: Experimental values of the relative sill height *S* and the incident Froude number F_1 for X/h_2 = 4; average value of C_o: 0.360

$X/h_2 = 4$		
S	F_1	C_o Eq. (8)
1	3.265	0.35988583
1.215	3.648	0.35969036
1.5	4.132	0.36000959
1.58	4.26	0.36069082
1.72	4.495	0.35993118
1.82	4.67	0.3582934
1.885	4.76	0.36001993
2	4.94	0.36029623
2.23	5.301	0.36003362
2.45	5.64	0.35976475
2.585	5.841	0.35999104
2.85	6.2	0.36294489
3.2	6.74	0.36017214
3.426	7.1	0.35737574
3.596	7.304	0.35999745
3.75	7.52	0.35993309
3.87	7.685	0.36002628
4	7.86	0.36029219
4.25	8.206	0.35997398
4.5	8.543	0.35998332
4.6	8.666	0.36061751

Table 5: Experimental values of the relative sill height S and the incident Froude number F_1 for X/h_2 = 4.5; average value of C_o: 0.390

X/h_2 = 4.5		
S	F_1	C_o Eq. (8)
1	3.125	0.38976365
1.21	3.474	0.38997409
1.5	3.938	0.3899658
1.6	4.094	0.38991469
1.75	4.323	0.39005435
1.88	4.522	0.38964684
2	4.698	0.39000607
2.25	5.063	0.39005333
2.35	5.209	0.38980213
2.45	5.35	0.38999094
2.6	5.56	0.39018206
2.8	5.85	0.389028
3.5	6.786	0.3900273
3.65	6.98	0.39031603
3.96	7.38	0.39054343
4	7.438	0.39005089
4.25	7.76	0.38990209
4.5	8.08	0.38964665
4.625	8.23	0.3901116
4.85	8.512	0.38998443
4.95	8.635	0.3900263
5.28	9.04	0.38999933

Table 6: Experimental values of the relative sill height S and the incident Froude number F_1 for $X/h_2 = 5$; average value of C_o: 0.421

$X/h_2 = 5$		
S	F_1	C_o Eq. (8)
1	3	0.42044821
1.25	3.389	0.42086211
1.5	3.763	0.42108052
1.65	3.98	0.42141852
1.7	4.054	0.42107808
1.75	4.127	0.42085084
2	4.48	0.42078082
2.25	4.83	0.41993691
2.5	5.16	0.42079769
2.7	5.422	0.42105647
2.85	5.62	0.4207677
3	5.8	0.42225005
3.5	6.443	0.4209884
3.7	6.69	0.4210284
3.95	7	0.42063754
4.25	7.357	0.4210403
4.65	7.82	0.42191259
4.83	8.05	0.42044649
5	8.24	0.42101425
5.245	8.522	0.42104531
5.55	8.86	0.42171087
5.85	9.209	0.42101062

Table 7: Experimental values of the relative sill height *S* and the incident Froude number F_1 for X/h_2 = 7; average value of C_o: 0.444

X/h_2 = 7		
S	F_1	C_o Eq. (8)
1.2	3.215	0.44408293
1.35	3.435	0.44382326
1.5	3.648	0.44406218
1.745	3.99	0.44381957
1.95	4.265	0.44430442
2.5	4.985	0.44402212
3	5.61	0.44411465
3.275	5.944	0.44423546
3.65	6.395	0.44391882
4	6.805	0.44392255
4.75	7.65	0.44480182
5	7.938	0.44404515
5.5	8.49	0.44387473
5.8	8.815	0.44388121

Table 8: Experimental values of the relative sill height *S* and the incident Froude number F_1 for X/h_2 = 8.5; average value of C_o: 0.453

X/h_2 = 8.5		
S	F_1	C_o Eq. (8)
1.5	3.605	0.45324352
1.85	4.082	0.45303414
2	4.281	0.45292074
2.5	4.92	0.45324438
3	5.538	0.45293996
3.5	6.135	0.45278547
4	6.71	0.45317387
4.58	7.365	0.4530201
5	7.828	0.45300512
5.5	8.37	0.45292712