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A SIMPLIFIED ANALYTICAL SOLUTION FOR THE DIVIDING MANIFOLD FLOW PROBLEM

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ABSTRACT

A simple analytical solution to the problem of dividing manifold flow has been developed in the present study. Based on simplifying hypotheses accepted in practice and using energy principle considerations, differential equations for pressure head variation over the manifold are derived for both turbulent and laminar flow regimes in the pipe and lateral port orifices. From that, simple analytical expressions are obtained for solving practical problems such as variations in the pressure head, residual flow, and lateral port flow distribution. A comparison with literature results related to an irrigation engineering problem shows excellent agreement despite the simplicity of the model. Additionally, a parametric analysis concerning the decay rate of the pressure head for both flow regimes is performed for illustration.

Keywords: Spatially varied flow, Manifold problem, Analytical solution.

INTRODUCTION

Basically, a manifold consists of a main pipe (called a barrel) along which numerous junctions of small pipes or ports are placed to allow a flow distribution as a dividing manifold or collect flow as a combining manifold (Larock et al., 2000). Manifold flow problems are present in several applications, such as irrigation systems, water supply networks, and even in chemical engineering.

The main issue facing practical designers when designing a manifold is the strong interdependence of flow parameters, mainly the flow in the barrel, lateral flow through each port, and the pressure head available at the considered point. Indeed, the lateral flow, considered an orifice flow, depends solely on the pressure head, which is a function of the flow rate in the manifold pipe at the considered section. The latter, called residual flow depends on the lateral issuing flow from the manifold entrance. It becomes obvious that the problem is far from the classical algebraic equations governing constant flow pipe calculations, but rather a differential problem that relates a variable to its variation rate.

Classically, to drastically simplify the problem, the first attempt to approach the problem consists of assuming a constant and uniform lateral flow at each orifice. This assumption degenerates the problem into a simple quadrature where a polynomial expression of the third degree is obtained (Nalluri and Featerstone, 2016). However, the uniformity of lateral flows is the main weakness of this approach.

To enhance the former approach, several analytical solutions have been proposed by many authors in which different modeling hypotheses are adopted but consider lateral flow as dependent on the local pressure head. One of the early contributions is due to Acrivos et al. (1959), where a second-order nonlinear differential equation is obtained and numerically solved for dividing and combining manifolds. Warrick and Yitayew (1987; 1988) considered both velocity head losses and variable discharge along the manifold in their analysis. The appropriate second-order nonlinear equation is solved analytically for two flow regimes, laminar and fully turbulent. Scaloppi and Allen (1993) analyzed the effect of ground slope and velocity head on pipeline hydraulics and an alternate simplified procedure that neglects velocity head effects was also presented. Valiantzas (1998; 2002) derived an analytical energy line for a single-diameter lateral, taking into account the effect of the number of outlets. Yıldırım (2007) presented an analytical procedure in which energy relations are improved based on the average friction drop approach with a simple exponential function to express the nonuniform outflow concept. More recently, Liu et al. (2017) analyzed a perforated fluid distribution pipe by the momentum equation for variable mass flow with a variable exchange coefficient and variable friction coefficient.

This simple literature survey shows that relying on energy principle considerations and other simplifying assumptions related to flow resistance, such as orifice flow exponent and outflow uniformity, more or less complicated analytical solutions have been obtained, allowing the solution of problems encountered in practical applications. Principally, these problems are how the pressure head, flow through lateral ports and carried flow within the main pipe vary throughout its length.

This paper aims to present a simplified analytical solution to the dividing manifold flow problem to solve practical hydraulic design issues with a good degree of accuracy. The theoretical analysis focuses on a horizontal manifold with a very large outlet number. Both turbulent and laminar flows are analyzed, considering a constant friction factor. The results are compared with an example taken from the literature for pressure head and flow variation along the main pipe. The present analytical solution forms a simple and direct approach, especially for design analysis compared to previous heavy numerical and/or analytical procedures.

THEORETICAL ANALYSIS

Let us consider a manifold pipe of constant diameter D and length L having equally spaced circular exit ports, all of the same diameter d (Fig. 1). The manifold is connected upstream to a tank or a reservoir under a total head H, and the downstream end of the main is a dead end. The exit flow rate from each port of the circular section is:

$$q = C_d S \sqrt{2gh} \tag{1}$$

where $q = q(x)$ is the flow rate of given exit ports along the manifold, $h = h(x)$ is the driving head, which is the vertical distance from the centerline of the port to the local hydraulic grade line above that port (Fig. 1), S is the cross-sectional area of the exit port, g is acceleration due to gravity, and C_d is the discharge coefficient, taking account of different energy losses through the orifices and eventually lateral small pipes such as branches and secondary pipes. Here, in Eq. (1), the flow from the lateral ports is presumed to exit as a jet into the atmosphere.

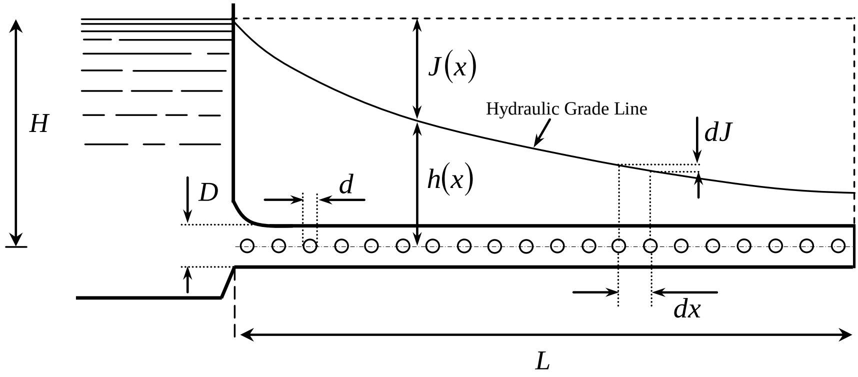

Figure 1: Definition sketch of the manifold flow problem

Considering an infinite number of exit ports spaced at an infinitesimal distance dx, the elementary head loss dJ along the manifold is expressed by the Darcy-Weisbach relationship (1854) as follows:

$$dJ = \frac{8f}{g\pi^2 D^5} Q_x^2 dx \tag{2}$$

where f represents the friction factor, D is the manifold diameter, and $Q_x = Q_x(x)$ is the residual flow rate in the manifold at the considered exit which can be expressed as:

$$Q_x = Q_T - \int_0^x q(x)dx \tag{3}$$

On the other hand, the total discharge transited through the manifold Q_T is:

$$Q_T = \int_0^L q(x)dx \tag{4}$$

Eq. (3) reads then:

$$Q_x = \int_0^L q(x)dx - \int_0^x q(x)dx \tag{5}$$

Combining definite integrals, Eq. (5) becomes:

$$Q_x = \int_x^L q(x)dx = \int_x^L K\sqrt{h}dx \tag{6}$$

where $K = C_d S\sqrt{2g}$.

On the other hand, according to the Bernoulli theorem, conservation of the total energy along the manifold requires the following equality:

$$H = h + \frac{Q_x^2}{2gA^2} + J \tag{7}$$

where $A = \pi D^2/4$ denotes the manifold cross-section area. Differentiating Eq. (7) with respect to the streamwise x-coordinate and neglecting convective acceleration (gradient of the kinetic energy), one obtains:

$$\frac{dH}{dx} = \frac{dh}{dx} + \frac{dJ}{dx} = 0 \tag{8}$$

Combining Eqs. (2) and (8) results in:

$$\frac{dh}{dx} + CQ_x^2 = 0 \tag{9}$$

where $C = 8f/g\pi^2 D^5$ is the elementary hydraulic resistance of the manifold. Replacing the expression of Q_x from Eq. (6) into Eq. (9) gives:

$$\frac{dh}{dx} + C\left[\int_x^L K\sqrt{h}\,dx\right]^2 = 0 \tag{10}$$

The problem of manifold flow is then governed by Eq. (10), which is a nonlinear first-order differential equation due to the varying coefficients K and C. The solution of Eq.

(10) allows the determination of pressure head variation along the manifold and, consequently, lateral and residual flow rates.

ANALYTICAL SOLUTION

To derive an analytical solution to Eq. (10), one must consider constant coefficients, that is, admit that both the friction factor *f* and the discharge coefficient C_d are independent of the flow along the *x*-coordinate. In this assumption, one considers that flow in the manifold is under a rough turbulent regime such that *f* can be considered constant at least along most of the manifold length.

Considering these simplifying hypotheses, integration of Eq. (10) is straightforward and, for the initial condition *h* (0) = *H*, the exact solution is then as follows:

$$h(x) = He^{-CK^2 x\,(L^2 - Lx + x^2/3)} \tag{11}$$

Eq. (11) shows that the hydraulic grade line from the reservoir to the dead end of the manifold is of an exponential decay form. It is easy to derive from Eq. (11), at the dead-end location *x* = *L*, that:

$$\left.\frac{dh(x)}{dx}\right|_{x=L} = 0 \tag{12}$$

Eq. (12) shows that the hydraulic gradient, and thus the discharge, is zero. On the other hand, the residual pressure at the dead end of the manifold is:

$$h(L) = He^{-CK^2 L^3/3} \tag{13}$$

For convenience, one can express Eq. (11) in a dimensionless form. Adopting the following normalization:

$$h^* = \frac{h}{H} \tag{14a}$$

$$x^* = \frac{x}{L} \tag{14b}$$

The exact solution of Eq. (10) in dimensionless form is expressed as follows:

$$h^* = e^{-\beta x^* (1 - x^* + x^{*2}/3)} \tag{15}$$

in which $\beta = CK^2L^3$ is a characteristic parameter of the manifold.

Equivalent flow rate

An interesting issue in the manifold flow problem is the determination of the equivalent flow rate. It consists of finding a hypothetical constant discharge for which the same head-loss is produced throughout the manifold by the real variable flow effectively carried out. This can be easily achieved by setting the following equality:

$$J_T = \frac{8 f L}{g\pi^2 D^5}\overline{Q}^2 = H - h(L) \tag{16}$$

Here J_T is the total head-loss throughout the manifold and $\overline{Q}$ the equivalent flow rate. Replacing Eq. (13) into Eq. (16) and rearranging, one obtains:

$$\overline{Q} = \sqrt{\frac{H}{CL}\left(1 - e^{-CK^2L^3/3}\right)} \tag{17}$$

Eq. (17) allows the reduction of the manifold flow problem to an analogous constant pipe flow problem. Note that this analogy does not consider the actual shape of the pressure head profile but gives only a tool for rapid hydraulic analysis.

Case of porous orifices

For creeping flow resulting from high liquid viscosity or very small flow rates through orifices, it is possible to deduce an exact solution to the problem. The approximation of the porous orifice for the relationship between discharge q and driving head h on the one hand and the assumption of laminar flow in the manifold pipe on the other hold in this case. This results in linear relationships for the variables q and h, and Eqs. (2) and (6) respectively as follows:

$$dJ = \breve{C} Q_x dx \tag{18}$$

$$Q_x = \int_x^L \breve{K}\, h\, dx \tag{19}$$

where $\breve{C} = 128\nu / g\pi D^4$ is the elementary hydraulic resistance of the manifold under laminar flow conditions (Rouse, 1938), and $\breve{K}$ is the hydraulic conductivity of the porous orifice element.

Note that the assumption of linear relationships between the variables q and h is valid for only small heads (Borutzky et al., 2002). Thus, the governing differential Eq. (10) becomes:

$$\frac{dh}{dx} + \breve{C}\breve{K}\int_x^L h\,dx = 0 \tag{20}$$

This is a simple linear differential equation for which the particular solution for $h(0) = H$ is expressed as follows:

$$h(x) = He^{-\breve{C}\breve{K}\,x\,(\,L-x/2\,)} \tag{21}$$

In dimensionless form, Eq. (21) reduces to:

$$h^{*} = e^{-\varphi x^{*}(1-x^{*}/2)} \tag{22}$$

where $\varphi = \breve{C}\breve{K}L^2$ represents a characteristic of the manifold.

When comparing Eqs. (11) and (21), it appears clear that the main difference in the exact solution of the manifold flow problem under turbulent or laminar conditions lies in the argument of the decay term. Whereas in the former regime, the x-coordinate is involved in a cubic order, in the case of porous orifices, a quadratic order is involved.

APPLICATION AND VERIFICATION

To verify the accuracy of the present analytical solution (Eq. 15 for turbulent flow), a comparison was performed with an example taken from Yıldırım (2007). The results of the analytical model are compared with those obtained from the accurate SBS (step-by-step) numerical method, which was developed by Hathoot et al. (1993).

The problem consists of determining the pressure head $h(x)$, discharge $Q_x(x)$ profiles, and head loss distribution along the pipe and the corresponding flow characteristics for a horizontal polyethylene trickle irrigation lateral with turbulent flow emitters. The total number of emitters is 151, equally spaced at 1.0 m. The required average emitter discharge is $\bar{q} = 5.555\times10^{-7}\ m^3/s$ which requires a total flow rate for the manifold $Q_T = 8.389\times10^{-5}\ m^3/s$. The pressure head at the entrance of the manifold is $H = 8.70\,m$.

It is worth noting that in the present application, the results taken from Yıldırım's study (2007) concern a turbulent flow emitter coefficient not equal to 0.5 (Eq.1) as for the proposed analytical model but rather equal to 0.54. It is obvious that no analytical solution is expected for such a value, but the comparison aims to analyze the proposed model and show its accuracy even in such cases.

Figs. 2a, 2b, and 2c show the variation in the relative pressure head $h^*(x^*)$, residual discharge $Q_x(x^*)$, and relative pressure drop $J/J_T(x^*)$ as functions of the relative distance length of the manifold. The present analytical solution expressed by Eq. (15) is compared with both the SBS numerical solution and Yıldırım's analytical model. Fig. 2a shows that the predicted pressure head profile agrees well with the other models. The proposed model shows a pronounced curvature profile, however. The residual pressure head at the dead end of the pipe (given by Eq. 13) is more compatible with the SBS model, where a relative deviation of only 0.6 % is noticed and 1.6 % compared to Yıldırım's model. Concerning the variation in the flow rate along the manifold (Fig. 2b), a high agreement is observed between the present solution model and the other reference models. The variation in the residual flow rate shows a linear trend from $Q_T = 8.389 \times 10^{-5}\, m^3/s$ to zero at the dead end. In this case, the present solution matches the SBS model with high accuracy.

On the other hand, the relative pressure drop J/J_T along the manifold pipe (Fig. 2c) exhibits a nonlinear growth from the inlet to the dead end. The function describing its variation is simply the complementary values of the pressure head $h(x)$ (Eq. 11) to the total head H. It follows from the figure that a similar trend between the proposed solution and reference models can be seen. As reported above, for the variation of the relative pressure head, a slight overestimation of the head losses along the manifold is noticed. These differences can be attributed to simplifying hypotheses such as the constancy of the friction factor and other assumptions and to the exponent of the turbulent flow emitters, which is different from the theoretical value of 0.5 applied for orifices. Therefore, the average emitter discharge computed in the present model is $\bar{q} = 8.306\text{x}10^{-7} m^3/s$ instead of the actual value of $5.555\text{x}10^{-7} m^3/s$.

Even though the proposed analytical solution has very slight variations when compared to other more precise models, the agreement is very satisfactory.

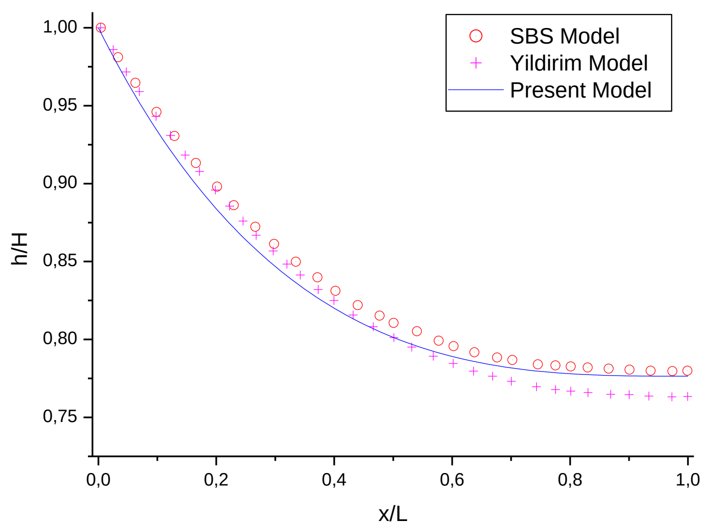

Figure 2a: Relative pressure head for horizontal trickle lateral with respect to the distance ratio from the inlet. Comparison of the present model with numerical SBS and Yıldırım models

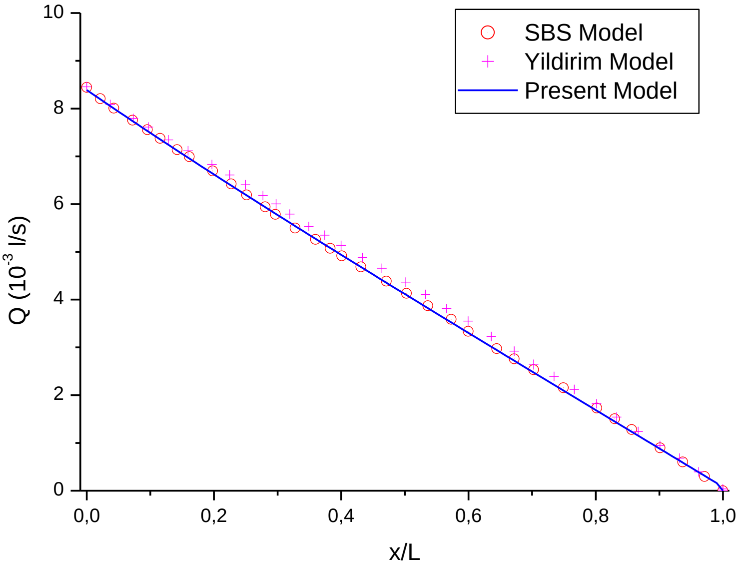

Figure 2b: Flow rate profile for horizontal trickle lateral with respect to the distance ratio from the inlet. Comparison of the present model with numerical SBS and Yıldırım models

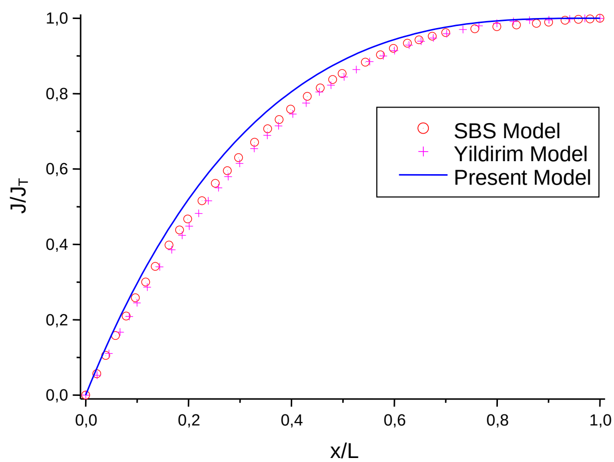

Figure 2c: Relative friction drop for horizontal trickle lateral with respect to the distance ratio from the inlet. Comparison of the present model with numerical SBS and Yıldırım models

To illustrate the effect of the characteristic parameters β and φ of the manifold pipe on the pressure head variation, several values were analyzed and the results are depicted in Figs. 3a and 3b.

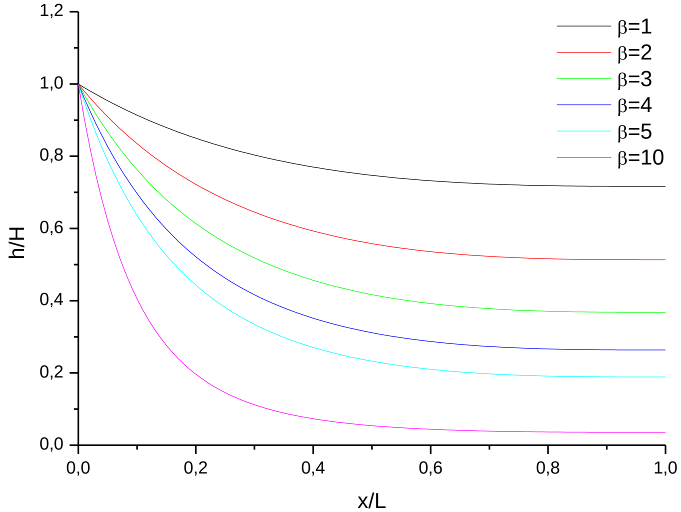

Figure 3a: Influence of manifold characteristic parameter on the relative pressure head along the manifold for turbulent flow

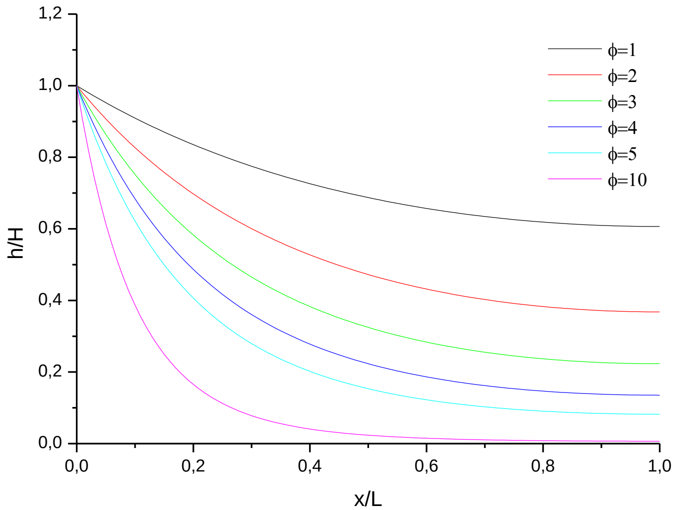

Figure 3b: Influence of the manifold characteristic parameter on the relative pressure head along the manifold for laminar flow

It follows from this parametric analysis for both flow regime hypotheses that characteristic parameters of the manifold, which are encompassed in β and φ, drastically control the decay rate of the pressure head profile along the pipe. One may deduce from Eqs. (15) and (22) governed by the exponential function, the decay follows the same pattern for both flow regimes, but at different rates. For high values of the β and φ parameters, the irregularity of the flow distribution through lateral ports is more pronounced. Far from the entrance, the pressure head vanishes significantly, which is not the case for low values of β and φ. This result, even though simple, gives more quantitative insight into the hydraulic behavior in terms of pressure variation and flow rate distribution throughout a dividing manifold.

CONCLUSION

In the present paper, a simplified analytical solution for a dividing manifold flow problem is presented. The solution aimed to provide an easy way to solve practical problems related to pipes with lateral ports, which are encountered principally in irrigation engineering and water distribution networks. The main issue is the determination of both the pressure head and flow rate variation throughout the manifold.

Under some simplifying hypotheses, such as a horizontal manifold pipe and constancy of friction factor, it was then possible to derive a differential equation for the space variation of pressure head over the uniformly perforated pipe length from the Bernoulli equation after neglecting the kinetic energy term.

The above simplifications are satisfied in most practical problems. Both the turbulent flow regime in the pipe and through the ports and the laminar regime case are treated. From that, simple and direct exact solutions are obtained as an exponential decay relationship of the characteristic manifold parameters and polynomial function of space coordinates along the pipe.

For verification and validation, a comparison with available data from the literature was performed for the case of horizontal polyethylene trickle irrigation lateral with turbulent emitters. The comparison models chosen for verification are the numerical Step-by-Step algorithm (SBS Model) and the analytical solution of Yıldırım (2007). The results showed excellent agreement for both pressure head variation and residual flow distribution over the pipe length, even for a slightly different emitter exponent from that adopted in the present model solution. However, a certain deviation is noticed concerning the distribution of lateral port flow principally due to the exponent of the emitters, among other simplifying hypotheses of the model.

It is worth recalling that other possible practical cases could be treated by generalizing the present analytical solution, but a more complex mathematical analysis treatment would be involved, which is not very useful for practical purposes, especially in the preliminary design step where a rapid, simple, and good approximation is needed.

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