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ANALYSING THE ANALYTICAL FORMULATION FOR NONLINEAR ROLL MOTION OF A SHIP WITH IRREGULAR WAVES UTILISING HOMOTOPY PERTURBATION METHOD

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ABSTRACT

Damping is unavoidable for the response of a ship's roll motion in waves. The roll motion of ships is represented as a second-order nonlinear differential equation that includes a nonlinear notation related to damping and restoring moments. In this paper, a new HPM process is implemented to address this real-time problem. Simple and closed-form analytical representations of the ship's roll motion and other physical properties have been generated. The results of the numerical simulation were compared to those of the analytical results. It is mentioned that there is satisfactory agreement.

Keywords: mathematical modeling**,** homotopy perturbation method, nonlinear damping, nonlinear roll response, irregular waves.

INTRODUCTION

The roll motion of a ship at sea also has a substantial impact on prevention and controllability; as a response, finding an exact solution for this kind of problem is not an easy task. The key to accurately predicting ship roll motion is to correctly recognize some physical properties of the given real line problem. Consequently, various authors have studied roll damping; there is currently no uniform method for predicting damping, which is mostly predicated on fluid viscosity. Several methods are available to predict the roll motion, i.e., the model test method, a semiempirical method, a numerical method based on CFD and a system identification method.

The roll damping of ships was identified using the system identification method. Fan et al. used the improved energy technique and the genetic algorithm. Different authors have implemented artificial neural networks to determine the roll motion of ships. To indicate the roll damping and restoring moment constants of ships, Mahfouz used an artificial neural network. Haddara (2006), Xing (2010), Mahfouz (2004), and Ueno (2013) also used ANNs to detect ship roll motion. By analyzing the free decay test, Kim and Park (2016) and Lee et al. (2010) utilized the Hilbert transform approach for evaluating the nonlinear roll damping and restoring moments of an FPSO. The R-MISO method was used by Somayajula and Falzarano (2016) to compute the roll parameters of an S-175 cargo ship. It has recently been applied to analyze the roll motion of ships due to the fast development of inverse problem theory. The deterministic inverse method was used by Jang et al. (2011) to determine the structural form of nonlinear roll damping in ships. Jang (2011) extended a certain approach by identifying some physical properties at the same time.

Han and Kinoshita (2012) described a stochastic inverse method application for investigating a ship's nonlinear roll damping. To solve engineering challenges, a variety of machine research methods have been broadly implemented. Maximum likelihood estimation is more suitable for large-scale sampling learning than traditional parameter estimation methods. To establish the mathematical model of ship maneuvering motion, Luo (2009) used least squares. SVR was also employed by Zhang and Zou (2009) and Wang et al. (2013) to calculate the hydrodynamic derivatives in the ship maneuvering motion equations. The least square SVR was utilized by (Xu et al., 2013; Hou and Zou, 2015), who preferred SVR to demonstrate physical parameters for both regular and irregular waves.

This paper investigated roll motion in irregular waves. To discover the damping and restoring moments for the nonlinear roll motion of ships, the homotopy perturbation method is presented.

MATHEMATICAL FORMULATION OF THE PROBLEM

A schematic diagram of the ship is shown in Fig. 1.

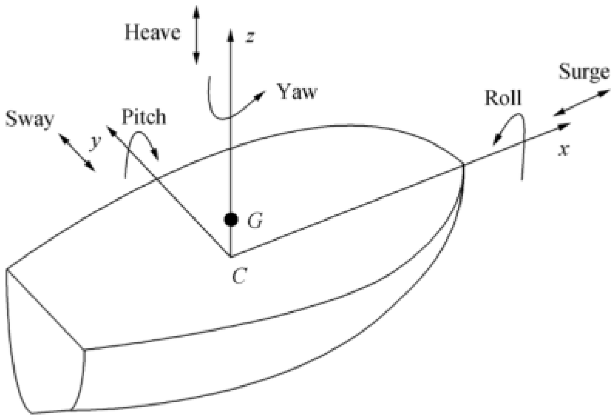

Figure 1: Schematic diagram of the ship

An ordinary second-order nonlinear differential equation of the system (Esperanc, 2008) can be used to represent the roll motion of a ship at sea for a single degree of freedom, according to rigid body dynamics.

$$(I_{XX} + J_{XX})\ddot{\theta} + D(\dot{\theta}) + R(\dot{\theta}) = G(t) \tag{1}$$

The above equation is explained elaborately (Gowthaman et al., 2019)

$$\ddot{\theta} + d(\dot{\theta}) + r(\theta) = k(t) \tag{2}$$

where

$$d(\dot{\theta}) = D(\dot{\theta})/(I_{xx} + J_{xx});\ r(\theta) = R(\theta)/(I_{xx} + J_{xx});$$

$$K(t) = M(t)/(I_{xx} + J_{xx}) \tag{3}$$

Several mathematical models have been developed for nonlinearity in waves that are not regular (Vapnik, 1999; Luo, 2009; Zhang, 2013; Wang, 2014). The nonlinear damping depending on the rolling speed is the sum of the linear and nonlinear terms. The two most common ways to express a nonlinear term are quadratic and cubic forms. Nonlinear damping is defined as follows in this study.

$$d(\theta) = d_1\dot{\theta} + f_1(\dot{\theta}) \tag{4}$$

The restoring moment (nonlinear term) is

$$r(\theta) = c_1\theta + c_3\theta^3 + c_5\theta^5 \tag{5}$$

To yield the random responses in this research article, two distinguished mathematical models with damping representations are utilized. The angular roll velocity has linear plus quadratic and linear plus cubic terms.

Model 1

The linear plus quadratic damping of the roll motion of a ship is given below (Xian-Rui Hou et al., 2018):

$$\ddot{\varphi} + .1627\dot{\varphi} + 0.5214\dot{\varphi}\left|\dot{\varphi}\right| + 11.4921\varphi + 1.7008\varphi^3 = K(t) \tag{6}$$

where

$$K(t) = \sum_{i=1}^{70} cos(\omega t) \tag{7}$$

with initial conditions

$$\varphi(0) = 0.07 and \dot{\varphi}(0) = 0 \tag{8}$$

Applying HPM to Eq. (6) by using the initial conditions, we obtain (Appendix-A)

$$\varphi(t) = 0.0700202e^{(-0.08135t)} sin(3.38902t + 1.5468) + 0.0093 cos(2t) - 0.0004 sin(2t) - 0.00005 \tag{9}$$

Eq. (9) is the analytical expression of roll angle. From the roll angle, the following results can be found:

The velocity of the roll angle

$$\dot{\varphi}(t) = 0.0057e^{-(0.08135t)} sin(3.38902t + 1.5468) + 0.2373e^{-0.08135t} cos(3.38902t) + 1.5468 - 0.0186 sin2t - 0.0008 cos2t \tag{10}$$

The acceleration is

$$\ddot{\varphi}(t) = -0.804e^{-0.08135t} sin(3.38902t + 1.5468) - 0.039e^{-0.08135t} cos(3.38902t + 1.5468) - 0.0372 cos(2t) + 0.0016 sin(2t) \tag{11}$$

The restoring and damping moment

$$r(\varphi) = 11.4921\varphi(t) + 1.7008(\varphi(t))^3 \tag{12}$$

$$d(\dot{\varphi}) = 0.1627\dot{\varphi}(t) + 0.5214(\dot{\varphi}(t))^2 \tag{13}$$

The comparison between the analytical and numerical results of the roll angle is clearly shown in Fig. 2, while Eqs. (10) to (13) are graphically represented in Figs. 3 to 6, respectively.

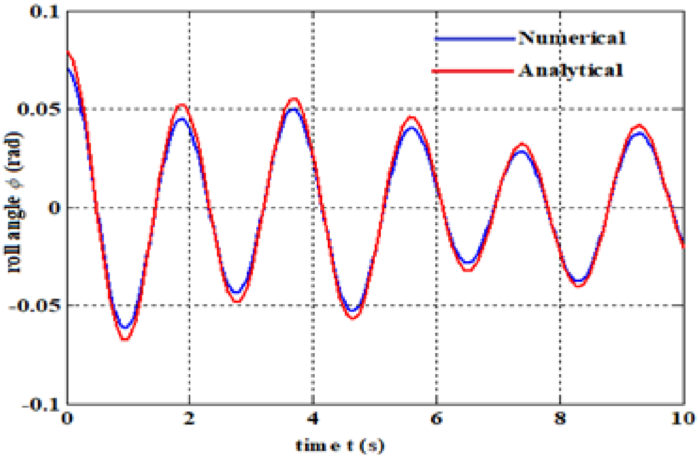

Figure 2: Comparison between the analytical and numerical results of the roll angle

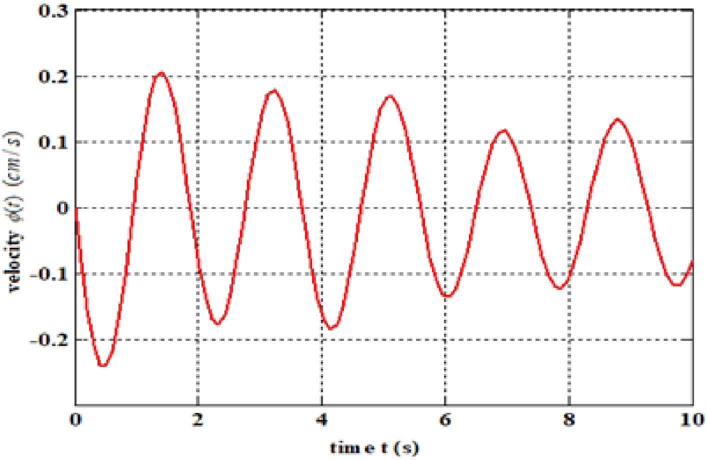

Figure 3: The velocity of the roll angle using Eq. (10)

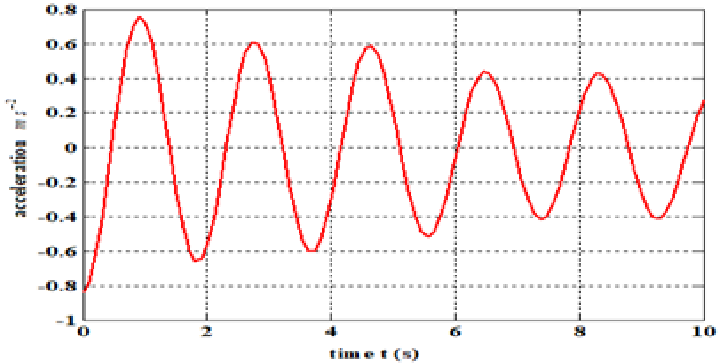

Figure 4: The acceleration of the roll angle using Eq. (11)

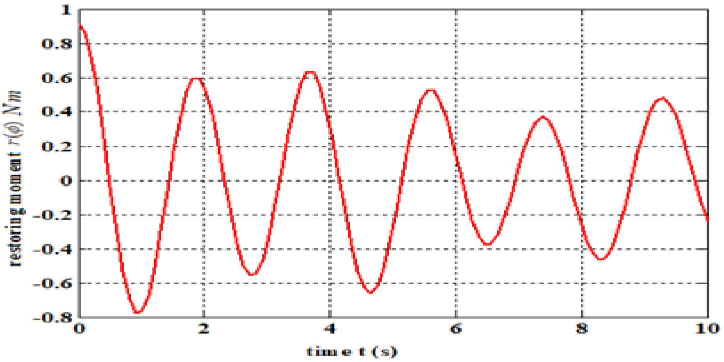

Figure 5: The restoring moment of the roll angle using Eq. (12)

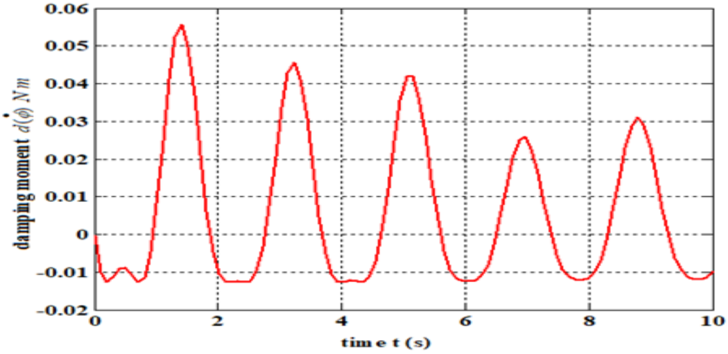

Figure 6: The damping moment of the roll angle using Eq. (13)

Model 2

The second mathematical model is expressed as a cubic term in the roll rate (JON-SWAP) (Xian-Rui Hou et al., 2018):

$$\ddot{\varphi} + 0.32\dot{\varphi} + 0.16\dot{\varphi}^3 + 16\varphi + 19.20\varphi^3 = K(t) \tag{14}$$

where

$$K(t) = \sum_{i=1}^{70} cos(\omega t) \tag{15}$$

The initial conditions for the above Eq. (14) are as follows:

$$\varphi(0) = 0.07 and \dot{\varphi}(0) = 0 \tag{16}$$

By applying the HPM to Eq. (14), the roll angle can be obtained as follows:

$$\varphi(t) = e^{-0.16t}(0.00049C + 0.0633A) - 0.00002C \rightleftarrows B\,cos(7.994t) - 0.000104BC\,sin(7.994t) + 0.002BC + 0.87 \times 10^{-5}CB\,cos(15.99t) - 0.000013CB\,sin(15.99t) + 0.00035C\,cos(1.9968t) + 0.000053AB\,sin(7.994t) - 0.00001265AB\,cos(15.99t) - 0.87 \times 10^{-5}AB\,sin(15.99t) + 0.00095AB + 0.0015AB\,cos(5.9968t) - 0.00004A\,sin(5.9968t) + 0.0044A\,cos(1.9968t) - 0.00035A\,sin(1.9968t) \tag{17}$$

where: $A = cos(3.9968t), B = e^{-0.48t}, C = sin(3.9968t)$

With the use of the roll angle, we can predict its physical properties, such as velocity, acceleration, restoring moment and damping moment.

Fig. 7 shows the comparison made between the analytical and numerical results of the roll angle, while the velocity, acceleration, restoring moment, and damping moment of the roll angle are graphically represented in Figs. 8 to 11, respectively, using Eq. (17).

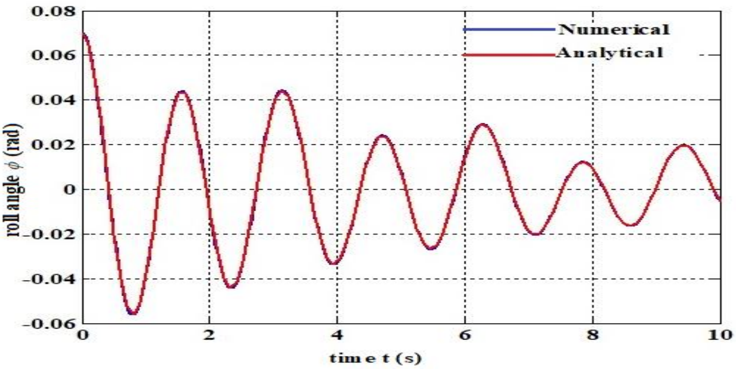

Figure 7: Comparison between the analytical and numerical results of the roll angle.

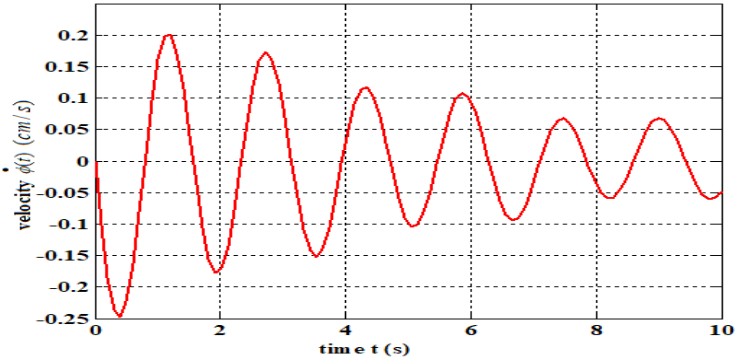

Figure 8: The velocity of the roll angle using Eq. (17)

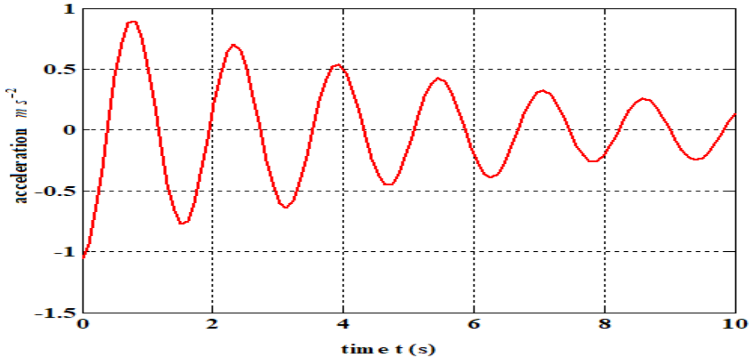

Figure 9: The acceleration of the roll angle using Eq. (17)

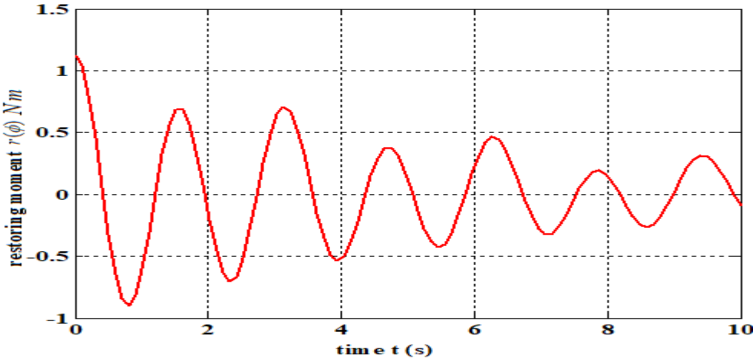

Figure 10: The restoring moment of the roll angle using Eq. (17)

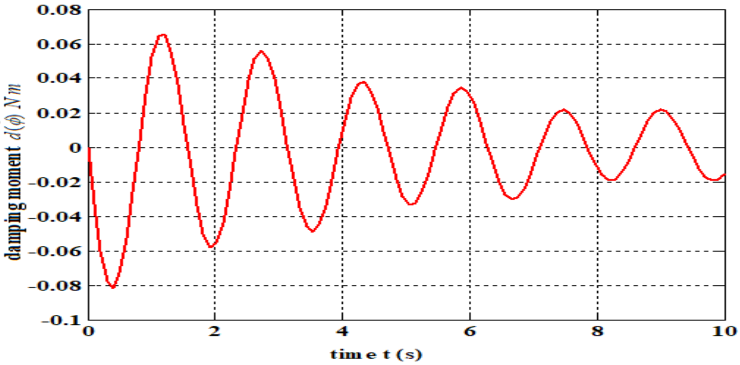

Figure 11: The damping moment of the roll angle using Eq. (17)

DISCUSSION

Eqs. (9) and (10) are two approximate analytical expressions of roll angle for the above two models of ship dynamics. Additionally, Eqs. (10) to (13) represent the velocity, acceleration, restoring and damping moment. The roll angle decay curve is shown in Figs. (2) and (7). The maximum amplitude and frequencies are obtained from the curves. Figs. (3), (4), (8), and (9) show the effect of time on the velocity and acceleration profiles.

The damping moment of ships is related to the diversity of factors such as hull shape, loading condition, *bilge keel*, rolling frequency, and range of rolling angle. Figs. (6) and (11) represent the damping and restoring moment. These two parameters are used to find the bounded motions (safe basin) and unbounded motions (i.e., capsizing). Additionally, in Fig. 2, the analytical results are compared with the simulation results.

CONCLUSION

In this research article, two different types of mathematical models for the nonlinear roll motion of ships are discussed. Approximate analytical results are obtained using the homotopy perturbation method. It is acknowledged that the analytical and simulation findings are compatible. These kinds of problems can be obtained from the proposed algorithm HPM.

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