



## COMPACTNESS OF HYDRAULIC JUMP RECTANGULAR STILLING BASINS USING A BROAD- CRESTED SILL

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### ABSTRACT

The study looks at the possibility of reducing the length of a rectangular hydraulic jump stilling basin by setting up a broad-crested sill. The conclusions of the study are based exclusively on laboratory tests using a specially designed hydraulic installation. The sills tested meet the hydraulic and geometric conditions of broad-crested sills suggested by the literature, in particular the Rao and Muralidhar study of 1963. The dimensionless parameters involved in the problem are the relative sill height  $S = s/h_1$ , the relative length of the stilling basin  $X/h_1$  corresponding also to the relative position of the sill, and the Froude number  $F_1$  of the incident flow of depth  $h_1$ . However, experimentation shows that the relative length  $X/h_1$  of the basin depends only on one of the parameters  $S$  or  $F_1$ . An explicit experimental relationship between the three parameters involved  $X/h_1$ ,  $S$  and  $F_1$  is derived, bearing in mind that  $S$  and  $F_1$  are related by a theoretical relationship according to a previous study conducted by the authors. The experimental tests involve the following wide ranges:  $1 \leq S \leq 6.15$ ,  $18 \leq X/h_1 \leq 60$ , and  $3.082 \leq F_1 \leq 9.20$ , which allow drawing quality conclusions.

The compactness of the stilling basin is evaluated by the ratio  $X/Lj^*$ , where  $Lj^*$  is the length of the classical hydraulic jump evolving freely on a horizontal apron without a sill. The sequent depth ratio  $Y^* = h_2^*/h_1$  of this type of hydraulic jump is related to the incident

Froude number  $F_1$  by the well-known Belanger's theoretical relationship successfully demonstrated in 1828, which remains unchanged.

Using the obtained appropriate experimental dimensionless relationship, combined with the Bradley and Peterka experimental equation giving the length of the classic hydraulic jump, it is shown that the ratio  $X/Lj^*$  is always less than unity regardless of the value of the Froude number  $F_1$ . This fact clearly indicates that the setup of a broad-crested sill has a reducing effect on the length  $X$  of the stilling basin. An in-depth study of the variation curve of  $X/Lj^*$  against  $F_1$  reveals that the best compactness is obtained for  $F_1 = 7.311$ , giving  $X/Lj^* \approx 0.765$ , which corresponds to a compactness rate of 23.5%.

**Keywords:** Control of hydraulic jumps, classical hydraulic jumps, CHJ, compactness, rectangular stilling basins, broad-crested sills, incident Froude number.

## INTRODUCTION

The hydraulic jump corresponds to a sudden change in the free surface of a flow passing from an initial depth  $h_1$  to a much greater final depth  $h_2$ . The flow of initial depth  $h_1$  is supercritical characterized by a high velocity, while the flow of final depth  $h_2$  is subcritical of rather low velocity. Thus, a hydraulic jump corresponds to a discontinuous transition from supercritical to subcritical flows.

When one carefully consults the literature, one realizes very quickly that the most complete information concerns what is commonly called the "classical hydraulic jump" (Bakhmetef, 1932, 1936; Escande, 1938; Bradley and Peterka, 1957; McCorquodale, 1986; Hager and Bremen, 1989; Hager et al., 1990). This list of references is by no means exhaustive. There is even more in the literature that deals with the classical hydraulic jump. The hydraulic jump is said to be "classical" when it occurs in a smooth prismatic rectangular channel with a horizontal apron. There is no longitudinal change in either the width or the shape of the channel, and its longitudinal axis is straight. It is often identified in the specialized literature by the three letters CHJ, which means "Classical Hydraulic Jump".

The literature is certainly abundant on the CHJ because of the simple rectangular shape of the channel, which is easy to implement, and the study of the flow can be fairly easily developed from the theoretical point of view. This is how Belanger (1828) developed his universally known and recognized equation, which translates the sequent depth ratio  $Y^* = h_2^*/h_1$  of the CHJ as a function of the Froude number  $F_1$  of the incident flow.

Belanger's relationship was obtained using the momentum theorem assuming that head losses other than those of the hydraulic jump are negligible.

It is customary to identify the characteristics of the CHJ by the symbol "\*" to distinguish them from those of the hydraulic jump evolving in other channel shapes. Hydraulic jumps can also be created and evolve in other channel shapes, such as trapezoidal, triangular, U-

shaped, and circular shapes (Advani, 1962; Ali and Ridgeway, 1977; Bourdon, 1963; Caric, 1977; Hager and Wanoschek, 1977; Achour, 2000; Achour and Debabèche, 2003), but herein, the previous list of references is not exhaustive. Other channel shapes can also be the site of a hydraulic jump, but these shapes are not of practical interest (Argyropoulos, 1957, 1961, 1962; Silvester, 1964).

The hydraulic jump can also be controlled by setting up a sill at the downstream end of the stilling basin at a distance  $X$  from the beginning of the hydraulic jump. The sill can be continuous or discontinuous, thin-crested or broad-crested. The fundamental role of the sill is to control the position of the jump during changes in flow parameters such as the flow rate. The height  $s$  of the sill must be such that the hydraulic jump moves neither upstream nor downstream. It must be stable and complete on the horizontal apron of the stilling basin. However, the hydraulic jump can also be forced, i.e., the hydraulic jump is located on either side of the sill.

Most of the studies on controlled or forced hydraulic jumps concern rectangular channels (Forster and Skrinde, 1950; Bretz, 1987; Achour and Amara, 2022a; Achour and Amara, 2022b; Achour and Amara, 2022c). Fewer studies have been devoted to the control of hydraulic jumps in other channel shapes, such as triangular and U-shapes (Achour and Debabèche, 2003a; Achour and Debabèche, 2003b).

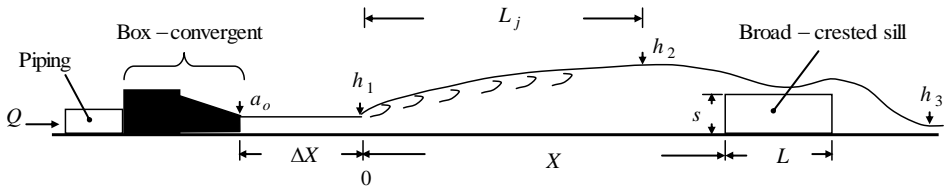
What is sought in practice when controlling the hydraulic jump by a sill is to find the minimum length  $X$  of the stilling basin for a given sill height  $s$  and initial Froude number  $F_1$ , such that the hydraulic jump is formed completely. The search for a minimum  $X$  is linked to the economic aspect of the problem since it will involve less volume of excavation and less surface area of the basin to be reinforced.

In this context, this paper examines in detail the influence that the setup of a broad-crested weir could have on the compactness of a rectangular stilling basin. The conclusions of the study are based on the results of carefully conducted experimental tests under various incident flow conditions. A hydraulic installation specially designed for this purpose was used.

## **DESCRIPTION OF THE TEST BENCH**

Fig. 1 schematically shows the somewhat special hydraulic installation used in the tests. A rectangular channel 0.293 m wide and 10 m long was used as a structure for the tests. To make the hydraulic jump appear on the horizontal apron of the channel, a supercritical flow must be caused at the initial section of the hydraulic jump of depth  $h_1$  and then transformed into a subcritical flow at the final section of the hydraulic jump where the depth is  $h_2$  (Fig. 1). Raising the depth from  $h_1$  to  $h_2$  is ensured by setting up a broad-crested sill of appropriate height  $s$ . To create a supercritical initial flow, a sluice gate is usually used. The installation used in this study is quite different since the incident supercritical flow is caused by a pressurized box-converging set equipped with guides to suitably and evenly direct the flow toward the exit. The  $\Delta X$  supercritical flow slice is short

enough to precisely assimilate the outlet opening  $a_o$  of the convergent to the initial depth  $h_1$ . This allows avoiding the very delicate measurement of the depth  $h_1$  by a point-gauge given the highly supercritical nature of the flow.



**Figure 1: Hydraulic jump controlled by a broad-crested sill; incident flow generated by a box-convergent set.**

To obtain various openings  $a_o$ , one operates a simple transverse cut of the convergent at the desired opening by the use of an appropriate metal plate cutting machine, provided the length of the convergent is sufficient.

In Fig. 4,  $X$  denotes the position of the sill counted from the initial section of the hydraulic jump whose length is  $L_j$ .

The flow rate  $Q$  was measured by a diaphragm flow meter with a deviation of 0.5 l/s, while the final depth  $h_2$  of the hydraulic jump was measured using a double precision Vernier point-gauge with an absolute error of 0.02 mm. All of these measuring instruments were described by the authors in a previous study (Achour and Amara, 2022c).

The installation thus described is supplied in a closed circuit with a maximum flow rate of 35 l/s. During the tests, the sill was kept unsubmerged and free so that the downstream depth  $h_3$  (Fig. 1) had no effect on either the flow rate  $Q$  or the flow over the horizontal apron.

## COMPACTNESS EFFECT

One of the objectives of the dimensioning calculation is the determination of the minimum length  $X$  of the stilling basin required for the complete formation of the hydraulic jump. The study looks at the possibility of highlighting any compactness effect of the stilling basin when the hydraulic jump is controlled by a broad-crested sill. Experimentation has shown that the relative length  $X/h_1$  only depends on the Froude number  $F_1$  of the incident flow or on the relative sill height  $S = s/h_1$ . Achour and Amara (2022b) theoretically derived the relationship between  $S$  and  $F_1$  as:

$$\frac{s}{h_1} = Y + \frac{1}{2} \left( \frac{F_1}{Y} \right)^2 - \frac{3}{2} F_1^{2/3} \quad (1)$$

where the sequent depth ratio  $Y$  is related to  $F_1$  by the well-known Belanger's relationship valid for the classical hydraulic jump. For  $Y = Y^*$ , Belanger's relationship is expressed as (1828):

$$Y^* = \frac{1}{2} \left( \sqrt{1 + 8F_1^2} - 1 \right) \quad (2)$$

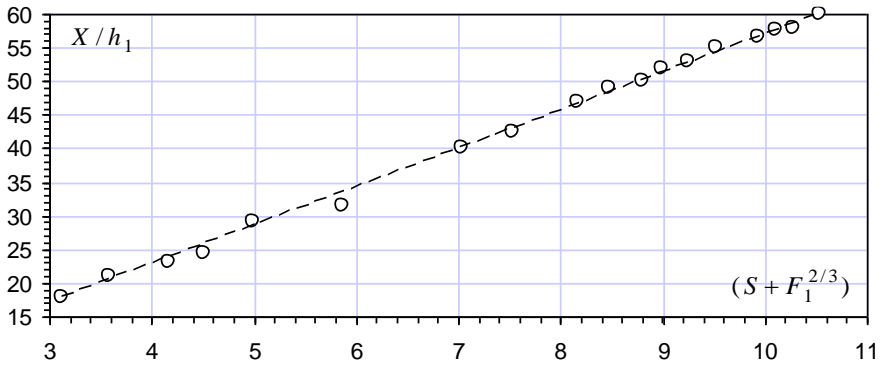
The analysis of the experimental measurements (Table 1) showed that a single dimensionless curve could be obtained when the variation in  $X/h_1$  was graphically represented as a function of the quantity  $(S + F_1^{2/3})$ , as shown in Fig. 2. The resulting average curve was in fact a straight line with a slope equal to 5.696 obtained over the whole wide range of incident Froude numbers  $F_1$  such that  $3 \leq F_1 \leq 9$ . Thereby:

$$\frac{X}{h_1} = 5.696 (S + F_1^{2/3}) \quad (3)$$

Eq. 3 was derived using the linear least-squares fitting method involving experimental data, with a coefficient of determination  $R^2 = 0.9976$  which is very acceptable in regard to experimental results.

**Table 1: Experimental values of the parameters  $F_1$ ,  $S$  and  $X/h_1$  during the control of the hydraulic jump by a broad-crested sill**

$F_1$	3.082	3.460	3.950	4.273	4.660	5.423	6.380	6.782	7.322
$S$	1	1.290	1.670	1.885	2.200	2.785	3.600	3.950	4.400
$X/h_1$	18	21	23	24.5	29	31.6	40	42.5	46.8
$F_1$	7.620	7.860	7.992	8.170	8.362	8.694	8.941	9.002	9.200
$S$	4.600	4.850	5	5.200	5.400	5.700	5.800	5.950	6.150
$X/h_1$	49	50	52	53	55	56.5	57.6	58	60



**Figure 2: Experimental variation in the relative length  $X/h_1$  of the stilling basin when controlling the hydraulic jump by a broad-crested sill; (o) Experimental points; (- -) Trend line**

On the other hand, the compactness of the stilling basin can be evaluated by the ratio  $X/L_j^*$ , where  $L_j^*$  is the length of the classical hydraulic jump. It is customary to use the sign "\*" to denote the characteristics of the classical hydraulic jump.

The reducing effect of the sill on the length  $X$ , or compactness effect, is felt when the ratio  $X/L_j^*$  is less than unity, i.e., when the length  $X$  of the stilling basin is less than the length  $L_j^*$  of the classical hydraulic jump. Notably, the ratio  $X/L_j^*$  can be written as follows:

$$X/L_j^* = (X/h_1)(L_j^*/h_1)^{-1} \tag{4}$$

The length  $L_j^*$  of the classical hydraulic jump could only be evaluated by laboratory experimentation. Its definition varies from author to author. The following three criteria have been proposed according to the author's point of view. Each considers that the length  $L_j^*$  must be measured in the section downstream of the hydraulic jump, where:

1. The free surface is practically or almost horizontal.
2. The turbulent surface of the roller of the hydraulic jump is small.
3. Gradually varied flow appears at the downstream end of the roller.

All of these definition criteria were intended to indicate the downstream limit of the hydraulic jump beyond which the protection or reinforcement of the stilling basin is no longer necessary. This limit defines the area where the flow is straightforwardly subcritical, characterized by low velocities and inconsequential tractive forces. In practice, the authors agree that the length  $L_j^*$  corresponds to the distance of the stilling

basin, which requires protection against scour that could occur due to tractive forces.

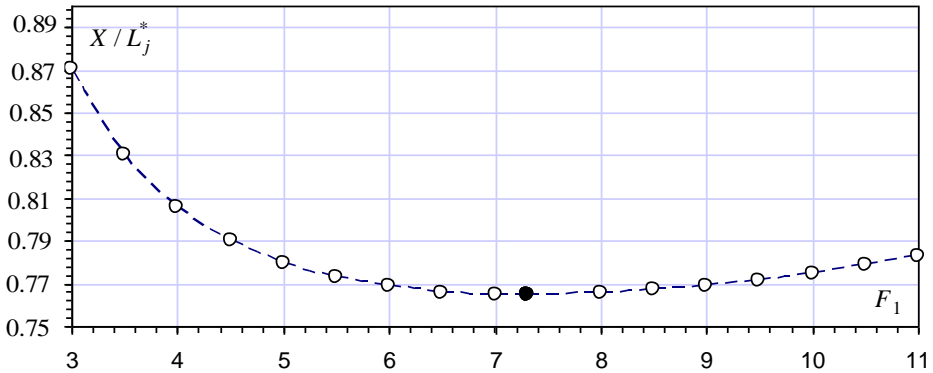
The tests by Peterka and Bradley (1957) aimed to derive an experimental relationship that could faithfully translate the length  $L_j^*$  of the classic hydraulic jump as much as possible. This, related to the initial height  $h_1$  of the jump, is written as:

$$\frac{L_j^*}{h_1} = 220 \tanh \left( \frac{F_1 - 1}{22} \right) \quad (5)$$

where “tanh” denotes the hyperbolic tangent. Inserting both Eq. 3 and Eq. 5 into Eq. 4 results in:

$$X / L_j^* = 0.0259 \frac{(S + F_1^{2/3})}{\tanh \left( \frac{F_1 - 1}{22} \right)} \quad (6)$$

Given that the relative sill height  $S$  depends only on the Froude number  $F_1$  of the incident flow according to Eq. 1 along with Eq. 2, it is therefore correct to conclude, with regard to Eq. 6, the ratio  $X / L_j^*$  also depends only on the Froude number  $F_1$ . Eq. 6 is represented graphically in Fig. 3.



**Figure 3: Variation in the relative length  $X/L_j^*$  as a function of the incident Froude number  $F_1$ . (●)  $X/L_j^*$  minimum**

What is obvious to observe from Fig. 3 is that regardless of the value of  $F_1$ , the ratio  $X/L_j^*$  is less than unity. This result allows the conclusion that the broad-crested sill does indeed have a compacting effect on the stilling basin. Moreover, Fig. 4 shows an asymmetrical bell curve with a first descending branch in the range  $3 \leq F_1 \leq 7.311$  and a

second ascending branch for  $F_1$  values greater than 7.311, i.e.,  $F_1 > 7.311$ . The curve thus passes through a minimum around  $F_1 = 7.311$  obtained after differentiating Eq. 6 with respect to  $F_1$  and solving the resulting implicit equation. Consequently, inserting this value into Eq. 6 gives  $X/L_j^*$  approximately equal to 0.765, which corresponds to the best possible compactness. Thus, for  $F_1 = 7.311$ , the broad-crested sill significantly reduces the length  $X$  of the stilling basin compared to the length  $X = L_j^*$  of the classical hydraulic jump. The compactness rate is then equal to 23.5%, which is significant.

## CONCLUSIONS

The study focused on the hydraulic jump controlled by a broad-crested sill. The main objective was to experimentally observe a probable compactness of the stilling basin by comparing its length  $X$  with the length  $L_j^*$  of the classical hydraulic jump. After setting up the problem, the experimental installation and the measuring equipment were described. The equipment was simple since the experiment required only a diaphragm flow meter for measuring the flow rate  $Q$  and a double-precision Vernier point-gauge intended in particular for measuring the final depth  $h_2$  of the hydraulic jump. The measurement of the initial depth  $h_1$  of the hydraulic jump was not carried out in a conventional way, i.e., using a water level gauge as is customary. Due to the strongly supercritical nature of the incident flow, the experimental measurement of  $h_1$  by the classical method is not easy, even imprecise. This difficulty was circumvented by setting up a pressurized box-convergent set intended to generate the incident flow (Fig. 1). The box-convergent assemblage is directly supplied by a pump operating in a closed circuit via a flexible pipe. The exit aperture  $a_0$  of the convergent, generating the incident flow, can be adjusted *ad libitum* according to the needs of the experiment. The position  $X$  of the sill, which also corresponds to the length of the stilling basin, was chosen so that the hydraulic jump originates very close to the opening  $a_0$  of the convergent, thus implying an undeveloped incident flow. In this way, the opening  $a_0$  can be reasonably assimilated to the initial depth  $h_1$  of the hydraulic jump, which was measured simply by the use of a Vernier caliper. In addition, the experimental installation thus designed allowed avoiding the use of an upstream supply basin and a sluice gate usually required to generate the incident flow.

The analysis of the experimental measurements first showed that the sequential depth ratio  $Y$  of the controlled jump faithfully satisfies the Belanger equation established for the classic hydraulic jump. This implies that the setup of a broad-crested sill for the control of the hydraulic jump in a rectangular channel has no reducing effect on the final depth  $h_2$  of the hydraulic jump.

The evaluation of both the length  $X$  of the stilling basin and the effect of compactness that could result from the setup of a broad-crested sill were the subject of particular attention during this study.



The analysis of the experimental measurements showed that the relative length  $X/h_1$  of the stilling basin ultimately depends only on the relative sill height  $S = s/h_1$  or on the Froude number  $F_1$  of the incident flow in accordance with Eq. 3, in which  $S$  and  $F_1$  are both related by the theoretical explicit Eq. 1.

To show a possible compactness effect on the stilling basin resulting from the setup of the broad-crested sill, the variation in the ratio  $X/Lj^*$  as a function of the incident Froude number  $F_1$  is graphically represented in Fig. 3 according to Eq. 6 along with Eq. 1. The asymmetrical bell curve obtained clearly showed that the ratio  $X/Lj^*$  is always less than unity regardless of the value of  $F_1$ , thus leading to the conclusion that there was undoubtedly a compactness effect. This reducing effect has been experimentally observed in the wide range  $3 \leq F_1 \leq 9$ . The best compactness was, however, observed for the value  $F_1 = 7.311$ , for which  $X/Lj^* = 0.765$ . The compactness rate was therefore 23.5%, which is not negligible.

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