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NORMAL DEPTH COMPUTATION IN A VAULTED RECTANGULAR CHANNEL USING THE ROUGH MODEL METHOD (RMM)

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ABSTRACT

Designing and maintaining open channels as well as studying nonuniform flow crucially depend on normal depth. In practical applications, the vaulted rectangular cross-section is widely applied. However, the form of the relationship determining the normal depth is implicit for this type of cross-section. Currently, trial and error procedures, the process of successively improving fitting, and regression-based fitting or the method of fitting curves are used to compute the normal depth for this type of channel. In this research, it is suggested to employ the rough model method (RMM) to estimate the normal depth in a vault-shaped rectangular cross-section. In this approach, knowing the Chezy or Manning coefficient will not be required in the process computation of normal depth in this conduit. However, it only needs measurable parameters, in particular, the impact of the absolute roughness. Based on well-known referential rough model properties, the RMM evaluates the normal depth using a dimensionless correction factor. The relationship that governs flow in the rough model allows for an explicit calculation of the normal depth in this kind of conduit. Calculation examples are given to illustrate how easy the calculation process is.

Keywords: Vaulted rectangular cross-section, Discharge, Normal depth, Manning's roughness coefficient, Rough model method, Uniform flow.

INTRODUCTION

The calculation of the normal depth is an essential task in the hydraulic design of open channels, and this parameter is important for the study and classification of nonuniform flow. (Swamee, 1994; Lakehal and Achour, 2017). The normal depth's governing equation is implicit for most of the channel sections; therefore, analytical solutions are not available (Vatankhah, 2012). In the past, due to the governing equation's implicit form, explicit computation approaches have been proposed in rectangular, trapezoidal, circular, egg-shaped, parabolic, and horseshoe channels (Swamee and Rathie, 2004; Vatankhah and Easa, 2011; Liu et al., 2010; Shang et al., 2020; Achour and Khattaoui, 2008).

The vault-shaped cross-section is used in irrigation and drainage systems in addition to being widely utilized as free surface water conveyance tunnels (Vatankhah, 2012; Liu and Wang, 2013). However, it is difficult to obtain an explicit solution for normal depth due to the complicated geometrical shape of the channel. In this respect, a few explicit equations with acceptable accuracy have been proposed for the computation of normal depth in this section. Recently, Vatankhah (2012) suggested an explicit equation for normal depth based on regression equations, with maximum relative errors of less than 0.05%. Using the principle of gradual optimization fitting of the data points, Liu and Wang (2013) proposed an explicit equation for the normal depth of the vault-shaped rectangular cross-section for different portions. The proposed equations have adequate accuracy with a maximum relative error of less than 1%. Shang et al., (2020) established an explicit formula for the normal depth for the vaulted rectangular cross-section using model parameters plus improved revised PSO algorithms in MATLAB, with maximum relative errors of less than 0.04%. Current methods to determine the normal depth used Manning's or Chezy's relations, with the Manning or Chezy coefficients considered to be a constant. Because these coefficients depend on geometric and hydraulic flow variables, including discharge, the longitudinal slope, kinematic viscosity, normal depth sought, and, more particularly, the absolute roughness that describes the state of the inner wall of the canal, this approach is not physically justified (Achour, 2015; Lakehal and Achour, 2017).

Therefore, this study aims to establish equations for determining normal depth in a vaulted rectangular cross-section based on practical data. The approach of computation is based on the rough model method (RMM), which has recently proven successful in the design of conduits and channels as well as in determining normal depth (Achour, 2014a; 2014b; 2015; Achour and Bedjaoui, 2012; Achour and Sehtal, 2014; Lakehal and Achour, 2017; Riabi and Achour, 2019). The RMM only requires measurable parameters in practice to calculate the normal depth in the vaulted rectangular cross-section, namely, the discharge Q, the longitudinal slope i, the diameter D of the conduit, the absolute roughness ε, and the kinematic viscosity ν of the flowing liquid. Moreover, unlike current methods of calculation, this one does not require the coefficients of Chezy and Manning. This approach is based on the geometric and hydraulic properties of a referential rough model with well-defined parameters. These parameters are employed to derive those of the study channel, particularly normal depth, using a nondimensional correction factor of linear dimension (Achour, 2014b). The resulting equations from the RMM are valid in the entire domain of turbulent flow, which corresponds to Reynolds number $R \geq 2300$ and relative roughness ε/D_h varying in the large range [0;0.05] (Achour, 2015). Examples are provided to help the reader comprehend the computation process and see how straightforward and effective it is.

GEOMETRIC CHARACTERISTICS

As shown in Fig. 1, the vaulted rectangular channel is composed of two different geometric parts. The water's surface is at the lower part, which is a rectangular section with a height of $D/2$ and a width when $0 \leq y_n \leq D/2$ (Fig. 1a). When $D/2 \leq y_n \leq D$, the water surface is situated at the upper part as a top arc section with a radius of $D/2$ (Fig. 1b). This conduit is defined by the aspect ratio $\eta = y_n / D$, commonly referred to as the nondimensional normal depth, where y_n is the normal depth and D is is the diameter of a circular cross-section. The longitudinal slope is i, the absolute roughness is ε, and the discharge is Q, with ν as the fluid's kinematic viscosity.

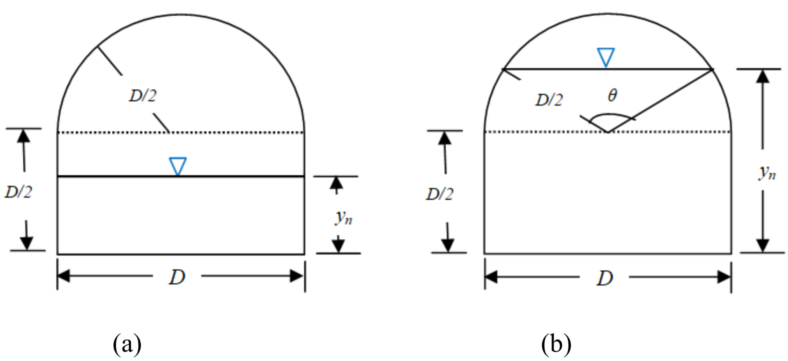

Figure 1: The vault-shaped rectangular cross-section at two flow depths

a) $0 \leq y_n \leq \frac{D}{2}$; b) $\frac{D}{2} \leq y_n \leq D$;

COMPUTATION OF NORMAL DEPTH BY MANNING'S EQUATION

The normal depth for uniform flow in channels and conduits is calculated by *Manning's* equation *(Chow, 1959)*:

$$Q = \frac{1}{n} A R_h^{\frac{2}{3}} \sqrt{i} \qquad (1)$$

where Q is the discharge (m^3/s), i is the longitudinal slope of the channel, n is the resistance coefficient of *Manning*, A is the water area (m^2), and R_h is the hydraulic radius (m).

According to the geometrical locus of the flow depth, the conduit's geometrical and hydraulic characteristics are expressed as follows:

1. For $0 \le y_n \le D/2$, i.e., $0 \le \eta \le 0.5$, the following are the formulas for the water area A and the wetted perimeter P:

$$A = D^2 \tag{2}$$

$$P = D(1 + 2\eta) \tag{3}$$

Hence, the hydraulic radius $R_h = A/P$ is:

$$R_h = D\frac{\eta}{(1+2\eta)} \tag{4}$$

Inserting Eqs. (2) and (4) into Eq. (1) and rearranging results in the following:

$$Q^* = 2^{\frac{8}{3}}\frac{\eta^{\frac{5}{3}}}{(1+2\eta)^{\frac{2}{3}}} \tag{5}$$

where Q^* is the relative conductivity expressed as follows:

$$Q^* = \frac{nQ}{\sqrt{i}(D/2)^{\frac{8}{3}}} \tag{6}$$

2. For $\frac{D}{2} \le y_n \le D$, i.e., $0.5 \le \eta \le 1$, from Figure 1b, the geometric properties can be expressed for the studied channel as follows:

The water area A is given by:

$$A = \frac{D^2}{4}\sigma(\eta)\varphi(\eta) \tag{7}$$

The wetted perimeter P is governed by the following relationship:

$$P = D\varphi(\eta) \tag{8}$$

where:

$$\sigma(\eta)=1+\frac{2(2\eta-1)\sqrt{\eta(1-\eta)}}{2+\frac{\pi}{2}-\cos^{-1}(2\eta-1)} \tag{9}$$

$$\varphi(\eta)=2+\frac{\pi}{2}-\cos^{-1}(2\eta-1) \tag{10}$$

The hydraulic radius $R_h = A/P$ is thus:

$$R_h=\frac{D}{4}\sigma(\eta) \tag{11}$$

Taking into account Eqs. (7) and (11), Eq. (1) can be rewritten as:

$$Q^*=\frac{1}{2^{\frac{2}{3}}}\varphi(\eta)\left[\sigma(\eta)\right]^{\frac{5}{3}} \tag{12}$$

The relative conductivity Q^* is determined by Eq. (6).

The problem posed consists of determining the normal depth y_n, which amounts to evaluating the aspect ratio η and consequently $y_n=\eta D$. When examining the form of relations (5) and (12), it appears that the aspect ratio η cannot be determined explicitly. Thus, the calculation requires a graphical method or an iterative procedure.

Approximate equations

Let Manning's coefficient *n* be a parameter that can be calculated. This essentially means that the relative conductivity Q^* of relation (6) is also a known parameter. To facilitate the calculation of the aspect ratio η, the following approximate relations are proposed:

1. For $0\le y_n\le\frac{D}{2}$

To directly compute the nondimensional normal depth η, we propose the following explicit relationship of the exact Eq. (5).

$$\eta=\left(\frac{Q^*}{2^{\frac{8}{3}}}\right)^{\frac{3}{5}}\left[1+1.08\left(\frac{Q^*}{2}\right)^{0.615}\left(1+2.05Q^{*0.941}\right)^{0.16}\right]^{\frac{2}{5}} \tag{13}$$

Eq. (13) is valid over the entire practical range of depth $0\le\eta\le0.5$ corresponding to the relative conductivity Q^* such that $0\le Q^*\le1.26$. The maximum relative error

$\left(\dfrac{\Delta\eta}{\eta(\%)} = 100\times\left(\dfrac{\eta_{exct}-\eta_{prop}}{\eta_{exct}}\right)\right)$ between the values of η calculated using the proposed equation (13) and the exact equation (5) is less than 0.093%.

2. For $\dfrac{D}{2} \le y_n \le D$

In the broad range $0.5 \le \eta \le 0.86$ and corresponding to $1.26 \le Q^* \le 2.32$, the following approximate relation of η was established:

$$\eta = 0.0000125{Q^*}^{9.939} + 0.2868Q^* + 0.1392 \tag{14}$$

The maximum relative error produced by Eq. (14) is less than 0.2%, which is more than sufficient for practical applications.

In relations (5) and (12), the main problem lies not in their implicit nature but in the fact that Manning's coefficient is required to calculate the normal depth sought on which depends this coefficient. As a result, it is difficult to know Manning's *n* since it depends on the normal depth sought.

In practice, the diameter *D* of the conduit, the absolute roughness ε, the slope *i* of the channel, the discharge *Q*, and the kinematic viscosity ν are the known parameters of the problem. Since Manning's coefficient *n* is not given in the problem data, it is impossible to determine the relative conductivity Q^* of Eq. (6). To solve the problem with only these data, the rough model method appears to be the most suitable computation tool. This is what this study's outcome aims to demonstrate.

COMPUTATION OF NORMAL DEPTH BY THE RMM

Characteristics of the reference rough model

To calculate the normal depth, the rough model method (RMM) is applied based on the reference rough model illustrated in Fig. 2.

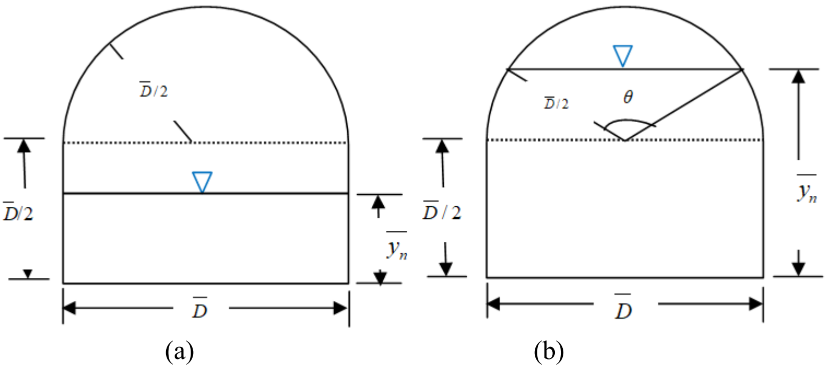

Figure 2: Rough model of a vaulted rectangular channel at two flow depths:
$$(\boldsymbol{a})\mathbf{0} \leq \overline{\boldsymbol{y_n}} \leq \frac{\overline{\boldsymbol{D}}}{\mathbf{2}}; \frac{(\boldsymbol{b})\overline{\boldsymbol{D}}}{\mathbf{2}} \leq \overline{\boldsymbol{y_n}} \leq \overline{\boldsymbol{D}}$$

Fig. 2 schematically shows the normal depth in a vaulted rectangular rough model channel. This channel is characterized by the diameter $\overline{D} \neq D$, and the normal depth $\overline{y_n}$ is such that $\overline{y_n} \neq y_n$ and even $\overline{y_n} > y_n$, and the aspect ratio $\overline{\eta} = \frac{\overline{y_n}}{\overline{D}} \neq \eta = \frac{y_n}{D}$.

The relative roughness value $\frac{\overline{\varepsilon}}{\overline{D_h}} = 0.037$, where $\overline{D_h}$ is the hydraulic diameter, is the distinctive feature of the rough model. The choice of this value is arbitrary. The relative roughness value that was selected is so great that the predominant flow regime is fully rough. Thus, according to the Colebrook-White equation (Colebrook, 1939), the friction factor is $\overline{f} = \frac{1}{16}$ because $R = \overline{R}$ tends to an infinitely large value (Achour, 2015).

To determine the normal depth, we admit the following conditions: $\overline{i} = i; \overline{Q} = Q$.

Applying Eq. (1) to the rough model leads to:

$$Q = \frac{1}{\overline{n}} \overline{A} \overline{R_h}^{\frac{2}{3}} \sqrt{i} \tag{15}$$

where $\overline{n}$ *is Manning's* resistance coefficient in the rough model, and its expression is (Achour, 2014a):

$$\overline{n} = \frac{\overline{R_h}^{\frac{1}{6}}}{\overline{C}} \tag{16}$$

According to the RMM, Chezy's coefficient $\overline{C}$ is given by the following equation (Achour and Sehtal, 2014):

$$\overline{C} = 8\sqrt{2g} \tag{17}$$

By inserting Eq. (17) into Eq. (16), we thus conclude:

$$\overline{n} = \frac{\overline{R_h}^{\frac{1}{6}}}{8\sqrt{2g}} \tag{18}$$

According to the ranges of values for the filling rate $\overline{\eta}$, the flow can be divided into two zones. These two zones of flow depths correspond to $0 \le \overline{\eta} \le 0.5$ and $0.5 \le \overline{\eta} \le 1$.

Determination of the normal depth for $0 \le \overline{\eta} \le 0.5$

The hydraulic parameters of this zone are as follows:

The following relation governs the wetted area $\overline{A}$:

$$\overline{A} = \overline{D}^2\overline{\eta} \tag{19}$$

The formula for the wetted perimeter $\overline{P}$ is:

$$\overline{P} = \overline{D}(1 + 2\overline{\eta}) \tag{20}$$

Thus, the hydraulic radius $\overline{R_h} = \overline{A}/\overline{P}$ is as follows:

$$\overline{R_h} = \overline{D}\frac{\overline{\eta}}{(1+2\overline{\eta})} \tag{21}$$

The following can be written by introducing Eq. (21) into Eq. (18):

$$\overline{n} = \frac{\overline{D}^{\frac{1}{6}}}{8\sqrt{2g}}\frac{\overline{\eta}^{\frac{1}{6}}}{(1+2\overline{\eta})^{\frac{1}{6}}} \tag{22}$$

By inserting Eqs. (19), (20), and (22) into Eq. (15) and rearranging, one can obtain the following:

$$\overline{Q}^* = 2^{\frac{5}{2}}\frac{\overline{\eta}^{\frac{3}{2}}}{(1+2\overline{\eta})^{\frac{1}{2}}} \tag{23}$$

where $\overline{Q}^*$ is the relative conductivity expressed as follows:

$$\overline{Q}^* = \frac{Q}{8\sqrt{2gi(\overline{D}/2)^5}} \tag{24}$$

Since all the parameters of Eq. (24) are known, it is possible to calculate the relative conductivity value $\overline{Q}^*$. The aspect ratio $\overline{\eta}$ can be calculated for the given value of $\overline{Q}^*$ by using equation (23).

When both sides of equation (23) are squared, the following is obtained:

$$\overline{Q}^{*2} = 2^5 \frac{\overline{\eta}^3}{(1+2\overline{\eta})} \tag{25}$$

We obtain a third-degree equation in η as follows:

$$\overline{\eta}^3 - \frac{1}{16}\overline{Q}^{*2}\overline{\eta} - \frac{1}{32}\overline{Q}^{*2} = 0 \tag{26}$$

Eq. (26) is a cubic equation that does not contain a second order. Its discriminant may be expressed as follows:

$$\Delta = \left(\frac{\overline{Q}^*}{8}\right)^4 \left(1 - \frac{\overline{Q}^{*2}}{27}\right) \tag{27}$$

Eq. (27) shows that two cases arise:

- $\overline{Q}^* \geq \sqrt{27}$, then $\Delta \leq 0$. The real root of the third-degree equation in $\overline{\eta}$ expressed by relation (26) is:

$$\overline{\eta} = \frac{\overline{Q}^*}{2\sqrt{3}}\cos\left(\frac{\beta}{3}\right) \tag{28}$$

where the angle β is such that:

$$\cos(\beta) = \frac{3\sqrt{3}}{\overline{Q}^*} \tag{29}$$

- $\overline{Q}^* \leq \sqrt{27}$, then $\Delta \geq 0$. The real root of the third-degree equation in $\overline{\eta}$ expressed by relation (26) is:

$$\overline{\eta} = \frac{\overline{Q}^*}{2\sqrt{3}}ch\left(\frac{\beta}{3}\right) \tag{30}$$

where the angle β is expressed as:

$$ch(\beta) = \frac{3\sqrt{3}}{\overline{Q}^*} \tag{31}$$

The exact value of the aspect ratio in the rough model is provided by Equations (28) and (30).

Nondimensional correction factor of the linear dimension

According to the rough model method, all linear dimensions L of a channel and its counterpart $\overline{L}$ of a rough model are connected by the following fundamental equation:

$$L = \psi\overline{L} \tag{32}$$

where ψ is a dimensionless correction factor with a value less than 1, which is determined by the following equation (Achour and Bedjaoui, 2006; 2012):

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D_h}}{4.75} + \frac{8.5}{\overline{R}}\right)\right]^{-\frac{2}{5}} \tag{33}$$

where $\overline{R}$ is the Reynolds number in the rough reference model, which can be given by:

$$\overline{R} = \frac{4Q}{\overline{P}\nu} \tag{34}$$

Steps for calculating the normal depth

To calculate the normal depth in a vaulted-shaped rectangular cross-section, when $0 \le \overline{Q}^* \le 1.41421356$, the following data must be given: Q, i, $\boldsymbol{D}$, ε and ν. It is important to first note that these parameters are practicably measured and that Manning's roughness coefficient is not imposed. The following steps are advised:

1. If we assume $\overline{D} = D$, the relative conductivity $\overline{Q}^*$ is given by:

$$\overline{Q}^* = \frac{Q}{8\sqrt{2gi(\overline{D}/2)^5}} = \frac{Q}{8\sqrt{2gi(D/2)^5}}$$

2. Depending on the sign of the discriminant Δ, use Eq. (28) or Eq. (30) to calculate the aspect ratio $\overline{\eta}$.

3. Eqs. (20) and (21) give $\overline{P}$ and $\overline{R_h}$, respectively. This allows us to calculate the

hydraulic diameter as $\overline{D}_h = 4\overline{R}_h$ and Reynolds number $\overline{R}$ by the use of Eq (34).

4. Apply Eq. (33) to determine the dimensionless correction factor ψ.
5. Let us use the values of the hydraulic radius $\overline{R_h}$ and correction factor ψ that have been determined to calculate the Manning coefficient n using the following relationship (Achour, 2014a):

$$n = \frac{\psi^{\frac{8}{3}} \overline{R_h}^{\frac{1}{6}}}{8\sqrt{2g}} \tag{35}$$

6. After calculating Manning's coefficient n, calculate the relative conductivity Q^* according to Eq. (6).
7. With the computed value of the relative conductivity Q^*, determine the aspect ratio η using the explicit Eq. (13).
8. Let us solve the problem by the rough model method. The new value of the relative conductivity $\overline{Q}^*$ is:

$$\overline{Q}^* = \frac{Q}{8\sqrt{2gi(D/2\psi)^5}}$$

9. Based on the sign of the discriminant Δ, we determine the aspect ratio $\overline{\eta} = \eta$ by applying Eq. (28) or Eq. (30), then the normal depth sought y_n is $y_n = \eta D$.

Determination of the normal depth for $0.5 \leq \overline{\eta} \leq 1$

The hydraulic parameters of this zone are as follows:

The wetted area $\overline{A}$ is governed by the following relation:

$$\overline{A} = \frac{\overline{D}^2}{4}\sigma(\overline{\eta})\varphi(\overline{\eta}) \tag{36}$$

The wetted perimeter $\overline{P}$ is given by:

$$\overline{P} = \overline{D}\varphi(\overline{\eta}) \tag{37}$$

where:

$$\sigma(\overline{\eta}) = 1 + \frac{2(2\overline{\eta}-1)\sqrt{\overline{\eta}(1-\overline{\eta})}}{(2+\frac{\pi}{2}-\cos^{-1}(2\overline{\eta}-1))} \tag{38}$$

$$\varphi(\overline{\eta}) = 2 + \frac{\pi}{2} - \cos^{-1}(2\overline{\eta} - 1) \tag{39}$$

The hydraulic radius $\overline{R_h} = \frac{\overline{A}}{\overline{P}}$ is then:

$$\overline{R_h} = \frac{\overline{D}}{4}\sigma(\overline{\eta}) \tag{40}$$

Inserting Eq. (40) into Eq. (18) leads to:

$$\overline{n} = \frac{1}{8\sqrt{2g}}\frac{\overline{D}^{1/6}}{4^{1/6}}\left[\sigma(\overline{\eta})\right]^{1/6} \tag{41}$$

By inserting Eqs. (36), (40) and (41) into Eq. (15) and rearranging, one may write:

$$\overline{Q}^{*} = \frac{1}{\sqrt{2}}\varphi(\overline{\eta})\left[\sigma(\overline{\eta})\right]^{3/2} \tag{42}$$

The relative conductivity $\overline{Q}^{*}$ is governed by Eq. (24).

Eq. (42) is implicit toward the aspect ratio $\overline{\eta}$. The known parameter is the relative conductivity $\overline{Q}^{*}$, and the determination of $\overline{\eta}$ is needed. The calculation involves a graphical procedure or an iterative method. One way to avoid this is to use the following derived explicit relationship:

$$\overline{\eta} = 0.0000045\overline{Q}^{*9.94} + 0.27847\overline{Q}^{*} + 0.1058$$
(43)

Eq. 43 was established in the wide range $0.5 \leq \overline{\eta} \leq 0.86$ corresponding to the relative conductivity $\overline{Q}^{*}$ such that $1.414 \leq \overline{Q}^{*} \leq 2.536$. The maximum relative deviation caused by Eq. (23) is less than 0.19%.

The calculation demonstrates that, in accordance with Eq. (42), the relative conductivity for the entire state $\overline{\eta} = 1$ corresponds to $\overline{Q}^{*} = 2.52493430$. As shown in figure 3, for this value of the relative conductivity $\overline{Q}^{*}$, Eq. (42) gives another value of the aspect ratio $\overline{\eta} = 0.854516733$.

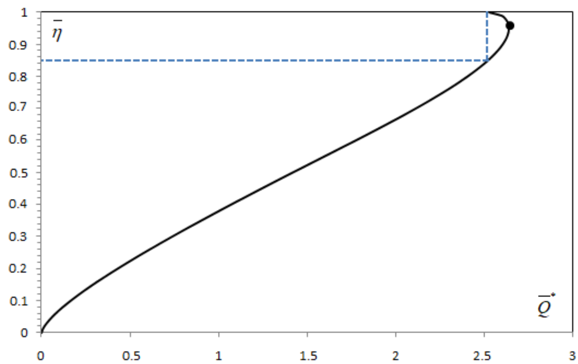

Figure 3: Plot of Eqs. (25) and (42). (•) maximum relative conductivity corresponding to $\overline{\eta} = 0.95$

Eqs. (37) and (40) permit the following writing for the aspect ratio $\overline{\eta} = 0.854516733$, respectively:

$$\overline{P} = 2.78813\overline{D} \tag{44}$$

$$\overline{R_h} = 0.29483\overline{D} \tag{45}$$

The hydraulic diameter $\overline{D_h} = 4\overline{R_h}$ is then:

$$\overline{D_h} = 1.17932\overline{D} \tag{46}$$

The diameter $\overline{\boldsymbol{D}}$ is given by Eq. (24) for $\overline{Q}^* = 2.52493430$. Hence:

$$\overline{D} = 0.523\left(\frac{Q}{\sqrt{gi}}\right)^{0.4} \tag{47}$$

The reference rough model's hydraulic and geometric properties are expressed by Equations (44) to (47). These equations will be applied to calculate the required value for the normal depth.

Steps for calculating the normal depth

To determine the normal depth y_n sought, when $\overline{Q}^* \geq 1.41421356$, the following data must be given: Q, D, i,ε and ν. It is advised to perform the following steps:

1. Determine the diameter $\overline{D}$ of the referential rough model using Eq. (47).
2. With the determined value of $\overline{D}$, the wetted perimeter $\overline{P}$ and the hydraulic diameter $\overline{D_h}$ are provided by equations (44) and (46), respectively.

3. In accordance with Eq. (34), compute the Reynolds number $\overline{R}$.

4. By inserting the values of $\overline{D_h}$ and $\overline{R}$ in Eq. (33), we obtain the nondimensional correction factor ψ.

5. Let us use the values of the hydraulic radius $\overline{\boldsymbol{R_h}}$ and correction factor ψ that have been determined to calculate the Manning coefficient n by the use of Eq. (35).

6. After calculating Manning's coefficient n, calculate the relative conductivity Q^* according to Eq (6).

7. With the computed value of the relative conductivity Q^*, determine the aspect ratio $\boldsymbol{\eta}$ using the explicit Eq. (13).

8. Let us solve the problem by the rough model method. According to the fundamental Eq. (32), apply the new linear dimension $\overline{D} = \frac{D}{\psi}$ to the referential rough model. Then, employ Eq. (24) to calculate the equivalent relative conductivity $\overline{Q}^*$.

9. Once the value of the relative conductivity $\overline{Q}^*$ has been calculated, let us use Eq. (43), to find the aspect ratio $\overline{\eta} = \eta$, the required normal depth sought y_n is $y_n = \eta D$

Proposed general formula

The equation below was obtained for direct computation of the normal depth for the vaulted rectangular cross-section (for two zones) by the RMM:

$$\overline{\eta} = \frac{0.3411\left(\frac{\overline{Q}^*}{2}\right)^{\frac{3}{5}} + 0.321\left(\frac{\overline{Q}^*}{2}\right)^{1.0385}}{1 - 0.009\left(\frac{\overline{Q}^*}{2}\right)^{8.235}} \tag{48}$$

Eq. (48) is valid over the entire practical range of depth $0.05 \le \eta \le 0.87$ corresponding to the relative conductivity Q^* such that $0.0603 \le Q^* \le 2.554$. The maximum relative error caused by Eq. (48) is less than 0.36% only, which is more than enough for practical applications.

Steps for calculating the normal depth by the general formula

When $0 \le \overline{Q}^* \le 1.41421356$

To compute the normal depth y_n, the following steps are recommended, provided that the parameters Q, i, $\boldsymbol{D}$, ε and ν are given.

1. If we assume $\overline{\mathbf{D}} = \mathbf{D}$, the relative conductivity $\overline{\mathbf{Q}}^*$ is given by:

$$\overline{Q}^* = \frac{Q}{8\sqrt{2gi(\overline{D}/2)^5}} = \frac{Q}{8\sqrt{2gi(D/2)^5}}$$

2. Depending on the sign of the discriminant Δ, use Eq. (28) or Eq. (30) to calculate the aspect ratio $\overline{\eta}$.
3. Eqs. (20) and (21) give, respectively: $\overline{P}$, $\overline{\boldsymbol{R_h}}$. This allows for the calculation of the hydraulic diameter by $\overline{D}_h = 4\overline{R}_h$ and Reynolds number *R* by the use of Eq. (34).
4. Apply Eq. (33) to determine the dimensionless correction factor ψ.
5. Assign to the rough model the following new linear dimension $\overline{D} = \frac{D}{\psi}$ according to the fundamental Eq. (32). Then, compute the new value of the relative conductivity $\overline{Q}^*$ using Eq. (24).
6. Applying then Eq. (48), results in $\overline{\eta} = \eta$.
7. Finally, the required normal depth y_n is then: $y_n = \eta D$.

When $\overline{Q}^* \ge 1.41421356$

The following data must be given: Q, D, i,ε and ν. To compute the required normal depth y_n, the following steps are recommended:

1. Determine the diameter $\overline{D}$ of the referential rough model using Eq. (47).

2. With the determined value of $\overline{D}$, the wetted perimeter $\overline{P}$ and the hydraulic diameter $\overline{D_h}$ are provided by equations (44) and (46), respectively.

3. In accordance with Eq. (34), compute the Reynolds number $\overline{R}$.

4. By inserting these values of $\overline{D_h}$ and $\overline{R}$ in Eq. (33), we obtain the nondimensional correction factor ψ.

5. According to the fundamental Eq. (32), apply the new linear dimension $\overline{D} = \frac{D}{\psi}$ to the referential rough model. Then, employ Eq. (24) to calculate the equivalent relative conductivity $\overline{Q}^*$

6. By inserting this value of $\overline{Q}^*$ into Eq. (48), we obtain $\overline{\eta} = \eta$. The required normal depth is then $y_n = D\eta$.

APPLICATION

The application of the RMM to compute normal depth in a vault-shaped rectangular cross-section is shown in the following examples.

Example 1

Compute the normal depth in a vaulted rectangular cross-section for the following data using the RMM:

$$Q = \frac{3m^3}{s}, D = 2m, i = 4.10^{-3}, \varepsilon = 10^{-3}, \nu = \frac{10^{-6}m^2}{s}.$$

1. If we assume $\boldsymbol{\overline{D} = D}$, according to Eq. (24), the relative conductivity $\overline{Q}^*$ is:

$$\overline{Q}^* = \frac{Q}{8\sqrt{2gi(D/2)^5}} = \frac{3}{8\sqrt{2\times 9.81\times 4.10^{-3}\times (2/2)^5}} = 1.33860293 < \sqrt{27}$$

2. According to the calculated value of $\overline{Q}^*$, the aspect ratio $\overline{\eta}$ in the rough model is governed by Eq. (30), along with Eq. (31). The angle β is as follows:

$$ch(\beta) = \frac{3\sqrt{3}}{\overline{Q}^*} = \frac{3\sqrt{3}}{1.33860293} = 3.88177279$$

Leading to $\beta = 2.03241903$radian

According to Eq. (30), the aspect ratio $\overline{\eta}$ in the rough model is then:

$$\overline{\eta} = \frac{\overline{Q}^*}{2\sqrt{3}} ch\left(\frac{\beta}{3}\right) = \frac{1.33860293}{2\sqrt{3}} \times ch\left(\frac{2.03241903}{3}\right) = 0.47854325$$

3. Using Eq. (20) and Eq. (21), the wetted perimeter $\overline{P}$ and the hydraulic radius $\overline{R_h}$ are:

$$\overline{P} = \overline{D}\left(1+2\overline{\eta}\right) = D\left(1+2\overline{\eta}\right) = 2\times\left(1+2\times 0.47854325\right) = 3.914173\text{m}$$

$$\overline{R_h} = \overline{D}\frac{\overline{\eta}}{(1+2\overline{\eta})} = 2\times\frac{0.47854325}{(1+2\times 0.47854325)} = 0.48903638\text{m}$$

The hydraulic diameter $\overline{D_h} = 4\overline{R_h}$ is then:

$$\overline{D_h} = 4\overline{R_h} = 4 \times 0.48903638 = 1.95614553\text{m}$$

4. Using Eq. (34), the Reynolds number $\overline{R}$ is:

$$\overline{R} = \frac{4Q}{\overline{P}\nu} = \frac{4\times 3}{3.914173\times 10^{-6}} = 3065781.71$$

5. According to Eq. (33), the nondimensional correction factor ψ was easily calculated as:

$$\psi \cong 1.35\left[-\log\left(\frac{\frac{\varepsilon}{\overline{D_h}}}{4.75}+\frac{8.5}{\overline{R}}\right)\right]^{-\frac{2}{5}} =$$

$$1.35\left[-\log\left(\frac{\frac{0.001}{1.95614553}}{475}+\frac{8.5}{3065781.71}\right)\right]^{-\frac{2}{5}} = 0.77872699$$

6. According to Eq. (35), the coefficient n is:

$$n = \frac{\psi^{\frac{8}{3}}\overline{R_h}^{\frac{1}{6}}}{8\sqrt{2g}} = \frac{0.77872699^{8/3}\times 0.48903638^{1/6}}{8\sqrt{2\times 9.81}} = 0.01285715\text{m}^{-1/3}s$$

7. Considering the determined value of n, the relative conductivity Q^* is governed by Eq.(6):

$$Q^* = \frac{nQ}{\sqrt{i}(D/2)^{\frac{8}{3}}} = \frac{0.01285715\times 3}{\sqrt{4.10^{-3}}(2/2)^{\frac{8}{3}}} = 0.60986808$$

8. According to Eq. (13), the aspect ratio η is:

$$\left(\frac{0.60986808}{2^{\frac{8}{3}}}\right)^{\frac{3}{5}}\left[1+1.08\left(\frac{0.60986808}{2}\right)^{0.615}\left(1+2.05\times 0.60986808^{0.941}\right)^{0.16}\right]^{\frac{2}{5}}$$

$$= 0.29544459 \approx 0.295$$

9. The required value of normal depth y_n is thus:

$$y_n = \eta D = 0.295 \times 2 = 0.59 \text{ m}$$

10. This step aims to verify the validity of the calculations by determining the discharge Q using Eq. (1). The calculated discharge should be equal to the discharge given in the problem statement.

The water area A was easily calculated using Eq. (2) such that:

$$A = D^2\eta = 2^2 \times 0.295 = 1.18m^2$$

According to Eq. (4), the hydraulic radius R_h is:

$$R_h = D\frac{\eta}{(1+2\eta)} = 2 \times \frac{0.295}{(1+2\times 0.295)} = 0.37106918\text{m}$$

Finally, according to Eq. (1), the discharge Q is:

$$Q = \frac{1}{n}AR_h^{\frac{2}{3}}\sqrt{i} = \frac{1}{0.01285715} \times 1.18 \times (0.37106918)^{\frac{2}{3}}\sqrt{4.10^{-3}} = 2.997 \approx \frac{3m^3}{s}$$

The estimated discharge, as is apparent, equals the discharge provided in the problem statement, demonstrating the accuracy of the computations.

Let us solve the problem by the rough model method. Let us calculate the new value of the relative conductivity:

$$\overline{Q}^* = \frac{Q}{8\sqrt{2gi(D/2\psi)^5}} = \frac{3}{8\sqrt{2\times 9.81 \times 4.10^{-3} \times (2/(2\times 0.77872699))^5}} = 0.7163328 < \sqrt{27}$$

According to the calculated value of $\overline{Q}^*$, the required value of the aspect ratio η is governed by Eq. (30), along with Eq. (31). The angle β is as follows:

$$ch(\beta) = \frac{3\sqrt{3}}{\overline{Q}^*} = \frac{3\sqrt{3}}{0.7163328} = 7.25382454$$

leading to $\beta = 2.66989058\text{radian}$

According to Eq. (30), the aspect ratio η is then:

$$\overline{\eta} = \eta = \frac{\overline{Q}^*}{2\sqrt{3}}ch\left(\frac{\beta}{3}\right) = \frac{0.7163328}{2\sqrt{3}} \times ch\left(\frac{2.66989058}{3}\right) = 0{,}29422874 \approx 0.294$$

The required value of normal depth y_n is thus:

$$y_n = \eta D = 0.294 \times 2 = 0.588 \approx 0.59 \text{ m}$$

This is indeed the value of y_n calculated in step 9

Let us solve the problem by Eq. (48), by inserting the calculated value $\overline{Q}^*$ into Eq. (48). This results in the equality of the relative normal depths in the rough model and in the current channel, i.e. $\overline{\eta} = \eta$. For the relative conductivity $\overline{Q}^* = 0.7163328$, the aspect ratio is:

$$\overline{\eta} = \eta = \frac{0.3411\left(\frac{\overline{Q}^*}{2}\right)^{\frac{3}{5}} + 0.321\left(\frac{\overline{Q}^*}{2}\right)^{1.0385}}{1 - 0.009\left(\frac{\overline{Q}^*}{2}\right)^{8.235}}$$

$$= \frac{0.3411 \times \left(\frac{0.7163328}{2}\right)^{\frac{3}{5}} + 0.321 \times \left(\frac{0.7163328}{2}\right)^{1.0385}}{1 - 0.009 \times \left(\frac{0.7163328}{2}\right)^{8.235}} = 0.29473403 \approx 0.295$$

This is indeed the value of the aspect ratio η calculated in step 8.

Example 2

Compute the normal depth in the vault-shaped rectangular cross-section for the following data using the RMM:

$$Q = \frac{4m^3}{s}, D = 3m, i = 10^{-4}, \varepsilon = 10^{-4}, \nu = \frac{10^{-6}m^2}{s}.$$

1. If we *assume* $\boldsymbol{\overline{D} = D}$, according to Eq. (24), the relative conductivity $\overline{Q}^*$ is:

$$\overline{Q}^* = \frac{Q}{8\sqrt{2gi(D/2)^5}} = \frac{4}{8\sqrt{2 \times 9.81 \times 10^{-4} \times (3/2)^5}} = 4.09630566 > 1.41421356$$

Thus, we apply the calculation steps in the following case: $0.5 \leq \overline{\eta} \leq 1$.

2. According to Eq. (47), the diameter $\overline{D}$ of the referential rough model is then:

$$\overline{D} = 0.523\left(\frac{Q}{\sqrt{gi}}\right)^{0.4} = 0.523 \times \left(\frac{4}{\sqrt{9.81 \times 10^{-4}}}\right)^{0.4} = 3.63908235\text{m}$$

3. Using Eq. (44) and Eq. (46), the wetted perimeter $\overline{P}$ and the hydraulic diameter $\overline{D_h}$ are:

$$\overline{P} = 2.78813\overline{D} = 2.78813 \times 3.63908235 = 10.1462347\text{m}$$

$$\overline{D_h} = 1.179332\overline{D} = 1.17932 \times 3.63908235 = 4.2916426\text{m}$$

4. According to Eq. (34), the Reynolds number $\overline{R}$ is:

$$\overline{R} = \frac{4Q}{\overline{P}\nu} = \frac{4 \times 4}{10.1462347 \times 10^{-6}} = 1576939.67$$

5. Using Eq. (33), the nondimensional correction factor ψ is as follows:

$$\psi \cong 1.35\left[-\log\left(\frac{\frac{\varepsilon}{\overline{D_h}}}{4.75} + \frac{8.5}{\overline{R}}\right)\right]^{-\frac{2}{5}} =$$

$$1.35\left[-\log\left(\frac{\frac{0.0001}{4.2916426}}{4.75} + \frac{8.5}{1576939.67}\right)\right]^{-\frac{2}{5}} = 0.70988175$$

6. With the previously calculated value of the correction factor ψ and given the hydraulic radius $\overline{R_h} = \frac{\overline{D_h}}{4} = 1.07291065\text{m}$, let us apply Eq. (35) to evaluate Manning's coefficient n, that is:

$$n = \frac{\psi^{\frac{8}{3}}\overline{R_h}^{\frac{1}{6}}}{8\sqrt{2g}} = \frac{0.70988175^{8/3} \times 1.07291065^{1/6}}{8\sqrt{2 \times 9.81}} = 0.01145539\text{m}^{-1/3}s$$

7. Considering the determined value of n, the relative conductivity Q^* is governed by Eq. (6):

$$Q^* = \frac{nQ}{\sqrt{i}(D/2)^{\frac{8}{3}}} = \frac{0.01145539 \times 3}{\sqrt{10^{-4}}(3/2)^{\frac{8}{3}}} = 1.55415144$$

8. According to Eq. (14), the aspect ratio η is:

$$\eta = 0.0000125 Q^{*9.939} + 0.2868 Q^{*} + 0.1392$$
$$= 0.0000125 \times 1.55415144^{9.939} + 0.2868 \times 1.55415144 + 0.1392 \approx 0.586$$

9. The required value of normal depth y_n is thus:

$$y_n = \eta D = 0.586 \times 3 = 0.758 \approx \ 0.76\text{m}$$

10. This step aims to verify the validity of the calculations by determining the discharge Q using Eq. (1). The calculated discharge should be equal to the discharge given in the problem statement.

Eqs. (9) and (10) give, respectively:

$$\sigma(\eta) = 1 + \frac{2(2\eta - 1)\sqrt{\eta(1-\eta)}}{\left(2 + \frac{\pi}{2} - \cos^{-1}(2\eta - 1)\right)} = 1 + \frac{2(2 \times 0.586 - 1) \times \sqrt{0.586 \times (1 - 0.586)}}{\left(2 + \frac{\pi}{2} - \cos^{-1}(2 \times 0.586 - 1)\right)}$$
$$= 1.07797866$$

$$\varphi(\eta) = \left(2 + \frac{\pi}{2} - \cos^{-1}(2\eta - 1)\right) = \left(2 + \frac{\pi}{2} - \cos^{-1}(2 \times 0.586 - 1)\right) = 2.17285957$$

The water area A was easily calculated using Eq. (7) such that:

$$A = \frac{D^2}{4}\sigma(\eta)\varphi(\eta) = \frac{3^2}{4} \times 1.07797866 \times 2.17285957 = 5.27016655\text{m}^2$$

According to Eq. (11), the hydraulic radius R_h is:

$$R_h = \frac{D}{4}\sigma(\eta) = \frac{3}{4} \times 1.07797866 = 0.80848399\text{m}$$

Finally, according to Eq. (1), the discharge Q is:

$$Q = \frac{1}{n} A R_h^{\frac{2}{3}} \sqrt{i} = \frac{1}{0.01145539} \times 5.27016655 \times (0.80848399)^{\frac{2}{3}} \sqrt{10^{-4}} = 3.994 \approx 4\text{m}^3/\text{s}$$

The estimated discharge, as is apparent, equals the discharge provided in the problem statement, demonstrating the accuracy of the computations.

Let us solve the problem by the rough model method. Assign to the rough model the following new linear dimension, according to Eq. (32):

$$\overline{D} = \frac{D}{\psi} = \frac{3}{0.70988175} = 4.22605599\text{m}$$

According to Eq. (24), the new value of the relative conductivity $\overline{Q}^*$ is then:

$$\overline{Q}^* = \frac{Q}{8\sqrt{2gi\left(\frac{D}{2\psi}\right)^5}} = \frac{4}{8\sqrt{2\times 9.81\times 4.10^{-3}\times\left(\frac{4.22605599}{2}\right)^5}} = 1.7392314$$

According to Eq. (43), the aspect ratio is thus:

$$\overline{\eta} = \eta = 0.0000045\overline{Q}^{*9.94} + 0.27847\overline{Q}^* + 0.1058$$

$$= 0.0000045\times 1.73923143^{9.94} + 0.27847\times 1.73923143 + 0.1058 \approx 0.591$$

Therefore, comparing the approximate value of the aspect ratio η from step 8 and the value we have just calculated, the relative deviation is as follows:

$$\frac{\Delta\eta}{\eta} = 100\times\frac{0.591-0.586}{0.591} = 0.85\%$$

The required value of normal depth y_n is thus:

$$y_n = \eta D = 0.591\times 3 \approx 1.77 \text{ m}$$

Let us solve the problem using Eq.48). By inserting the calculated value of $\overline{Q}^*$ into Eq. (48), this results in the equality of the relative normal depths in the rough model and in the current channel, i.e., $\overline{\eta} = \eta$. For the relative conductivity $\overline{Q}^* = 1.73923082$, the aspect ratio is:

$$\overline{\eta} = \eta = \frac{0.3411\left(\frac{\overline{Q}^*}{2}\right)^{\frac{3}{5}} + 0.321\left(\frac{\overline{Q}^*}{2}\right)^{1.0385}}{1-0.009\left(\frac{\overline{Q}^*}{2}\right)^{8.235}}$$

$$= \frac{0.3411\times\left(\frac{1.73923143}{2}\right)^{\frac{3}{5}} + 0.321\times\left(\frac{1.73923143}{2}\right)^{1.0385}}{1-0.009\times\left(\frac{1.73923143}{2}\right)^{8.235}} \approx 0.593$$

The required value of normal depth y_n is thus:

$$y_n = \eta D = 0.593\times 3 \approx 1.78 \text{ m}$$

The deviation between the value of normal depth y_n computed in this way and the value estimated at step 9 is less than 1.13%.

CONCLUSION

The computation of the normal depth in a vault-shaped rectangular cross-section by RMM showed the extent of the calculation's effectiveness and the possibility of doing so without the Manning coefficient value in the problem data.

In this study, the Manning equation is applied to a referential rough model whose characteristics are surmounted by the symbol "¯". When $0 \leq \overline{\eta} \leq 0.5$, this resulted in the construction of a third-degree explicit relationship between the aspect ratio and the relative conductivity, which was analytically solved using hyperbolic and trigonometric functions as well as approximate equations (13) and (48). When $0.5 \leq \overline{\eta} \leq 1$, it led to the creation of an approximate equation representing the aspect ratio, dependent upon the relative conductivity in the referential rough model. The diameter of this is the same as that of the full-model state, and the filling rate is $\overline{\eta} = 0.854516733$. The aspect ratio in the studied conduit and, accordingly, the normal depth were calculated from the known value of the aspect ratio in the rough model. This was made possible due to the nondimensional correction factor. The application of the rough model method was demonstrated through practical examples, which also demonstrated how simple it was to use.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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